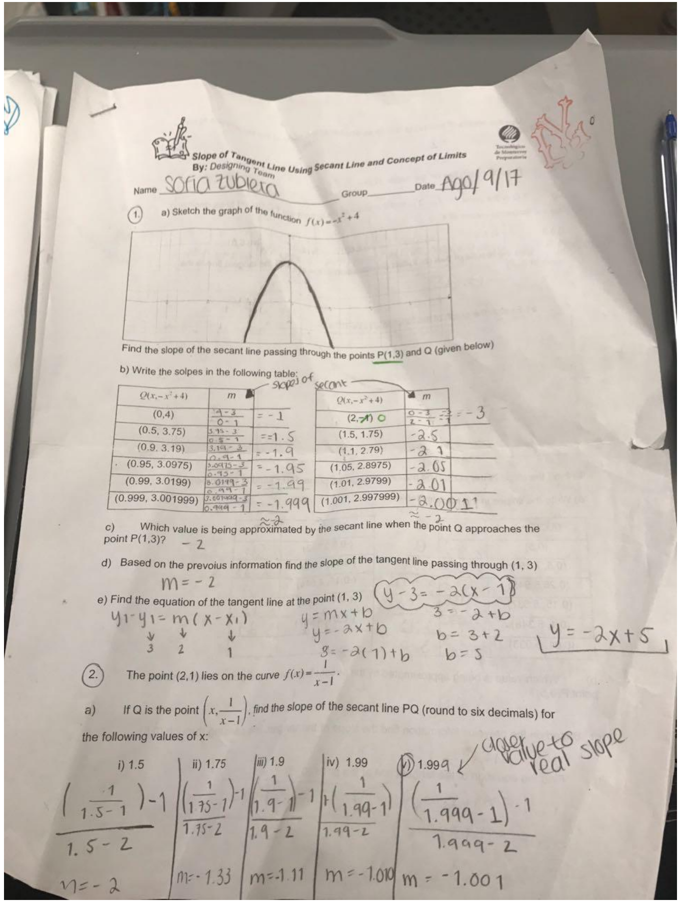
PORTAFOLIO

Slope of Tangent Line Using Secant Lines



Slope of Tangent Line Using Secant Line and Concept of Limits
By: Designing Team

Name Sofia Zubieta Group ____ Date Ago/9/17

1. a) Sketch the graph of the function $f(x)=-x^2+4$

Find the slope of the secant line passing through the points P(1,3) and Q (given below)

b) Write the solpes in the following table: slope of secant

$Q(x,-x^2+4)$	m
(0,4)	$\frac{4-3}{0-1} = -1$
(0.5, 3.75)	$\frac{3.75-3}{0.5-1} = -1.5$
(0.9, 3.19)	$\frac{3.19-3}{0.9-1} = -1.9$
(0.95, 3.0975)	$= -1.95$
(0.99, 3.0199)	$= -1.99$
(0.999, 3.001999)	$= -1.999$
	≈ -2

$Q(x,-x^2+4)$	m
(2, -1)	$\frac{0-3}{2-1} = -3$
(1.5, 1.75)	-2.5
(1.1, 2.79)	-2.1
(1.05, 2.8975)	-2.05
(1.01, 2.9799)	-2.01
(1.001, 2.997999)	-2.001
	≈ -2

c) Which value is being approximated by the secant line when the point Q approaches the point P(1,3)? -2

d) Based on the prevoius information find the slope of the tangent line passing through (1, 3) $m=-2$

