

Lesson 3: Add & subtract decimals with few non-zero digits

Goals

- Add or subtract decimals, and explain the reasoning (using words and other representations).
- Comprehend the term ungroup means to decompose a larger base-ten unit into 10 units of lower place value (e.g., 1 tenth as 10 hundredths).
- Recognise and explain (orally) that writing additional zeros or removing zeros after the last non-zero digit in a decimal does not change its value.

Learning Targets

- I can tell whether writing or removing a zero in a decimal will change its value.
- I know how to solve subtraction problems with decimals that require ungrouping or decomposing.

Lesson Narrative

As with addition, in KS2 students have used various ways to subtract decimals to hundredths. Base-ten diagrams and vertical calculations are likewise used for subtracting decimals. Ungrouping, which students have previously used to subtract whole numbers, is a key idea here. They recall that a base-ten unit can be expressed as another unit that is $\frac{1}{10}$ its size. For example, 1 tenth can be ungrouped into 10 hundredths or into 100 thousandths. Students use this idea to subtract a larger digit from a smaller digit when both digits are in the same base-ten place, e.g., 0.012 - 0.007. Rather than thinking of subtracting 7 thousandths from 1 hundredth and 2 thousandths, we can view the 1 hundredth as 10 thousandths and subtract 7 thousandths from 12 thousandths.

Ungrouping also suggests that we can write a decimal in several equivalent ways. Because 0.4 can be viewed as 4 tenths, 40 hundredths, 400 thousandths, or 4 000 tenthousandths, it can also be written as 0.40, 0.400, 0.4000, and so on; the additional zeros at the end of the decimal do not change its value. They use this idea to subtract a number with more decimal places from one with fewer decimal places (e.g., 2.5 - 1.028). These calculations depend on making use of the structure of base-ten numbers.

The second activity is optional; it gives students additional opportunities to practise summing decimals.

Building On

• Understand the place value system.

Addressing

• Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.



Instructional Routines

- Stronger and Clearer Each Time
- Collect and Display
- Think Pair Share

Required Preparation

Students draw base-ten diagrams in this lesson. If drawing them is a challenge, consider giving students access to:

- Commercially produced base-ten blocks, if available.
- Paper copies of squares and rectangles (to represent base-ten units), cut outs from copies of the blackline master of the second lesson in the unit.
- Digital applet of base-ten representations https://ggbm.at/zqxRkhMh.

Some students might find it helpful to use graph paper to help them align the digits for vertical calculations. Consider having graph paper accessible for these activities: Representing Decimal Subtraction and Enough to Subtract?

Student Learning Goals

Let's add and subtract decimals.

3.1 Do the Zeros Matter?

Warm Up: 5 minutes

This warm-up prompts students to reason about regrouping and about when the zeros in a decimal affect the number that it represents. The mathematical work of interest is how students combine two decimals (e.g., in analysing 1.009 + 0.391, do they see that 0.009 + 0.001 = 0.010?) and how they write the sum (e.g., do they know 1.4 = 1.40 = 1.400?).

Instructional Routines

Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet time to mentally add the decimals in the first problem and then another minute to discuss their answer and strategy with a partner. Briefly discuss their strategies as a class, and then ask students to complete the true-or-false questions.

Anticipated Misconceptions

Some students may say, "You can take the zeros away after the decimal point and the number stays the same." Although students could mean, for instance, that 12.9 is equal to



12.90, they might also mistakenly think 12.09 is equal 12.9. Ask these students to be more precise in their statement. Ask if the zero can be in any place after the decimal point or only in certain places.

Student Task Statement

- 1. Evaluate mentally: 1.009 + 0.391
- 2. Decide if each equation is true or false. Be prepared to explain your reasoning.
 - a. 34.56000 = 34.56
 - b. 25 = 25.0
 - c. 2.405 = 2.45

Student Response

- 1. 1.4 or 1.40 or 1.400. Strategies vary.
- 2.
- a. true
- b. true
- c. false (405 thousandths does not have the same value as 45 hundredths)

Activity Synthesis

Ask students to indicate whether they think each equation is true or false and ask for an explanation for each. Students may simply say that we can or cannot just remove the zeros. Encourage them to use what they know about place values or comparison strategies to explain why one number is greater than, less than, or equal to the other. If students do not notice that the two numbers in the true-or-false questions have the same digits except for the missing zeros, point that out after each question.

