

TRIGONOMETRIJA TROKUTA

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$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$$

POUČEK O SINUSU

* DODATNO

$$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$$

$$a^2 = b^2 + c^2 - 2bc \sin \alpha$$

$$b^2 = a^2 + c^2 - 2ac \sin \beta$$

$$c^2 = a^2 + b^2 - 2ab \sin \gamma$$

POUČEK O KOSINUSU

$$P_{\Delta} = \frac{a \cdot v_a}{2} \quad P_{\Delta} = \frac{b \cdot v_b}{2} \quad P_{\Delta} = \frac{c \cdot v_c}{2}$$

$$P_{\Delta} = \frac{1}{2} ab \sin \gamma \quad P_{\Delta} = \frac{1}{2} ac \sin \beta \quad P_{\Delta} = \frac{1}{2} bc \sin \alpha$$

$$P_{\Delta} = \frac{a^2 \sin \beta \sin \gamma}{2 \sin \alpha} \quad P_{\Delta} = \frac{b^2 \sin \alpha \sin \gamma}{2 \sin \beta} \quad P_{\Delta} = \frac{c^2 \sin \alpha \sin \beta}{2 \sin \gamma}$$

$$P_{\Delta} = r \cdot s \quad P_{\Delta} = \frac{abc}{4R}$$

$$P_{\Delta} = \sqrt{s(s-a)(s-b)(s-c)} \quad s = \frac{a+b+c}{2}$$

ROMB

$$p = \frac{e \cdot f}{2}$$

$$o = 4a$$

$$\alpha + \beta = 180^\circ$$

TRAPEZ

$$e^2 = a^2 + b^2 - 2ab \cos \beta$$

$$f^2 = a^2 + d^2 - 2ad \cos \alpha$$

$$b^2 = (a-c)^2 + d^2 - 2(a-c)d \cos \alpha$$

$$p = \frac{(a+c) \cdot v}{2}$$

PRAVOKUTNIK

$$o = 2a + 2b$$

$$p = a \cdot b$$

KVADRAT

$$o = 4a$$

$$p = a^2$$

PARALELOGRAM

$$p = a \cdot v$$

$$o = 2a + 2b$$

$$p = a \cdot b \cdot \sin \alpha$$

$$p = \frac{1}{2} e \cdot f \cdot \sin \beta \quad \beta = 180^\circ - \alpha$$

$$e^2 = a^2 + b^2 - 2ab \cos \beta$$

$$c^2 = a^2 + b^2 + 2ab \cos \alpha$$

$$f^2 = a^2 + b^2 - 2ab \cos \alpha$$

$$e^2 + f^2 = 2(a^2 + b^2)$$

$$a^2 = \left(\frac{e}{2}\right)^2 + \left(\frac{f}{2}\right)^2 - 2 \cdot \frac{e}{2} \cdot \frac{f}{2} \cdot \cos \beta$$

POUČAK O SINUSIMA

* DODATNO

* Odredi duljinu stranice a i kutove trokuta ABC ako je $b = 7,5 \text{ cm}$, $c = 6,2 \text{ cm}$ te $\beta - \gamma = 17^\circ$

$$b = 7,5 \text{ cm}$$

$$c = 6,2 \text{ cm}$$

$$\beta - \gamma = 17^\circ$$

$$\alpha = 64^\circ 32'$$

$$\alpha = 180^\circ - \beta - \gamma$$

$$\alpha = 180^\circ - 66^\circ 14' - 49^\circ 14'$$

$$\alpha = 64^\circ 32'$$

$$\beta = 17^\circ + \gamma$$

$$\beta = 17^\circ + 49^\circ 14'$$

$$\beta = 66^\circ 14'$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$

$$a = \frac{b \cdot \sin \alpha}{\sin \beta}$$

$$a = \frac{7,5 \cdot \sin(64^\circ 32')}{\sin(66^\circ 14')}$$

$$\underline{a = 7,4 \text{ cm}}$$

$$\frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$\frac{7,5}{\sin(17^\circ + \gamma)} = \frac{6,2}{\sin \gamma}$$