e) Find the equation of the tangent line at the point (1, 3) $y-3=-2(x-1)$

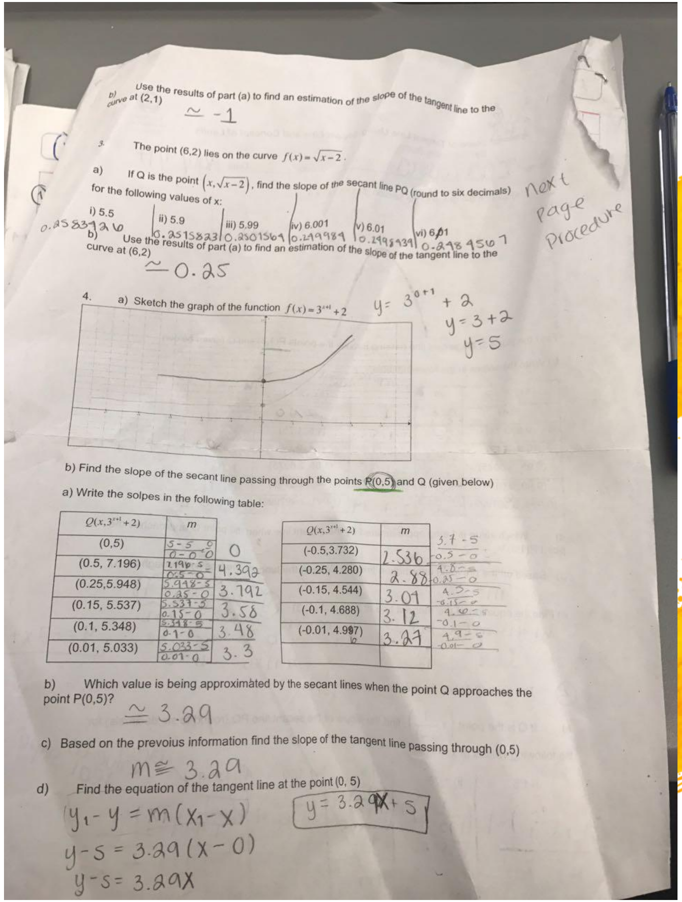
$y_1-y_1=m(x-x_1)$ $y=mx+b$ $y=-2x+b$ $3=-2(1)+b$ $3=-2+b$ $b=3+2$ $b=5$ $y=-2x+5$

2. The point (2,1) lies on the curve $f(x)=\frac{1}{x-1}$.

a) If Q is the point $\left(x,\frac{1}{x-1}\right)$, find the slope of the secant line PQ (round to six decimals) for the following values of x:

i) 1.5 ii) 1.75 iii) 1.9 iv) 1.99 v) 1.999 closer value to real slope

$m=-2$ $m=-1.33$ $m=-1.11$ $m=-1.010$ $m=-1.001$



b) Use the results of part (a) to find an estimation of the slope of the tangent line to the curve at (2,1) ≃ -1

3. The point (6,2) lies on the curve $f(x)=\sqrt{x-2}$.

a) If Q is the point $(x,\sqrt{x-2})$, find the slope of the secant line PQ (round to six decimals) for the following values of x:

i) 5.5 0.2583426 ii) 5.9 0.2515823 iii) 5.99 0.2501564 iv) 6.001 0.2499844 v) 6.01 0.2498439 vi) 6.01 0.2484507

Next page procedure

b) Use the results of part (a) to find an estimation of the slope of the tangent line to the curve at (6,2) ≃ 0.25

4. a) Sketch the graph of the function $f(x)=3^{x+1}+2$ $y=3^{0+1}+2$ $y=3+2$ $y=5$

b) Find the slope of the secant line passing through the points P(0,5) and Q (given below)

a) Write the solpes in the following table:

$Q(x,3^{x+1}+2)$	m
(0,5)	0
(0.5, 7.196)	4.392
(0.25, 5.948)	3.792
(0.15, 5.537)	3.58
(0.1, 5.348)	3.48
(0.01, 5.033)	3.3

$Q(x,3^{x+1}+2)$	m
(-0.5, 3.732)	2.536
(-0.25, 4.280)	2.88
(-0.15, 4.544)	3.04
(-0.1, 4.688)	3.12
(-0.01, 4.967)	3.27

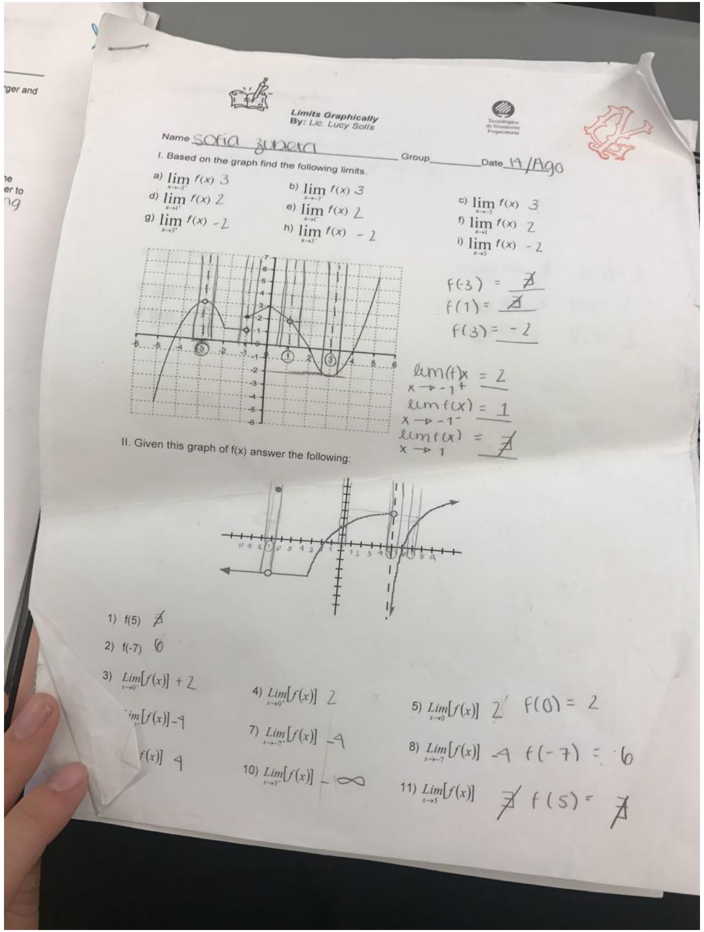
b) Which value is being approximated by the secant lines when the point Q approaches the point P(0,5)? ≃ 3.29