If not mentioned by students in their explanations, ask:

- "Can zeros be written at the end of a decimal without changing the number that it represents?"
- "Can zeros be eliminated from the end of a decimal without changing the value?"
- "Can zeros be written or erased in the middle of a decimal without changing the value?"

3.2 Calculating Sums

Optional: 15 minutes (there is a digital version of this activity)



Here students continue to use diagrams to represent sums of decimals, but they also transition to writing addition calculations vertically. They think about the alignment of the digits in column addition and subtraction to help ensure that correct values are combined. This activity is optional, so students have the option to spend more time subtracting decimals in the next activity.

As in the previous activity, consider having students use physical base-ten blocks (if available), a paper version of the base-ten figures (from the blackline master), or this digital applet ggbm.at/FXEZD466, as alternatives to drawing diagrams.

Instructional Routines

- Stronger and Clearer Each Time
- Think Pair Share

Launch

Remind students that the term "sum" means the result of an addition. Refer to the image from the previous activity that shows how several squares and rectangles were used to represent base-ten units. Tell students to use the same representations in this activity and to keep in mind that in the process of grouping they find more sums of decimals.

Arrange students in groups of 2. Give students 10 minutes of quiet work time, but encourage them to briefly discuss their responses with their partner after completing the second question and before continuing with the rest. Follow with a whole-class discussion.

Classes using the digital activities have an interactive applet with virtual blocks. In this activity, students must redefine the value of each block to represent the place values in each problem. To use the grouping and ungrouping features, the pieces must be aligned on the light blue grids. To bring a piece into the workspace, select one of the green tool icons and then click on the workspace. To move it, you must click on the Move tool

Action and Expression: Develop Expression and Communication. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Encourage students to begin with physical representations before drawing a diagram. Provide access to physical or virtual base-ten blocks to support drawing diagrams.

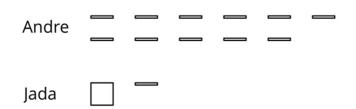
Supports accessibility for: Conceptual processing Conversing: Stronger and Clearer Each Time. Use this routine to give students an opportunity to refine their explanation about which calculation is correct for 0.2 + 0.05. At the appropriate time, give students time to meet with 2–3 partners to share their response. Display prompts students can use to provide feedback such as, "How did the alignment of the decimals change the sum?", "Why is aligning decimal places important here?" and "A detail (or word) you could add is _____, because" As students discuss, listen to how they talk about combining units, aligning digits of the same place values, and whether it is necessary to add zeros. Give students 1–2 minutes to revise their writing based on their conversations.

Design Principle(s): Optimise output (for justification)



Student Task Statement

1. Andre and Jada drew base-ten diagrams to represent 0.007 + 0.004. Andre drew 11 small rectangles. Jada drew only two figures: a square and a small rectangle.



- a. If both students represented the sum correctly, what value does each small rectangle represent? What value does each square represent?
- b. Draw or describe a diagram that could represent the sum 0.008 + 0.07.
- 2. Here are two calculations of 0.2 + 0.05. Which is correct? Explain why one is correct and the other is incorrect.

	0.2		0.2
+	0.05	+	0.05
	0.25		0.07

3. Compute each sum. If you get stuck, consider drawing base-ten diagrams to help you.

a.

$$\begin{array}{r}
0.11 \\
+ 0.005 \\
0.209 + 0.01
\end{array}$$

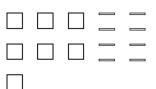
c. 10.2 + 1.1456

Student Response

b.

1.

- a. A square represents 1 hundredth. A small rectangle represents 1 thousandth.
- b. 7 squares (for 7 hundredths) and 8 small rectangles (for 8 thousandths)





2. The first response is correct, and the second response is incorrect. Sample reasoning: Digits that represent unlike units were combined, so the sum would be off. Adding 2 tenths and 5 hundredths would not produce 7 hundredths.

3.