$$7,5 \sin \gamma = 6,2 \sin(17^\circ + \gamma)$$

$$7,5 \sin \gamma = 6,2 (\sin 17^\circ \cos \gamma + \cos 17^\circ \sin \gamma)$$

$$7,5 \sin \gamma = 6,2 (0,29 \cdot \cos \gamma + 0,96 \cdot \sin \gamma)$$

$$7,5 \sin \gamma = 1,798 \cos \gamma + 5,952 \sin \gamma$$

$$7,5 \sin \gamma - 5,952 \sin \gamma = 1,798 \cos \gamma$$

$$1,548 \sin \gamma = 1,798 \cos \gamma \quad | : \cos \gamma$$

$$\frac{1,548 \sin \gamma}{\cos \gamma} = 1,798 \quad | : 1,548$$

$$\frac{\sin \gamma}{\cos \gamma} = 1,16$$

$$\tan \gamma = 1,16$$

$$\underline{\gamma = 49^\circ 14'}$$

POUČAK O KOSINUSU

* Dužine stranica trokuta u omjeru su 4:3:6. Koliki je najmanji kut ovog trokuta?

$$4:3:6 = a:b:c$$

$$a = 4$$

$$b = 3$$

$$c = 6$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$\cos \beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos \beta = \frac{4^2 + 6^2 - 3^2}{2 \cdot 4 \cdot 6}$$

$$\cos \beta = \frac{16 + 36 - 9}{2 \cdot 4 \cdot 6}$$

$$\cos \beta = \frac{43}{48}$$

$$\beta = 26^\circ 23'$$

$$\beta < \alpha < \gamma$$

$\beta \rightarrow$ najmanji kut (nasuprot najkraće stranice najmanji je kut)

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos \alpha = \frac{3^2 + 6^2 - 4^2}{2 \cdot 3 \cdot 6}$$

$$\cos \alpha = \frac{29}{36}$$

$$\alpha = 36^\circ 20'$$

$$\gamma = 180^\circ - 36^\circ 20' - 26^\circ 23'$$

$$\gamma = 117^\circ 17'$$

$$\gamma = 180^\circ - \alpha - \beta$$

* Za dužine stranica trokuta vrijedi $c-b=b-a=2$ cm, a jedan kut trokuta iznosi 120° . Koliki je opseg trokuta?

$$\begin{array}{l} c-b=b-a=2 \\ \gamma=120^\circ \end{array} \begin{array}{l} \leftarrow c-b=2 \rightarrow \\ \leftarrow b-a=2 \rightarrow \end{array} \begin{array}{l} c=b+2 \text{ NAJDUŽJA STR.} \\ a=b-2 \text{ NAJKRAĆA STR.} \end{array}$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$(b+2)^2 = (b-2)^2 + b^2 - 2(b-2) \cdot b \cos \gamma$$

$$b^2 + 4b + 4 = b^2 - 4b + 4 + b^2 + (-2b^2 + 4b) \cos \gamma$$

$$4b + 4b - b^2 = (-2b^2 + 4b) \cos \gamma$$

$$8b - b^2 = b^2 - 2b$$

$$8b - b^2 - b^2 + 2b = 0$$

$$-2b^2 + 10b = 0 \quad /: (-2) \quad \Delta \text{ — Kvadratna jednačina}$$

$$b^2 - 5b = 0$$

$$b(b-5) = 0$$

$$b_1 = 5 \quad \begin{array}{l} \swarrow \\ b_2 = 0 \end{array} \quad \Delta \text{ — stranica ne može biti } 0.$$

* U trokutu ABC je $c = 11\text{cm}$, $R = 12\text{cm}$, te $\beta = 50^\circ 33' 28''$.
 Kolika je površina trokuta?