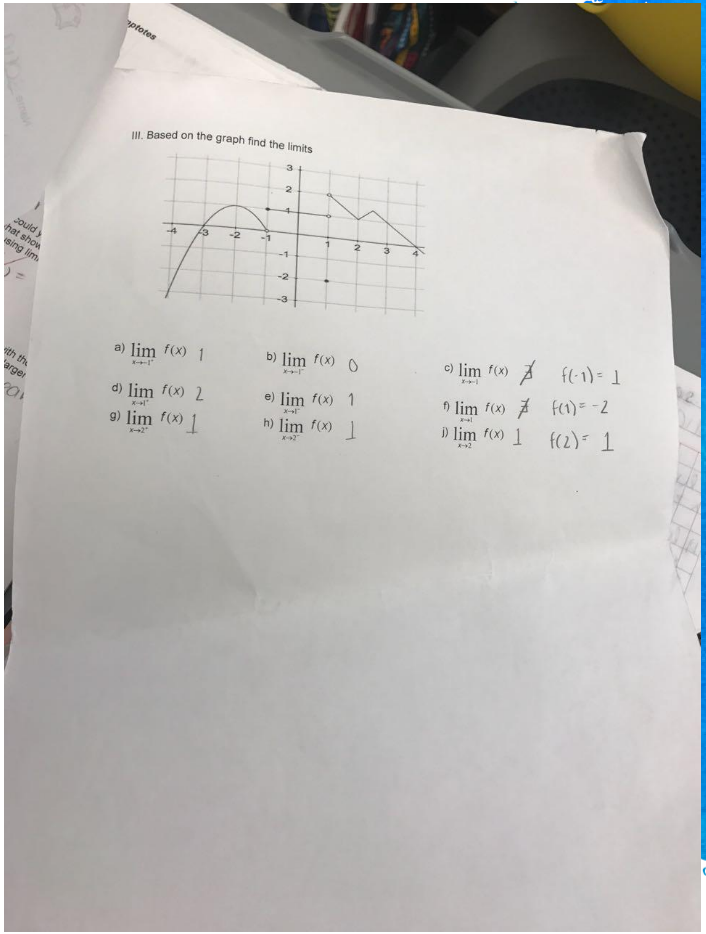
c) Based on the prevoius information find the slope of the tangent line passing through (0,5) $m \cong 3.29$

d) Find the equation of the tangent line at the point (0, 5) $y=3.29x+5$

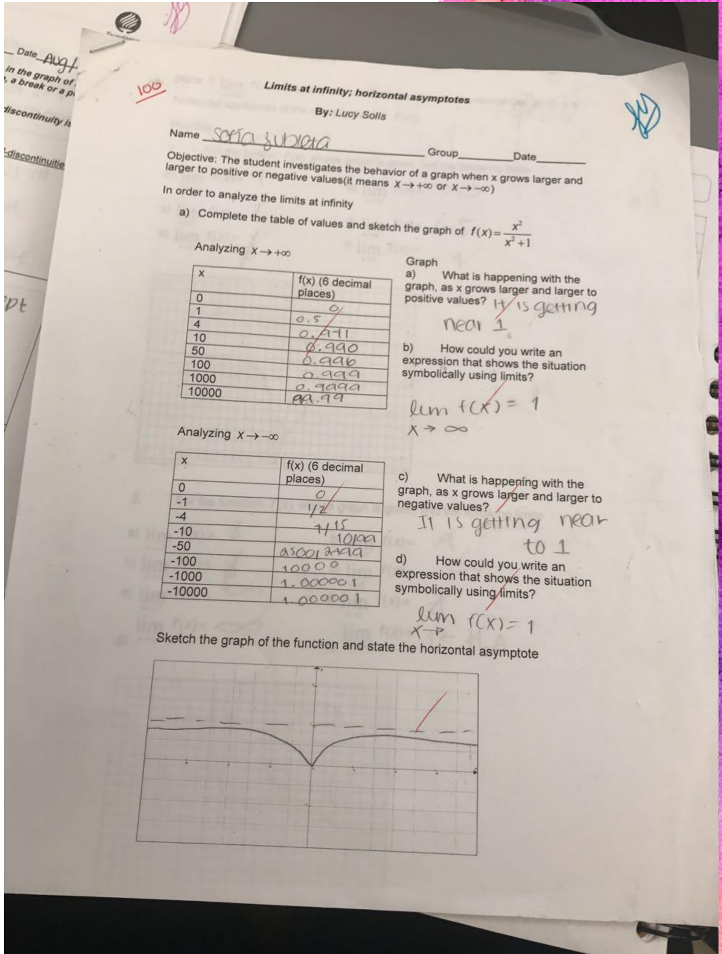
$y_1-y=m(x_1-x)$ $y-5=3.29(x-0)$ $y-5=3.29x$