- a. 0.115
- b. 0.219
- c. 11.3456

Activity Synthesis

The goal of this discussion is to help students understand that vertical calculation is an efficient way to find the sums of decimals. Discuss:

- "When finding 0.008 + 0.07, why do we not combine the 8 thousands and 7 hundredths to make 15?" (Hundredths and thousandths are different units. If each hundredth is ungrouped into 10 thousandths, we can add 70 thousandths and 8 thousandths to get 78 thousandths).
- "How do we use representations of base-ten numbers to add effectively and efficiently?" (Make sure to put together tenths with tenths, hundredths with hundredths, etc. Also, make sure that if a large square represents $\frac{1}{10}$ for one summand,

it also represents $\frac{1}{10}$ for the other.)

• "When adding numbers without using base-ten diagrams or other representations, what can we do to help add them correctly?" (Pay close attention to place value so we combine only like units. It is helpful to line up the digits of the numbers so that numerals that represent the same place value are placed directly on top of one another.)

Additionally, consider using colour coding to help students visualise the place-value structure, as shown here.

0.2 + 0.05 0.25

3.3 Subtracting Decimals of Different Lengths

25 minutes (there is a digital version of this activity)

In this activity, students encounter two variations of decimal subtraction in which regrouping is involved. They subtract a number with more decimal places from one with fewer decimal places (e.g., 0.1 - 0.035), and subtract two digits that represent the same place value but where the value in the second number is greater than that in the starting



number (e.g., in 1.12 - 0.47, both the tenth and hundredth values in the second number is larger than those in the first).

Students represent these situations with base-ten diagrams and study how to perform them using vertical calculations. The big idea here is that of ungrouping or of decomposing a unit with 10 of another unit that is $\frac{1}{10}$ its size to make it easier to subtract. In some cases, students would need to decompose twice before subtracting (e.g., a tenth into 10 hundredths, and then 1 hundredth into 10 thousandths).

Use the whole-class discussion to highlight the correspondences between the two methods and to illustrate how writing zeros at the end of a decimal helps us perform subtraction.

As in previous activities, consider having students use physical base-ten blocks (if available), a paper version of the base-ten figures (from the blackline master), or this digital applet https://gbm.at/FXEZD466, as alternatives to drawing diagrams.

Instructional Routines

• Collect and Display

Launch

Keep students in the same groups of 2. Give partners 4–5 minutes to complete the first two questions. Then, give students 4–5 minutes of quiet time to complete the last question and follow with a whole-class discussion.

Classes using the digital activities have an interactive applet with virtual blocks. Note that students may need to reassign the values of the blocks to answer the questions. To use the grouping and ungrouping features, the pieces must be aligned on the light blue grids. To bring a piece into the workspace, select one of the green tool icons and then click on the workspace. To move it, you must click on the Move tool. Subtract by deleting with the delete tool.

Listen for and capture two or three different ways students refer to the idea of ungrouping as they work on problems 1 and 2.

Action and Expression: Develop Expression and Communication. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Encourage students to begin with physical representations before drawing a diagram. Provide access to physical or virtual base-ten blocks to support drawing diagrams. Supports accessibility for: Conceptual processing

Anticipated Misconceptions

If students struggle to subtract two numbers that do not have the same number of decimal digits (such as in questions 3a and 3d), consider representing the subtraction with base-ten diagrams. For example, to illustrate 0.3 - 0.05, start by drawing 3 large rectangles to represent 3 tenths. Replace 1 rectangle with 10 squares, each representing 1 hundredth. Cross out 5 squares to show subtraction of 5 hundredths. Alternatively, replace 3

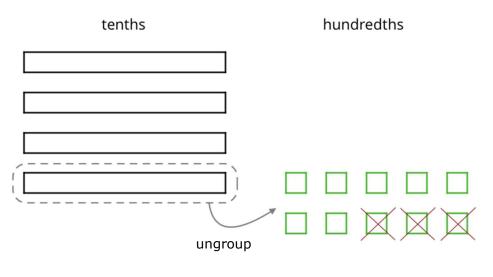


rectangles (3 tenths) with 30 squares (30 hundredths), cross out 5 squares to show subtraction of 5 hundredths.

Student Task Statement

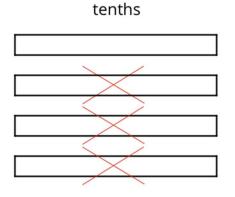
Diego and Noah drew different diagrams to represent 0.4 - 0.03. Each rectangle represents 0.1. Each square represents 0.01.

