Grades 5-8 (S), 9-12 (S)
Duration: 10-20 minutes
Tools: one Logifaces Set / class

Individual work

Keywords: Edges

604 - Variety of Blocks


MATHS / COMBINATORICS


LOGIFACES Erasmus+

DESCRIPTION

The vertical edges of the Logifaces blocks are 1, 2 or 3 units high. Students consider the number of the different possible Logifaces blocks.

LEVEL 1 Given the table of the 27 possible variations of the numbers 1,2 and 3 , find the variations in the table that represent the same blocks.

LEVEL 2 Students examine the blocks 222, 133, 123, then consider the number of the different possible Logifaces blocks, given that each vertical edge has length 1, 2 or 3, but without any further instructions.

## SOLUTIONS / EXAMPLES

Notes on formulating what we mean by different blocks:

- We recommend that the teacher does not specify this when raising the question, as students may ask the question themselves.
- If students are confused whether 123 and 132 are different the teacher can play the following game: the teacher presents the two blocks and students memorise them. Then the teacher hides both, puts one forward and asks: can you tell which one is this? The idea of orientation can also be discussed, see Part b) of exercise 601 - Matchmaking for more details.

LEVEL 1 The overall number of possibilities of three digit numbers with digits 1,2 and 3 can be calculated with the formula of the variation with repetition and it is equal to $3^{3}=27$. These cases are shown in the table below:

| 111 | 121 | 131 | 211 | 221 | 231 | 311 | 321 | 331 |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| 112 | 122 | 132 | 212 | 222 | 232 | 312 | 322 | 332 |
| 113 | 123 | 133 | 213 | 223 | 233 | 313 | 323 | 333 |

Some of these three digit numbers represent the same blocks.
Three digit numbers with three identical digits are listed only once in the table:

| 111 | 121 | 131 | 211 | 221 | 231 | 311 | 321 | 331 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 112 | 122 | 132 | 212 | 222 | 232 | 312 | 322 | 332 |
| 113 | 123 | 133 | 213 | 223 | 233 | 313 | 323 | 333 |

From this type there are 3 different blocks.
Three digit numbers with two identical and one different digit that represent the same block are listed three times in the table. For example 112, 121 and 211 represent the same block. We mark the three digit numbers that represent the same block with identical colours.

| 111 | 121 | 131 | 211 | 221 | 231 | 311 | 321 | 331 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 112 | 122 | 132 | 212 | 222 | 232 | 312 | 322 | 332 |
| 113 | 123 | 133 | 213 | 223 | 233 | 313 | 323 | 333 |

From this type there are 6 different blocks.
Three digit numbers with three different digits that represent the same block are listed three times in the table. For example 123, 231 and 312 represent the same block.

| 111 | 121 | 131 | 211 | 221 | 231 | 311 | 321 | 331 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| 112 | 122 | 132 | 212 | 222 | 232 | 312 | 322 | 332 |
| 113 | 123 | 133 | 213 | 223 | 233 | 313 | 323 | 333 |

From this type there are 2 different blocks.
Hence altogether there are $3+6+2=11$ different blocks.
LEVEL $2 \quad$ There are 11 different blocks: 111, 222, 333, 122, 112, 133, 113, 223, 233, 123, 132.
PRIOR KNOWLEDGE
None
RECOMMENDATIONS / COMMENTS
The goal of the exercise is to develop systematic counting and thinking, spatial vision and the notion of equivalence of polyhedra by rotations. Before the counting, a short and practical notation should be introduced for the different pieces. It can be proposed either by the students or by the teacher. The Level 1 exercise helps understand that one block can have multiple notations. The Level 2 exercise can be viewed as a more difficult exercise without the Level 1 part.

It is not necessary to know the complete Logifaces set for this task. Let's show a few pieces first (for example $222,133,123$ ), then ask the question of this exercise!

During the discussion of the difference between the blocks 123 and 132 the idea of orientation can be discussed, see Part b) of exercise 601 - Matchmaking for more details.

We recommend exercise 618 - Variety with Formulas as an additional question, where the task is to find a formula for the number possibilities.

