

You must remember the following rule:

$$\frac{d(\ln x)}{dx} = \frac{1}{x} \quad \text{or} \quad \frac{d(\ln a)}{dx} = \frac{1}{a} \cdot \frac{da}{dx} = \frac{a'}{a}$$

I. Find the derivative for each of the following functions, frame your final answer.

1) $y = \frac{\ln 2x}{\ln 3x} = \frac{1}{x} \ln(2x) - \frac{1}{x} \ln(3x)$ $\frac{d}{dx} \left(\frac{\ln(2x)}{\ln(3x)} \right) = \frac{\ln(2/3)}{(\ln(3x))^2} = \frac{\ln(2/3)}{(\ln 3x)^2}$	2) $f(x) = \sqrt{1 + \ln x}$ $f'(x) = \frac{1}{2\sqrt{1 + \ln x}} \cdot \left(\frac{1}{x} \right) = \frac{1}{2x\sqrt{1 + \ln x}}$
3) $h(x) = \ln(e^x)$ $h'(x) = \frac{1}{e^x} \cdot e^x = 1$	4) $g(x) = x^2 + \ln x$ $g'(x) = 2x + \frac{1}{x}$
5) $y = \frac{\ln x}{x^2 + 1}$ $u = \ln x, v = x^2 + 1$ $v' = 2x$	6) $f(x) = \frac{\ln(x^2 + 1)}{(x^2 + 1)^2}$

II. Solve the following exercises, remember to show your procedure

1. The demand for a certain article is given by $p = 50 - 10 \ln x$, where x represents the price of the article and $x \geq 1$. Determine the rate of change of the demand with respect to the price.

$$p = 50 - 10 \left(\frac{1}{x} \right) = p' = -\frac{10}{x}$$

2. The revenue by selling " x " hundreds of articles is given by $R(x) = 60 \sqrt{\ln x}$, where $x \geq 1$ and R is measured in hundreds of dollars. Find the marginal revenue for 400 items.

$$60 (\ln x)^{1/2}$$

$$= 30 (\ln x)^{-1/2} \left(\frac{1}{x} \right) = \frac{30}{x \sqrt{\ln x}} = 0.0306$$

$$\frac{60}{400 \sqrt{\ln x}} = \frac{30}{200 \sqrt{\ln x}}$$

Rules of Differentiation → Not in order

1) $9x^2 (2x+5)^4$
 $u = 9x^2, v = (2x+5)^4$
 $u' = 18x, v' = 4(2x+5)^3 (2) = 8(2x+5)^3$
 $f'(x) = 18x(2x+5)^3 + 9x^2(8(2x+5)^2) = (2x+5)^2 (12x^2 + 32x + 72)$

3) $(4x+1)^3 (3x^2)^4$
 $u = (4x+1)^3, v = (3x^2)^4$
 $u' = 3(4x+1)^2 (4) = 12(4x+1)^2$
 $v' = 4(3x^2)^3 (6x) = 24x(3x^2)^3 = 24x(27x^6) = 648x^7$
 $f'(x) = (12(4x+1)^2)(3x^2)^4 + (4x+1)^3(648x^7)$

7) $f(x) = \frac{(2x-1)^3}{2x^2}$
 $u = (2x-1)^3, v = 2x^2$
 $u' = 3(2x-1)^2 (2) = 6(2x-1)^2$
 $v' = 4x$
 $f'(x) = \frac{(6(2x-1)^2)(2x^2) - (2x-1)^3(4x)}{(2x^2)^2} = \frac{12x^2(2x-1)^2 - 4x(2x-1)^3}{4x^4}$

12) $g(x) = (x^2+1)^8 (1-3x)^4$
 $u = (x^2+1)^8, v = (1-3x)^4$
 $u' = 8(x^2+1)^7 (2x) = 16x(x^2+1)^7$
 $v' = 4(1-3x)^3 (-3) = -12(1-3x)^3$
 $f'(x) = (16x(x^2+1)^7)(1-3x)^4 - (x^2+1)^8(12(1-3x)^3)$

I. Circle the right answer. (5 point each)

1) Find the slope for $f(x) = 5x^2$ at $x=3$
A) 30 B) -75 C) -45 D) 30

2) What is the equation of the tangent line for the curve $y = x^2 + 2$ at the point $(-1, 1)$?
A) $y = -3x + 4$ B) $y = 3x - 4$ C) $y = 3x + 4$ D) $y = -3x - 4$

3) The following functions is not differentiable at $x = -4$
a) $f(x) = |x+4|$ b) $f(x) = x^2 - 4$ c) $f(x) = \frac{x+2}{x-4}$ d) $f(x) = \sqrt{x+4}$

4) The following function is not differentiable at $x = 1$
a) $f(x) = \frac{1}{x+1}$ b) $f(x) = (x-1)^2$ c) $f(x) = |x+1|$ d) $f(x) = \sqrt{x-1}$