• Diego started by drawing 4 rectangles to represent 0.4. He then replaced 1 rectangle with 10 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 3 rectangles and 7 squares in his diagram.



Diego's Method

• Noah started by drawing 4 rectangles to represent 0.4. He then crossed out 3 rectangles to represent the subtraction, leaving 1 rectangle in his diagram.

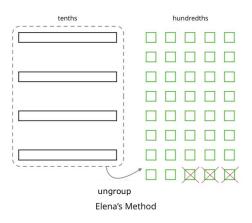


Noah's Method

1. Do you agree that either diagram correctly represents 0.4 - 0.03? Discuss your reasoning with a partner.



Elena also drew a diagram to represent 0.4 – 0.03. She started by drawing 4 rectangles. She then replaced all 4 rectangles with 40 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 37 squares in her diagram. Is her diagram correct? Discuss your reasoning with a partner.



- 3. Find each difference. Explain or show your reasoning.
 - a. 0.3 0.05
 - b. 2.1 0.4
 - c. 1.03 0.06
 - d. 0.02 0.007

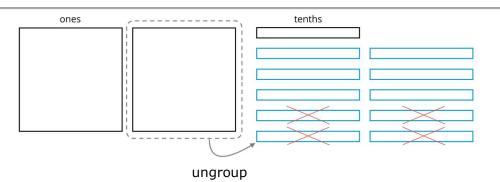
Student Response

- 1. Answers vary. Sample reasoning:
 - I agree with Diego. Since 10 hundredths is 1 tenth, 1 rectangle can be replaced with 10 squares. Subtraction of 0.03 means taking away 3 hundredths or 3 small squares.
 - I disagree with Noah's representation. He removed 3 tenths not 3 hundredths.
- 2. Yes, her diagram is correct. Sample reasoning: Four rectangles is 4 tenths, which is equal to 40 hundredths. She correctly removed 3 hundredths from 40 hundredths.

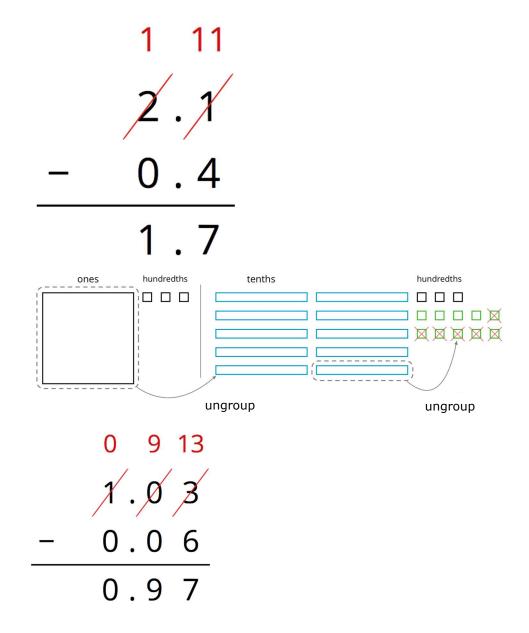
3.

- a. 0.25. Sample reasoning: 0.3 is 3 tenths or 30 hundredths. Subtracting 0.05 or 5 hundredths from 30 hundredths leaves 25 hundredths, which is 0.25.
- b. 1.7. Sample reasoning:





c. 0.97. Sample reasoning:





d. 0.013. Sample reasoning: 0.02 is 2 hundredths. One of the hundredths could be ungrouped into 10 thousandths so that 7 thousandths could be subtracted. What remain are 1 hundredth and 3 thousandths, which is 0.013.

Are You Ready for More?

A distant, magical land uses jewels for their bartering system. The jewels are valued and ranked in order of their rarity. Each jewel is worth 3 times the jewel immediately below it in the ranking. The ranking is red, orange, yellow, green, blue, indigo, and violet. So a red jewel is worth 3 orange jewels, a green jewel is worth 3 blue jewels, and so on.

At the Auld Shoppe, a shopper buys items that are worth 2 yellow jewels, 2 green jewels, 2 blue jewels, and 1 indigo jewel. If they came into the store with 1 red jewel, 1 yellow jewel, 2 green jewels, 1 blue jewel, and 2 violet jewels, what jewels do they leave with? Assume the shopkeeper gives them their change using as few jewels as possible.