Limits Graphically





Limits at infinity: Horizontal A.



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Limits at infinity; horizontal asymptotes

By: Lucy Solis

Name Sofia zubieta Group________ Date________

Objective: The student investigates the behavior of a graph when x grows larger and larger to positive or negative values(it means $x \to +\infty$ or $x \to -\infty$)

In order to analyze the limits at infinity

a) Complete the table of values and sketch the graph of $f(x)=\frac{x^2}{x^2+1}$

Analyzing $x \to +\infty$

x	f(x) (6 decimal places)
0	0
1	0.5
4	0.941
10	0.990
50	0.996
100	0.999
1000	0.9999
10000	99.99

Graph

a) What is happening with the graph, as x grows larger and larger to positive values? It is getting near 1

b) How could you write an expression that shows the situation symbolically using limits?

$\lim_{x \to \infty} f(x) = 1$

Analyzing $x \to -\infty$

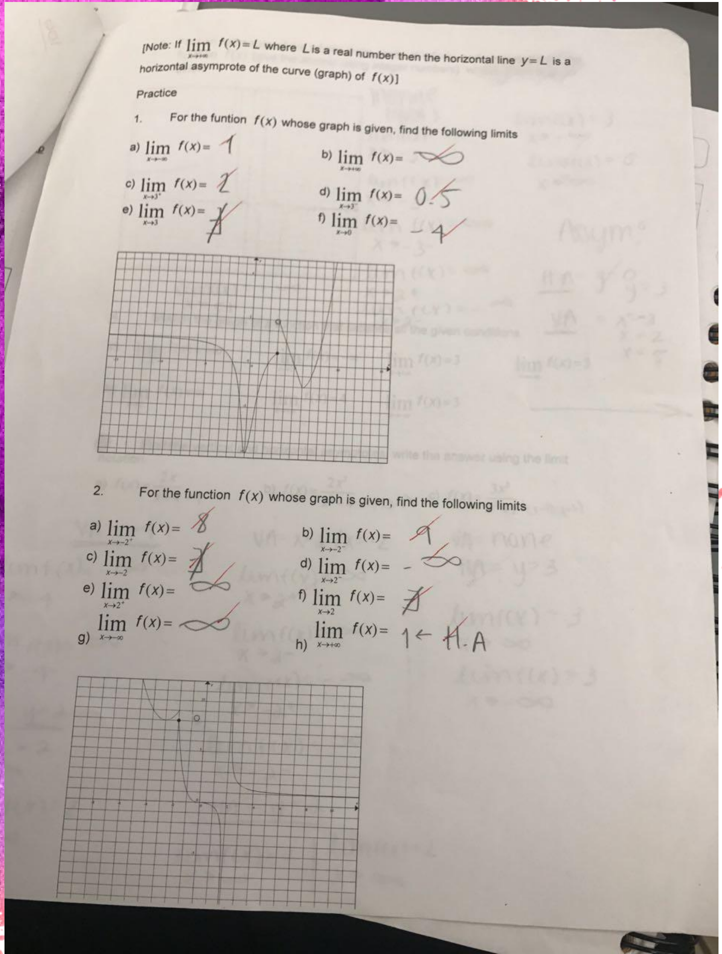
x	f(x) (6 decimal places)
0	0
-1	1/2
-4	1/15
-10	10/99
-50	2500/2499
-100	10000
-1000	1.000001
-10000	1.000001

c) What is happening with the graph, as x grows larger and larger to negative values? It is getting near to 1

d) How could you write an expression that shows the situation symbolically using limits?