II. Answer the following questions.

1. The position of an object s at any time t , is given by: (15 points)
 $s(t) = -18t^2 + 15t + 8$ where s is measured in feet and t is measured in seconds.
Find the equation of acceleration at any time t .
 $s''(t) = -36t + 15 \Rightarrow -7 = -108t$

2. The following graph shows the function $y = f(x)$ (20 points)

a) Find the values of " x " where the function is not continuous. $x = 4, -2$

b) Find the values of " x " where the function is not differentiable. $x = 4, -2, 1$

I. Determine if true or false for each of the following statements (5 points each)

1. The derivative of $y = e^{-x}$ is $y' = e^{-x}$. (False, $y' = -e^{-x}$)

2. The derivative of $y = \ln(x-4)^5$ is $y' = 5 \ln(x-4)^4$. (False, $y' = \frac{5}{x-4}$)

3. If $s(t)$ is the function of position of an object in motion, then $a(t) = s''(t)$ is equal to the function of the acceleration of the object. (True)

4. If the velocity of the car is a function of time, then the derivative of this function with respect to time, describes the acceleration of the car. (True)

II. Circle the right answer. (10 point each)

1. (D) The derivative for $y = 2x^5$ is:
A) $y' = 2x^5$ B) $y' = 2e^x$ C) $y' = \frac{6x^5}{x}$ D) $y' = 6x^4e^x$

2. (B) The derivative for $y = \ln \sqrt{2x-4}$ is:
A) $y' = \frac{1}{2x-4}$ B) $y' = \frac{1}{2} \ln(2x-4)^{1/2}$ C) $y' = \frac{1}{2} \ln \frac{2}{\sqrt{2x-4}}$ D) $y' = \frac{1}{x-2}$

3. If the equation that gives the velocity of an object is $v(t) = 2t^2 e^{6t}$, then the equation that gives the acceleration is:
A) $a(t) = 6t^2 e^{6t} (2t+1)$ B) $a(t) = 6t^2 e^{6t}$ C) $a(t) = 36t^2 e^{6t}$ D) $a(t) = 12t^2 e^{6t}$

If $f(x) = e^x$ then $f'(x) = (e^x)'$

1) $y = \frac{3}{4x}$
 $\frac{3 \cdot 4x^{-2}}{4x^2} = \frac{12x^{-2}}{4x^2} = \frac{3}{x^2}$

2) $y = \frac{e^x}{2x}$
 $2x(e^x)' - (2x)'e^x = 2xe^x - 2e^x = 2e^x(x-1)$

3) $y = e^{2x}(2x-1)^3$ Product rule
 $4e^{2x}(3)(2x-1)^2 + (2x-1)^3 \cdot 2e^{2x} = 12e^{2x}(2x-1)^2 + 2e^{2x}(2x-1)^3$

4) $f(x) = \frac{e^{2x}}{6} + 2x^2$
 $\frac{2e^{2x}}{6} + 10x^9 = \frac{1}{3}e^{2x} + 10x^9$

5) $f(x) = e^{(x^2+3x)^2}$
 $4e^{(x^2+3x)^2} \cdot (2x+3) \cdot (2x+3) = 8e^{(x^2+3x)^2} \cdot (x^2+3x)$

BS

1. If $f(5) = 1, f'(5) = 0, g(5) = 11$
a) $(f \cdot g)'(5) = f'(5)g(5) + f(5)g'(5) = 0 \cdot 11 + 1 \cdot 12 = 12$
b) $(f/g)'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{g(5)^2} = \frac{11 \cdot 0 - 1 \cdot 12}{11^2} = -\frac{12}{121}$
c) $(f/g)'(5) = \frac{f(5)g'(5) - f'(5)g(5)}{g(5)^2} = \frac{1 \cdot 12 - 0 \cdot 11}{11^2} = \frac{12}{121}$

2. If $f(3) = 4, g(3) = 2, f'(3) = 10$ and $g'(3) = 5$, find the following values
a) $(f+g)'(3) = f'(3) + g'(3) = 10 + 5 = 15$
b) $(f \cdot g)'(3) = f'(3)g(3) + f(3)g'(3) = 10 \cdot 2 + 4 \cdot 5 = 20 + 20 = 40$
c) $(f/g)'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{g(3)^2} = \frac{2 \cdot 10 - 4 \cdot 5}{2^2} = \frac{20 - 20}{4} = 0$

x	f(x)	f'(x)	g(x)	g'(x)
-1	2	1	1	2
0	-1	0	-1	3
1	2	-1	0	5

3. If $h(x) = f(x)g(x)$, use the table to find $h'(-1), h'(0)$ and $h'(1)$

x	f(x)	f'(x)	g(x)	g'(x)
-1	2	1	1	2
0	-1	0	-1	3
1	2	-1	0	5

$h'(-1) = (1)(1) - (2)(2) = 1 - 4 = -3$

$h'(0) = (-1)(0) - (-1)(3) = 0 - (-3) = 3$

$h'(1) = (2)(-1) - (2)(5) = -2 - 10 = -12$
 $\frac{-12}{4} = -3$