$$C = 11\text{cm}$$

$$R = 12$$

$$\beta = 50^\circ 30' 28''$$

$$p = ?$$

$$\frac{c}{\sin \gamma} = 2R$$

$$\sin \gamma = \frac{c}{2R}$$

$$\sin \gamma = \frac{11}{2 \cdot 12}$$

$$\sin \gamma = \frac{11}{24}$$

$$\gamma = 24^\circ 16' 47''$$

$$\alpha = 180^\circ - \gamma - \beta$$

$$\alpha = 180^\circ - 50^\circ 30' 28'' - 24^\circ 16' 47''$$

$$\alpha = 102^\circ 13' 45''$$

$$a = 2R \cdot \sin \alpha$$

$$a = 2 \cdot 12 \cdot \sin(102^\circ 13' 45'')$$

$$a = 23.46$$

$$\frac{b}{\sin \beta} = 2R$$

$$b = 2R \cdot \sin \beta$$

$$b = 2 \cdot 12 \cdot \sin(50^\circ 30' 28'')$$

$$b = 18.52$$

$$P_D = \frac{a \cdot b \cdot c}{4R}$$

$$P_D = \frac{23.46 \cdot 18.52 \cdot 11}{4 \cdot 12}$$

$$P_D = 99.57 \text{ cm}^2$$

* Površina trokuta iznosi 20cm^2 . Dva su njegova kuta 30° i 45° . Kolike su dužine stranica ovog trokuta?

$$P_{\Delta} = 20\text{cm}^2$$

$$\alpha = 30^\circ$$

$$\beta = 45^\circ$$

$$a, b, c = ?$$

$$\gamma = 180^\circ - \alpha - \beta$$

$$\gamma = 180^\circ - 30^\circ - 45^\circ$$

$$\gamma = 105^\circ$$

$$P_{\Delta} = \frac{a^2 \sin \beta \sin \gamma}{2 \sin \alpha}$$

$$a^2 = \frac{2 P_{\Delta} \sin \alpha}{\sin \beta \sin \gamma}$$

$$a^2 = \frac{2 \cdot 20 \cdot \sin(30^\circ)}{\sin(45^\circ) \cdot \sin(105^\circ)}$$

$$a^2 = 29.28/\sqrt{}$$

$$a = 5.41$$

$$P_{\Delta} = \frac{b^2 \sin \alpha \sin \gamma}{2 \sin \beta}$$

$$b^2 = \frac{2 P_{\Delta} \sin \beta}{\sin \alpha \cdot \sin \gamma}$$

$$b^2 = \frac{2 \cdot 20 \cdot \sin(45^\circ)}{\sin(30^\circ) \cdot \sin(105^\circ)}$$

$$b^2 = 58.56/\sqrt{}$$

$$b = 7.65\text{cm}$$

$$P_{\Delta} = \frac{c^2 \sin \alpha \sin \beta}{2 \sin \gamma}$$

$$c^2 = \frac{2 P_{\Delta} \sin \gamma}{\sin \alpha \cdot \sin \beta}$$

$$c^2 = \frac{2 \cdot 20 \cdot \sin(105^\circ)}{\sin(30^\circ) \cdot \sin(45^\circ)}$$

$$c^2 = 109.28/\sqrt{}$$

$$c = 10.45\text{cm}$$

* Ako je trokutu ABC zadano $a = 32\text{cm}$, $R = 18\text{cm}$,
 $\gamma = 33^\circ$, odredi ostale stranice i kutove trokuta.

$$\begin{array}{l} a = 32\text{cm} \\ R = 18 \\ \gamma = 33^\circ \\ \hline b, c = ? \end{array}$$

$$\frac{c}{\sin \gamma} = 2R$$

$$c = 2R \cdot \sin \gamma$$

$$c = 2 \cdot 18 \cdot \sin(33)$$

$$\underline{c = 19,61\text{cm}}$$

$$\frac{a}{\sin \alpha} = 2R$$
$$\sin \alpha = \frac{a}{2R}$$
$$\sin \alpha = \frac{32}{2 \cdot 18}$$

$$\sin \alpha = 0,89$$

$$\alpha = 62^\circ 52'$$

$$\alpha_2 = 180 - \alpha_1$$

$$\alpha_2 = 117^\circ 8'$$

$$\beta = 180 - \alpha - \gamma$$
$$\beta = 180 - 62^\circ 52' - 33^\circ$$
$$\underline{\beta = 84^\circ 8'}$$