Student Response

2 orange jewels, 1 yellow jewel, 1 green jewel, and 2 indigo jewels

Activity Synthesis

The purpose of this discussion is for students to make connections between two different and correct ways to subtract decimals when ungrouping is required. Ask:

- "What is the difference between Diego's method and Elena's method?" (Diego only breaks up 1 tenth into 10 hundredths, whereas Elena breaks up all 4 tenths into hundredths.)
- "What are some advantages to Diego's method?" (Diego's method is quicker to draw. It shows the 3 tenths and 7 hundredths. Elena would need to count how many hundredths she has.)
- "What are some advantages to Elena's method?" (Elena's diagram shows a difference of 37 hundredths, which matches how we say 0.37 in words.)

Consider connecting the diagrams for Diego's and Elena's work with numerical equations as shown here.

Diego's	Elena's
0.4 - 0.03 = 0.3 + 0.10 - 0.03 = 0.3 + 0.07 = 0.37	0.4 - 0.03 = 0.40 - 0.03 = 0.37

We can also show both calculations by arranging the numbers vertically. On the left, the 3 and 10 in red show Diego's unbundling of the 4 hundredths. The calculation on the right illustrates Elena's representation: the 0 written in blue helps us see 4 tenths as 40



hundredths, from which we can subtract 3 hundredths to get 37 hundredths. Use this example to reinforce that writing an additional zero at the end of a non-zero decimal does not change its value.

	3 10	
	0.4	0.40
_	0.03	- 0.03
	0.37	0.37

To deepen students' understanding, consider asking:

- "In the problem 2.1 0.4, how does ungrouping in the diagram show up in the vertical calculation with numbers?" (In the diagram, we ungroup 1 whole to make 10 tenths. With the calculation, we rewrite 1 whole as 10 tenths (over the tenth place) and combine it to the given 11 tenths before subtracting 4 tenths.)
- "When might it be really cumbersome to subtract using base-ten diagrams? Can you give examples?" (When the numbers involve many decimal places, such as 113.004 6.056802, or a problem with large digits, such as 7.758 0.869.)

Emphasise that the algorithm (column subtraction) for subtraction of decimals works like the algorithm for subtraction of whole numbers. The only difference is that the values involved in the subtraction problems can now include tenths, hundredths, thousandths, and so on. The key in both cases is to pay close attention to the place values of the digits in the two numbers.

Lesson Synthesis

In this lesson, we saw that decimal subtraction problems can be done with base-ten diagrams or with vertical calculations. In both cases, it is important to subtract the values that are in the same decimal place. We also saw that zeros can be written to or removed from the end of a decimal without changing the value of the number.

- When using base-ten blocks to represent subtraction of decimals, how do we remove a larger value from a smaller value that are in the same decimal place? For example, to find 4.5 2.7, how do we remove 7 tenths from 5 tenths? (We ungroup a larger unit into 10 of a smaller unit; in this case, we exchange a 1 with 10 tenths, which allows us to subtract 7 tenths.)
- When calculating differences of decimals, why should we line up the decimal points or digits in the same decimal places? (The value of any digit in a base-ten number depends on its place. Lining up the decimal points and like units help us subtract correctly.)



- How do we subtract a number with more decimal places with one with fewer decimal places (e.g., 4.1 1.0935)? (We can write zeros at the end of the shorter decimal to help us subtract.)
- Which are more efficient for finding differences, using base-ten diagrams or using column subtraction? (It depends on the length of the number and the size of the digits. Base-ten diagrams may take a while to draw.)

3.4 Calculate the Difference

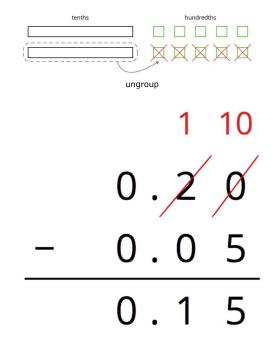
Cool Down: 5 minutes

Student Task Statement

- 1. Find the sum 1.56 + 0.083. Show your reasoning.
- 2. Find the difference 0.2 0.05. Show your reasoning.
- 3. You need to be at least 39.37 inches tall (about a metre) to ride on a bumper car. Diego's cousin is 35.75 inches tall. How many more inches will he need to grow before Diego can take him on the bumper car ride? Explain or show your reasoning.

