

2.  $R$  is the radius of the semicircle,  $CX = r$ , and  $OC = a$ .

Since  $r$  is the radius of the little circle, we can construct a right triangle with sides  $r$ ,  $(a+r)$ , and  $R - r$  (from the center of the circle to the semicircle is  $r$  and the dotted line from  $O$  to the outside of the semicircle is  $R$ ). We can then use the Pythagorean Theorem and we would have

$$r^2 + (a + r)^2 = (R - r)^2$$

After foiling:  $r^2 + a^2 + 2ar + r^2 = R^2 - 2Rr + r^2$

After subtracting  $a^2$  from both sides, and adding  $2Rr$  and  $r^2$  to both sides:

$$r^2 + 2ar + r^2 + 2Rr - r^2 = R^2 - a^2$$

After combining like terms and factoring:  $r^2 + 2r(R + a) = R^2 - a^2$

Looking at the right side of our equation ( $R^2 - a^2$ ), and knowing  $POC$  is a right triangle from the construction, we know that  $R^2 - a^2 = CP^2$ . Side  $OP$  is equal to  $R$  because it is the radius of the semicircle.

Let's start with  $(R + a + r)^2$  which is equal to  $(R + a)^2 + CP^2$  and based on the triangle  $ACP$

We get that by the Pythagorean theorem  $(R + a)^2 + CP^2 = AP^2$ .

By taking the square root of both sides,  $AP = R + a + r$  which is also equal to  $AX$ . Thus,  $AP = AX$ .