$\lim_{x \to} f(x) = 1$

Sketch the graph of the function and state the horizontal asymptote



[Note: If $\lim_{x \to +\infty} f(x) = L$ where L is a real number then the horizontal line $y = L$ is a horizontal asymprote of the curve (graph) of $f(x)$]

Practice

1. For the funtion $f(x)$ whose graph is given, find the following limits

a) $\lim_{x \to -\infty} f(x) =$ 1

b) $\lim_{x \to +\infty} f(x) =$ ∞

c) $\lim_{x \to 3^+} f(x) =$ 2

d) $\lim_{x \to 3^-} f(x) =$ 0.5

e) $\lim_{x \to 3} f(x) =$ ∄

f) $\lim_{x \to 0} f(x) =$ -4

2. For the function $f(x)$ whose graph is given, find the following limits

a) $\lim_{x \to -2^+} f(x) =$ 8

b) $\lim_{x \to -2^-} f(x) =$ 9

c) $\lim_{x \to -2} f(x) =$ ∄

d) $\lim_{x \to 2^-} f(x) =$ $-\infty$

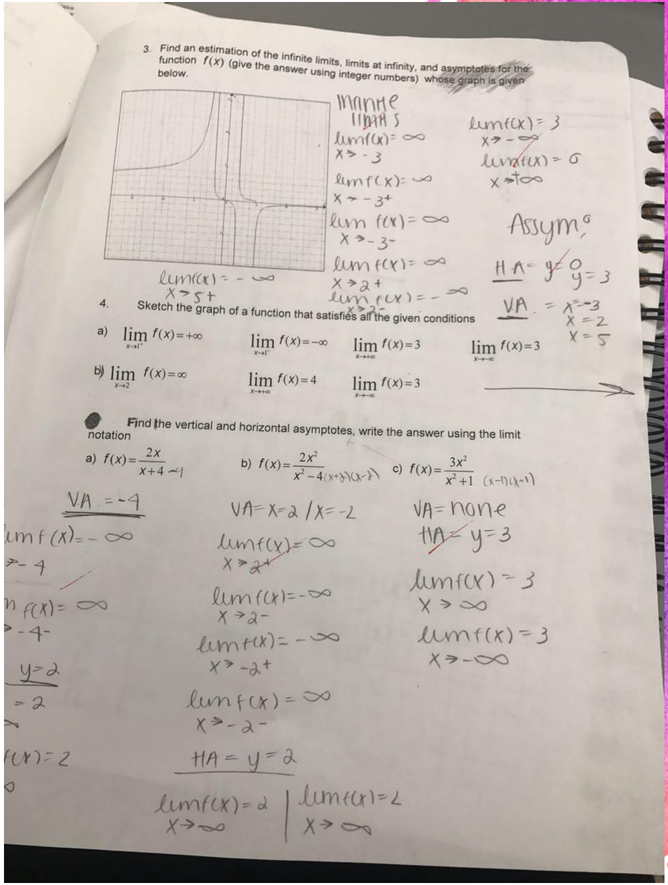
e) $\lim_{x \to 2^+} f(x) =$ ∞

f) $\lim_{x \to 2} f(x) =$ ∄

g) $\lim_{x \to -\infty} f(x) =$ ∞

h) $\lim_{x \to +\infty} f(x) =$ 1 ← H.A

Limits at infinity: Horizontal A



3. Find an estimation of the infinite limits, limits at infinity, and asymptotes for the function $f(x)$ (give the answer using integer numbers) whose graph is given below.

infinite limits

$\lim_{x \to -3} f(x) = \infty$

$\lim_{x \to -3^+} f(x) = \infty$

$\lim_{x \to -3^-} f(x) = \infty$

$\lim_{x \to 2^+} f(x) = \infty$

$\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to 5^+} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = 3$

$\lim_{x \to +\infty} f(x) = 0$

Assym:

HA = ~~y = 0~~ y = 3

VA = x = -3, x = 2, x = 5

4. Sketch the graph of a function that satisfies all the given conditions

a) $\lim_{x \to 1^+} f(x) = +\infty$ $\lim_{x \to 1^-} f(x) = -\infty$ $\lim_{x \to +\infty} f(x) = 3$ $\lim_{x \to -\infty} f(x) = 3$

b) $\lim_{x \to 2} f(x) = \infty$ $\lim_{x \to +\infty} f(x) = 4$ $\lim_{x \to -\infty} f(x) = 3$