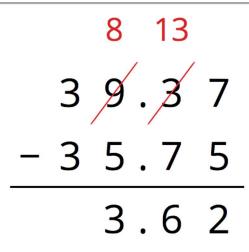
Student Response

- 1. 1.56 + 0.083 = 1.643
- 2. 0.2 0.05 = 0.15



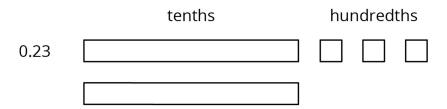
3. 3.62 inches taller



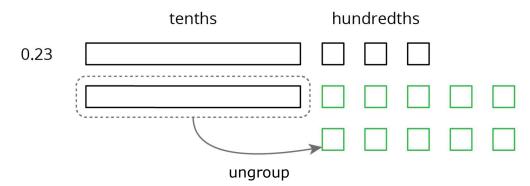


Student Lesson Summary

Base-ten diagrams can help us understand subtraction as well. Suppose we are finding 0.23 - 0.07. Here is a diagram showing 0.23, or 2 tenths and 3 hundredths.



Subtracting 7 hundredths means removing 7 small squares, but we do not have enough to remove. Because 1 tenth is equal to 10 hundredths, we can ungroup(or decompose) one of the tenths (1 rectangle) into 10 hundredths (10 small squares).

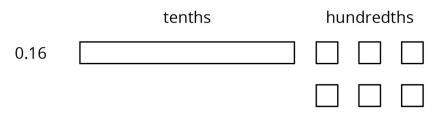


We now have 1 tenth and 13 hundredths, from which we can remove 7 hundredths.



	tenths	hundredths
0.23		
		$\Box \Box \Box \boxtimes \boxtimes \boxtimes$
		$\boxtimes \boxtimes \boxtimes \boxtimes \boxtimes \boxtimes$
		subtract 0.07

We have 1 tenth and 6 hundredths remaining, so 0.23 - 0.07 = 0.16.



Here is a vertical calculation of 0.23 - 0.07.

	1	13
	0.2	3
_	0.0	7
	0.1	6

0.023

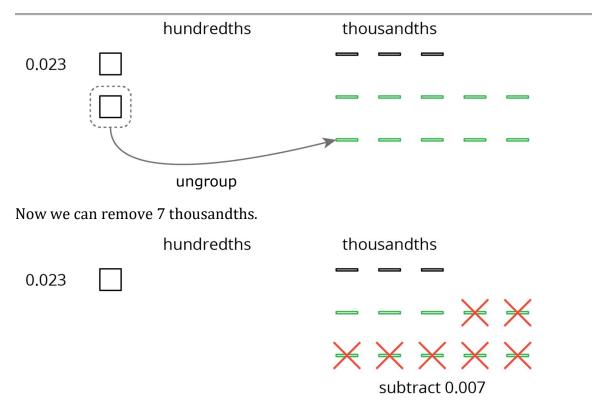
Notice how this representation also shows a tenth is ungrouped (or decomposed) into 10 hundredths in order to subtract 7 hundredths.

This works for any decimal place. Suppose we are finding 0.023 - 0.007. Here is a diagram showing 0.023.

hundredths	thousandths

We want to remove 7 thousandths (7 small rectangles). We can ungroup (or decompose) one of the hundredths into 10 thousandths.





We have 1 hundredth and 6 thousandths remaining, so 0.023 - 0.007 = 0.016.

	hundredths	thousandths
0.016		

Here is a vertical calculation of 0.023 - 0.007.

		1	13
	0.0	Z	Z
_	0.0	0	7
	0.0	1	6

Lesson 3 Practice Problems

1. **Problem 1 Statement**

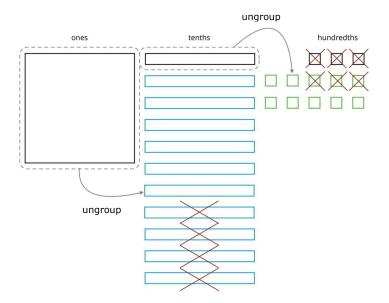
Here is a base-ten diagram that represents 1.13. Use the diagram to find 1.13 - 0.46.

