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Limits at infinity; horizontal asymptotes

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Objective: The student investigates the behavior of a graph when x grows larger and larger to positive or negative values(it means $x \to +\infty$ or $x \to -\infty$)

In order to analyze the limits at infinity

a) Complete the table of values and sketch the graph of $f(x) = \frac{x^2}{x^2+1}$

Analyzing $x \to +\infty$

x	f(x) (6 decimal places)
0	0
1	0.5
4	0.941176
10	0.990099
50	0.999600
100	0.999900
1000	0.999999
10000	0.999999

Graph

a) What is happening with the graph, as x grows larger and larger to positive values?

It approaches to the asymptote

b) How could you write an expression that shows the situation symbolically using limits?

$\lim_{x \to \infty} = 1$

Analyzing $x \to -\infty$

x	f(x) (6 decimal places)
0	0
-1	0.5
-4	0.941176
-10	0.99099
-50	0.999600
-100	0.999900
-1000	0.999999
-10000	0.999999

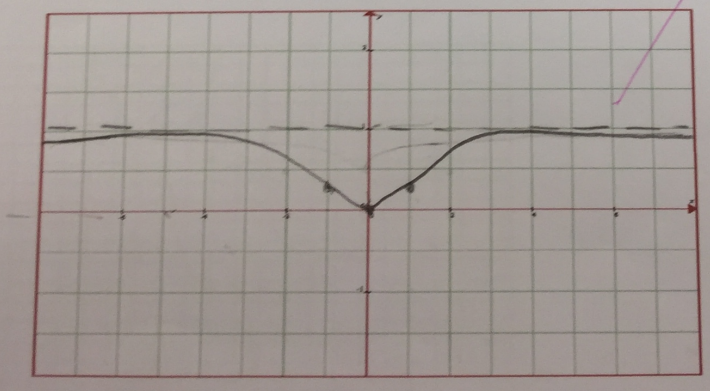
c) What is happening with the graph, as x grows larger and larger to negative values?

It approaches to the asymptote

d) How could you write an expression that shows the situation symbolically using limits?

$\lim_{x \to -\infty} = 1$

Sketch the graph of the function and state the horizontal asymptote



$f(x) = \frac{x^2}{x^2+1}$

[Note: If $\lim_{x\to+\infty} f(x) = L$ where L is a real number then the horizontal line $y = L$ is a horizontal asymprote of the curve (graph) of $f(x)$]