Find the vertical and horizontal asymptotes, write the answer using the limit notation

a) $f(x) = \frac{2x}{x+4}$ b) $f(x) = \frac{2x^2}{x^2-4}$ $(x+2)(x-2)$ c) $f(x) = \frac{3x^2}{x^2+1}$ $(x-1)(x+1)$

a) VA = -4

$\lim_{x \to -4} f(x) = -\infty$

$\lim f(x) = \infty$, $x \to -4^-$

y = 2

$\lim f(x) = 2$

b) VA = x = 2 / x = -2

$\lim_{x \to 2^+} f(x) = \infty$

$\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to -2^+} f(x) = -\infty$

$\lim_{x \to -2^-} f(x) = \infty$

HA = y = 2

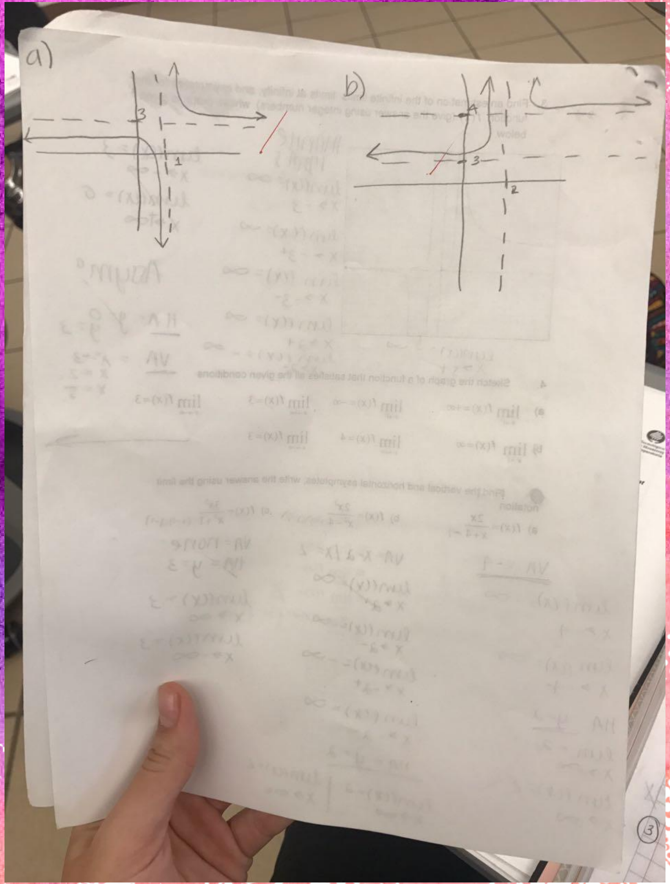
$\lim_{x \to \infty} f(x) = 2$ | $\lim_{x \to \infty} f(x) = 2$

c) VA = none

~~HA~~ y = 3

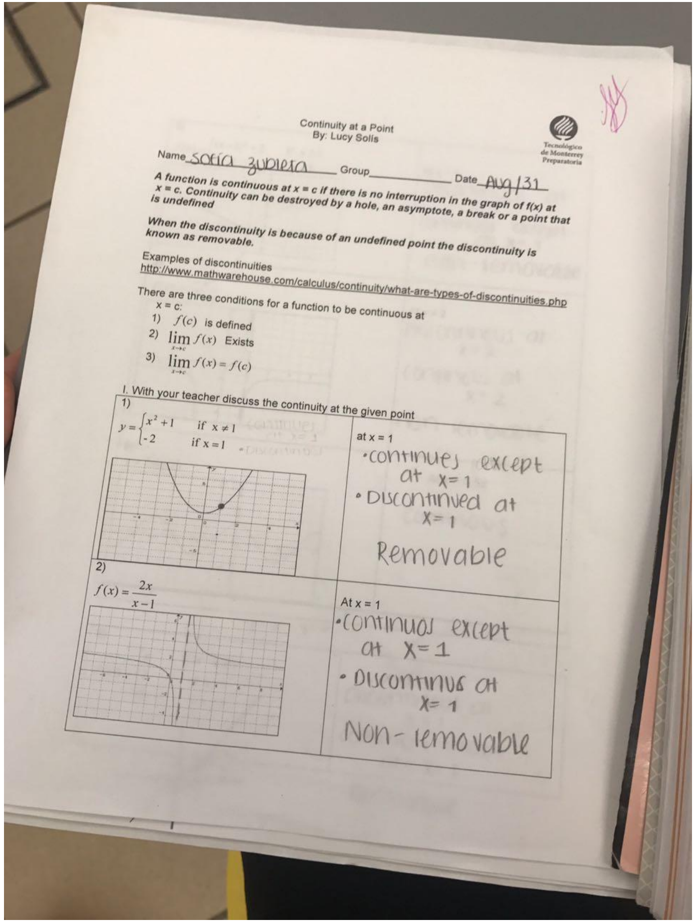
$\lim_{x \to \infty} f(x) = 3$

$\lim_{x \to -\infty} f(x) = 3$



Application Limits

Continuity at a Point



Continuity at a Point
By: Lucy Solis

Tecnológico de Monterrey Preparatoria

Name Sofia Zubiera Group ______ Date Aug/31

A function is continuous at x = c if there is no interruption in the graph of f(x) at x = c. Continuity can be destroyed by a hole, an asymptote, a break or a point that is undefined

When the discontinuity is because of an undefined point the discontinuity is known as removable.