Explain or show your reasoning.



Solution

0.67. Sample response: First, ungroup 1 tenth into 10 hundredths and then take away 6 hundredths from the 13 hundredths, leaving 7 hundredths. Next, ungroup the 1 one as 10 tenths. After taking away 4 tenths, 6 tenths are left. So the answer is 0.67.



2. Problem 2 Statement

Compute the following sums. If you get stuck, consider drawing base-ten diagrams.

- a. 0.027 + 0.004
- b. 0.203 + 0.01
- c. 1.2 + 0.145

Solution

- a. 0.031
- b. 0.213
- c. 1.345



(Diagrams for b and c shown here.)

b.	=	
c.		

3. Problem 3 Statement

A student said we cannot subtract 1.97 from 20 because 1.97 has two decimal digits and 20 has none. Do you agree with him? Explain or show your reasoning.

Solution

Disagree. Sample explanation: The number 1.97 is equal to 197 hundredths. 20 can be written as 20.00 or 2000 hundredths. We can subtract 197 from 2000 to get 1803 hundredths, so 20 - 1.97 = 18.03.

4. Problem 4 Statement

Decide which calculation shows the correct way to find 0.3 - 0.006 and explain your reasoning.

А	В	С	D
0.3	0.3	0.3 0	0.3 0 0
-0.006	-0.006	-0.0 0 6	-0.006
0.3 0 6	0.097	0.0 2 4	0.294

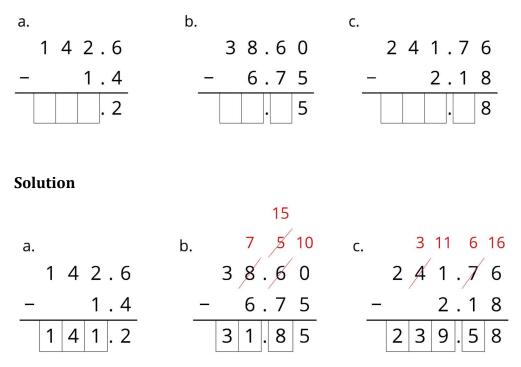


Solution

D. Sample reasoning: It is the only one that shows the decimal points correctly lined up so that the same base-ten units are aligned vertically.

5. Problem 5 Statement

Complete the calculations so that each shows the correct difference.



6. Problem 6 Statement

The school store sells pencils for £0.30 each, hats for £14.50 each, and binders for £3.20 each. Elena would like to buy 3 pencils, a hat, and 2 binders. She estimated that the cost will be less than £20.

- a. Do you agree with her estimate? Explain your reasoning.
- b. Estimate the number of pencils could she buy with £5. Explain or show your reasoning.

Solution

- a. Disagree. Sample reasoning: The hat costs more than £14, and two binders cost more than £6. Even without the pencils the cost is already more than £20.
- b. Answers vary, but should be around 15 or 16. Sample reasoning: She could buy 3 pencils for every pound, so for £5, she could buy around 15 pencils.



7. Problem 7 Statement

A cuboid measures $7\frac{1}{2}$ cm by 12 cm by $15\frac{1}{2}$ cm.

- a. Calculate the number of cubes with edge length $\frac{1}{2}$ cm that fit in this cuboid.
- b. What is the volume of the cuboid in cm³? Show your reasoning. If you are stuck, think about how many cubes with $\frac{1}{2}$ cm edge lengths fit into 1 cm³.

Solution

- a. 11, 160 cubes
- b. 1395 cm³. Sample reasoning: Eight $\frac{1}{2}$ cm cubes fit in a 1 cm cube and 11, 160 of these $\frac{1}{2}$ cm cubes fit in the prism. So, 11, 160 ÷ 8 of the 1 cm cubes fit in the cuboid. That means the volume of the cuboid in cm³ is 11, 160 ÷ 8 = 1395.

8. Problem 8 Statement

At a constant speed, a car travels 75 miles in 60 minutes. How far does the car travel in 18 minutes? If you get stuck, consider using the table.

minutes	distance in miles
60	75
6	
18	

Solution

22.5 miles (or equivalent). Possible strategy:

minutes	distance in miles
60	75
6	7.5
18	22.5



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