Practice

1. For the funtion $f(x)$ whose graph is given, find the following limits

a) $\lim_{x\to-\infty} f(x) =$ 1

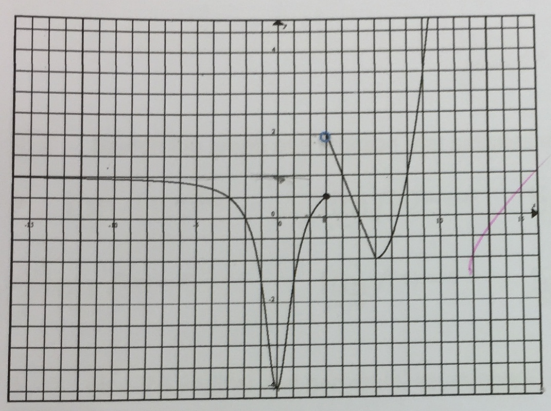
b) $\lim_{x\to+\infty} f(x) = \infty$

c) $\lim_{x\to3^+} f(x) =$ 0.5 = 2

d) $\lim_{x\to3^-} f(x) = 2$ = 5

e) $\lim_{x\to3} f(x) =$ ∄

f) $\lim_{x\to0} f(x) = -4$



2. For the function $f(x)$ whose graph is given, find the following limits

a) $\lim_{x\to-2^+} f(x) =$ 8

b) $\lim_{x\to-2^-} f(x) =$ 9

c) $\lim_{x\to-2} f(x) =$ ∄

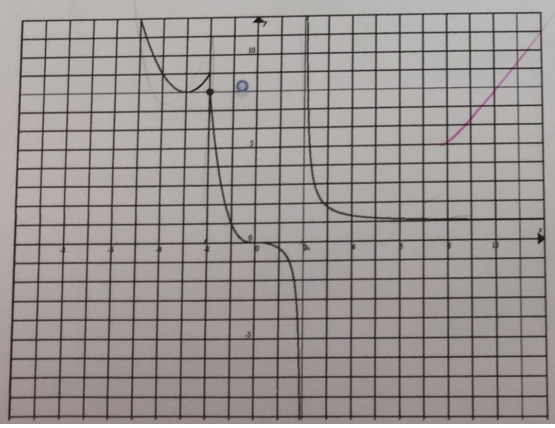
d) $\lim_{x\to2^-} f(x) = -\infty$

e) $\lim_{x\to2^+} f(x) = \infty$

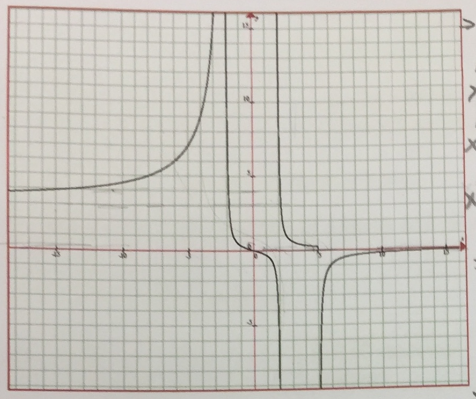
f) $\lim_{x\to2} f(x) =$ ∄

g) $\lim_{x\to-\infty} f(x) = \infty$

h) $\lim_{x\to+\infty} f(x) = 1$



3. Find an estimation of the infinite limits, limits at infinity, and asymptotes for the function $f(x)$ (give the answer using integer numbers) whose graph is given below.



→ Infinite limits (V.A)

$\lim_{x\to -3} f(x) = \infty$

$\lim_{x\to -3^+} f(x) = \infty$

$\lim_{x\to -3^-} f(x) = \infty$

$\lim_{x\to 2^+} f(x) = \infty$

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 5^+} f(x) = -\infty$

(H. A)

→ Limits at Infinity

$\lim_{x\to -\infty} f(x) = 3$

$\lim_{x\to \infty} f(x) = 0$

→ Asymptotes

- Horizontal: $y=0, y=3$
- Vertical: $x=-3$, $x=2$, $x=+5$.

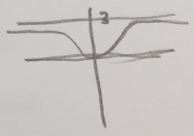
4. Sketch the graph of a function that satisfies all the given conditions

a) $\lim_{x\to 1^+} f(x) = +\infty$ $\lim_{x\to 1^-} f(x) = -\infty$ $\lim_{x\to +\infty} f(x) = 3$ $\lim_{x\to -\infty} f(x) = 3$

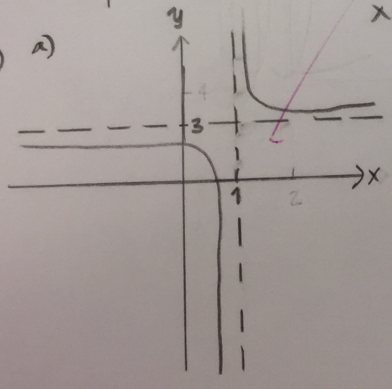
b) $\lim_{x\to 2} f(x) = \infty$ $\lim_{x\to +\infty} f(x) = 4$ $\lim_{x\to -\infty} f(x) = 3$

5. Find the vertical and horizontal asymptotes, write the answer using the limit notation

a) $f(x) = \frac{2x}{x+4}$	b) $f(x) = \frac{2x^2}{x^2-4}$	c) $f(x) = \frac{3x^2}{x^2+1}$
Horizontal A: $y=2$; $\lim_{x\to\infty} f(x) = 2$; $\lim_{x\to -\infty} f(x) = 2$	$y=2$; $\lim_{x\to\infty} f(x) = 2$; $\lim_{x\to -\infty} f(x) = 2$	$y=3$; $\lim_{x\to\infty} f(x) = 3$; $\lim_{x\to -\infty} f(x) = 3$
Vertical Asymptote: $x=-4$; $\lim_{x\to -4^+} f(x) = -\infty$; $\lim_{x\to -4^-} f(x) = +\infty$	$x=2$, $x=-2$; $\lim_{x\to 2^+} f(x) = \infty$; $\lim_{x\to 2^-} f(x) = -\infty$; $\lim_{x\to -2^-} f(x) = \infty$; $\lim_{x\to -2^+} f(x) = -\infty$	None



(4) a)



b)