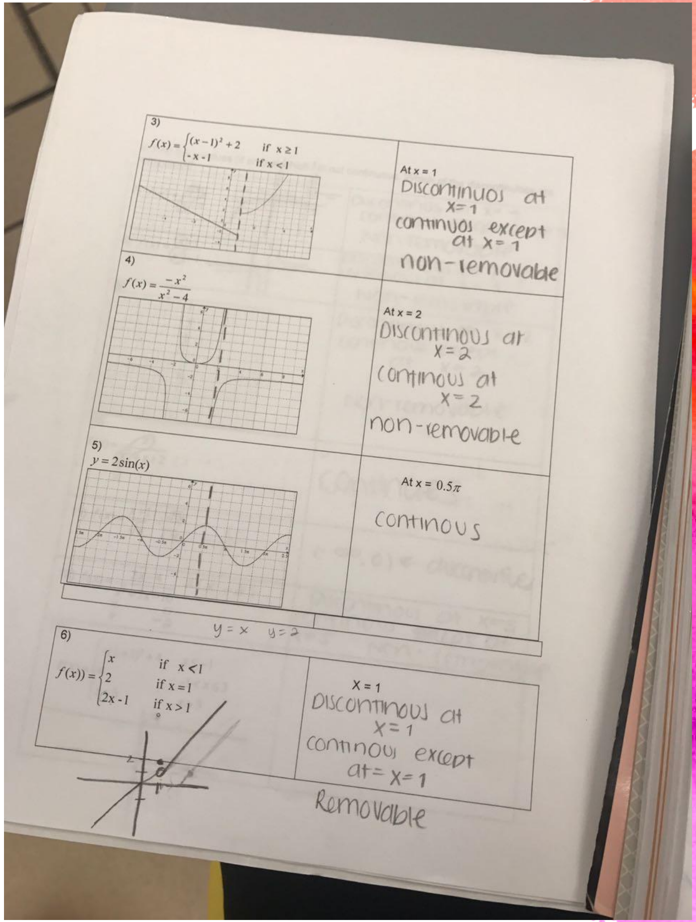
Examples of discontinuities
http://www.mathwarehouse.com/calculus/continuity/what-are-types-of-discontinuities.php

There are three conditions for a function to be continuous at x = c:

1) $f(c)$ is defined
2) $\lim_{x\to c} f(x)$ Exists
3) $\lim_{x\to c} f(x) = f(c)$

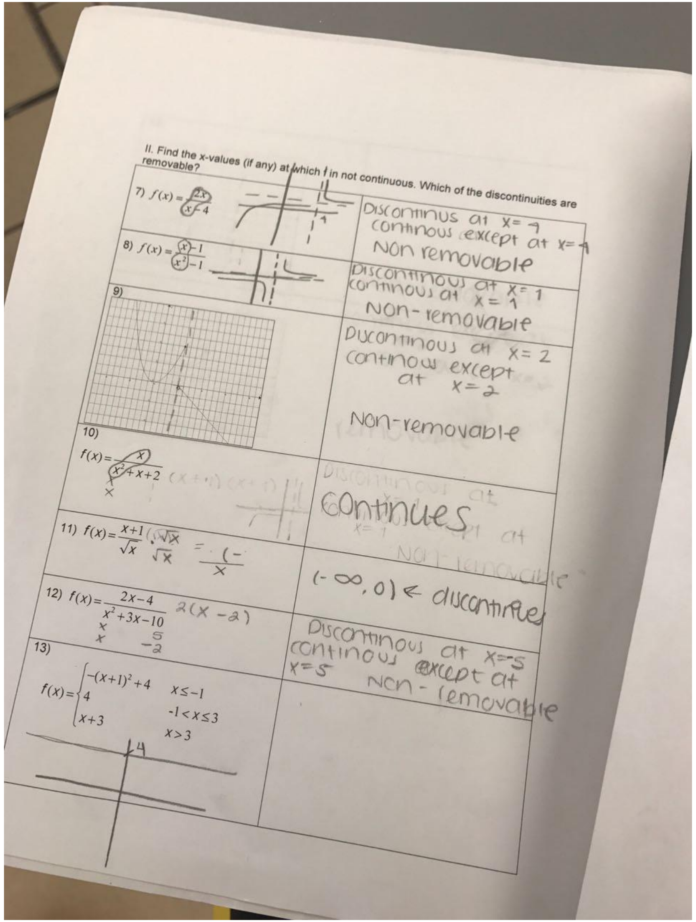
I. With your teacher discuss the continuity at the given point