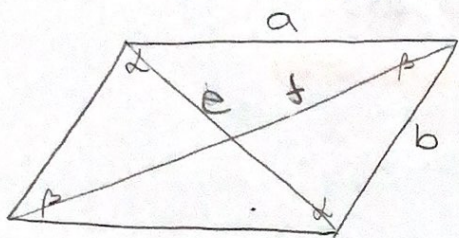
$$\frac{b}{\sin \beta} = 2R$$

$$b = 2R \cdot \sin \beta$$

$$b = 2 \cdot 18 \cdot \sin(84^\circ 8')$$

$$\underline{b = 35,81\text{cm}}$$

* Duzine stranica paralelograma su 11.5 i 16.8 cm, a jedan unutarnji kut paralelograma iznosi $135^{\circ} 16'$. Kolike su duzine dijagonala paralelograma?



$$\begin{aligned} a &= 16.8 \\ b &= 11.5 \\ \alpha &= 135^{\circ} 16' \end{aligned}$$

$$e^2 = a^2 + b^2 + 2ab \cos \alpha$$

$$e^2 = 16.8^2 + 11.5^2 + 2 \cdot 16.8 \cdot 11.5 \cdot \cos(135^{\circ} 16')$$

$$e^2 = 140.15$$

$$e = \underline{11.83 \text{ cm}}$$

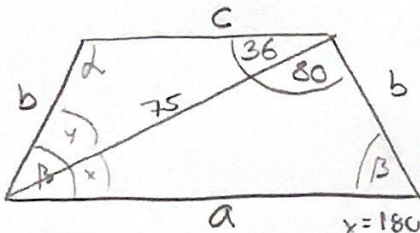
$$f^2 = a^2 + b^2 - 2ab \cos \alpha$$

$$f^2 = 16.8^2 + 11.5^2 - 2 \cdot 16.8 \cdot 11.5 \cdot \cos(135^{\circ} 16')$$

$$f^2 = 688.9815$$

$$f = \underline{26.25 \text{ cm}}$$

* dijagonala jednakokravnog trapeza dugačka je 75cm i dijeli unutarnji kut trapeza na dva dijela od 36° i 80° . Kolike su dužine stranica trapeza?



$$\alpha = 36^\circ + 80^\circ$$

$$\alpha = 116^\circ$$

$$\beta = 180^\circ - \alpha$$

$$\beta = 180^\circ - 116^\circ$$

$$\beta = 64^\circ$$

$$x = 180^\circ - 80^\circ - \beta$$

$$x = 180^\circ - 80^\circ - 64^\circ$$

$$x = 36^\circ$$

$$y = \beta - x$$

$$y = 64^\circ - 36^\circ$$

$$y = 28^\circ$$

$$\frac{a}{\sin(80^\circ)} = \frac{75}{\sin \beta}$$

$$a = \frac{75 \cdot \sin(80^\circ)}{\sin \beta}$$

$$a = \frac{75 \cdot \sin(80^\circ)}{\sin(64^\circ)}$$

$$a = 82,18$$

$$\frac{c}{\sin y} = \frac{75}{\sin \alpha}$$

$$c = \frac{75 \sin y}{\sin \alpha}$$

$$c = \frac{75 \cdot \sin(28^\circ)}{\sin(116^\circ)}$$

$$c = 39,16$$

$$\frac{b}{\sin(86^\circ)} = \frac{75}{\sin \alpha}$$

$$b = \frac{75 \cdot \sin(36^\circ)}{\sin \alpha}$$

$$b = \frac{75 \cdot \sin(36^\circ)}{\sin(116^\circ)}$$

$$b = 49,05$$