1) $y = \begin{cases} x^2+1 & \text{if } x \neq 1 \\ -2 & \text{if } x = 1 \end{cases}$	at x = 1 •continues except at x=1 •Discontinued at x=1 Removable
2) $f(x) = \frac{2x}{x-1}$	At x = 1 •continuos except at x=1 •Discontinus at x=1 Non-removable



3) $f(x) = \begin{cases} (x-1)^2+2 & \text{if } x \geq 1 \\ -x-1 & \text{if } x < 1 \end{cases}$	At x = 1 Discontinuos at x=1 continuos except at x=1 non-removable
4) $f(x) = \frac{-x^2}{x^2-4}$	At x = 2 Discontinous at x=2 continous at x=2 non-removable
5) $y = 2\sin(x)$	At x = 0.5π continous
6) $f(x) = \begin{cases} x & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2x-1 & \text{if } x > 1 \end{cases}$ y = x y = 2	X = 1 Discontinous at x=1 continous except at x=1 Removable

Continuity at a Point



II. Find the x-values (if any) at which f in not continuous. Which of the discontinuities are removable?

7) $f(x) = \frac{2x}{x-4}$

Discontinus at x = 4
continous except at x = 4
Non removable

8) $f(x) = \frac{x-1}{x^2-1}$

Discontinous at x = 1
continous at x = 1
Non-removable

9)

Discontinous at x = 2
continous except at x = 2

Non-removable

10) $f(x) = \frac{x}{x^2+x+2}$

continues

11) $f(x) = \frac{x+1}{\sqrt{x}}$

$(-\infty, 0)$ ← discontinues

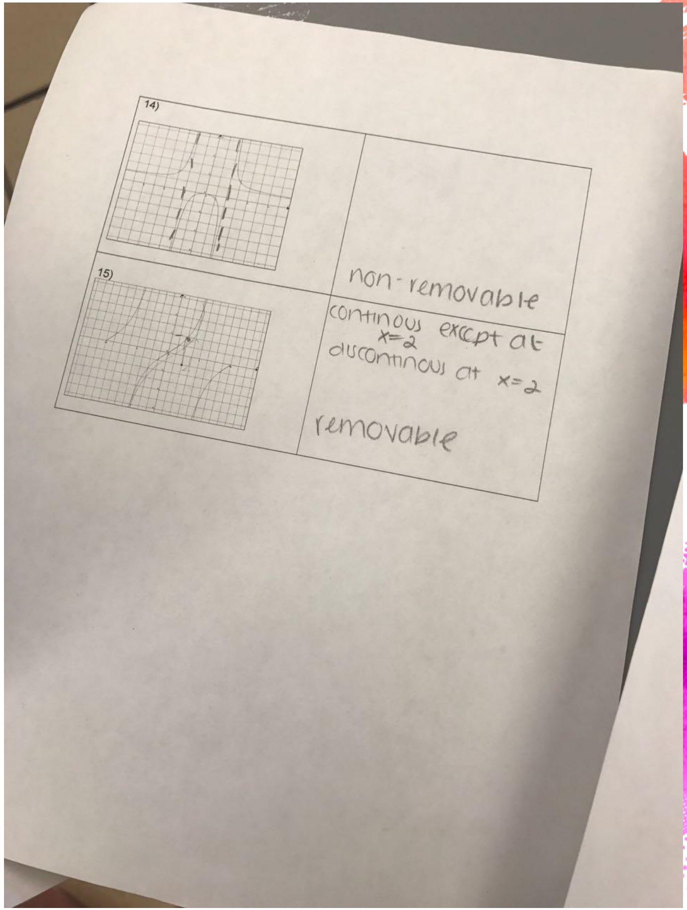
12) $f(x) = \frac{2x-4}{x^2+3x-10}$

$2(x-2)$

Discontinous at x = -5
continous except at x = 5
Non-removable

13)

$$f(x) = \begin{cases} -(x+1)^2+4 & x \le -1 \\ 4 & -1 < x \le 3 \\ x+3 & x > 3 \end{cases}$$



14)

non-removable

15)

continous except at x = 2
discontinous at x = 2

removable