Sainik School Examination Board
PRE-SEE 2078

Time: 3:00 hrs Optional Mathematics **F.M. :** 100

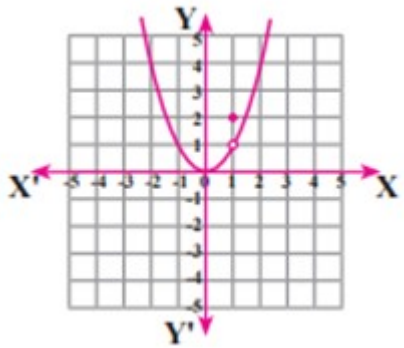
Candidates are required to answer in their own words as far as practicable. The figures in the margin represent the full marks.

Attempt **all** the questions.

Group 'A' $[5 \times (1+1) = 10]$

1. (a) What is the period of the function $f(x) = \sin x$?
   (b) State remainder theorem.

2. (a) State with reason whether the graph of the function is continuous or discontinuous at $x = 1$.



   (b) Write down the condition under which the matrix is singular.

3. (a) What is the condition of perpendicularity of two straight lines $y = m_1x + c_1$ and $y = m_2x + c_2$?
   (b) Name the conic section which is formed when the intersecting plane is parallel to the base of cone ?

4. (a) Express $\sin A$ in terms of $\tan\frac{A}{2}$.
   (b) Write down the product of $2\sin\alpha.\cos\beta$ as the sum or difference of sine or cosine.

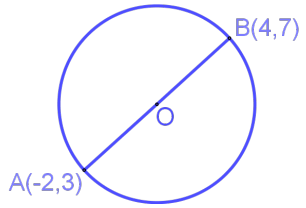
5. (a) TThe position vectors of vertices of a triangle ABC are $\vec{a}, \vec{b}$ and $\vec{c}$. What is the position vector of its centroid G ?
   (b) To what transformation is the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ associated?

Group 'B' $[13 \times 2 = 26]$

6. (a) By using synthetic division method, find the quotient and remainder when $5x^3 + 4x^2 - 8x - 1$ is divided by $(x+2)$.
   (b) If $(x - 2p)$ is one of the factors of $x^3 - 2px^2 - 5x + 4p + 18$, find the value of p.
   (c) Find the co-ordinates of points on x-axis at which graph of parabola $y = x^2 - 4x - 5$ cut off.

7. (a) Show that the matrices $P = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$ and If $Q = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$ are inverse to each other.
   (b) According to Cramer's rule, find the value of D_1 and D_2 for the system of equations $2x - 3y = 5$ and $3x + y = 2$.

8. (a) Find the slopes of lines $4x + 5y + 1 = 0$ and $\frac{x}{10} + \frac{y}{8} = 1$ then write the relation between them.
   (b) Find the equation of circle given alongside.



9. (a) If $\cos 30° = \frac{\sqrt{3}}{2}$, prove that:

$$\cos 15° = \frac{1}{2}\sqrt{2+\sqrt{3}}$$

   (b) Prove that: $\frac{\cos 20° - \cos 80°}{\sin 20° + \sin 80°} = \frac{1}{\sqrt{3}}$
   (c) Solve: $\sqrt{3}\sec\frac{\theta}{3} - 2 = 0$ $[0° \le \theta \le 180°]$

10. (a) If $\overrightarrow{OA} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} k \\ k+2 \end{pmatrix}$ and $\angle AOB = 90°$, find the value of k.
    (b) The position vectors of the points P and Q are $8\vec{i} - \vec{j}$ and $3\vec{j}$ respectively. Find the position vector of point M such that $\overrightarrow{PM} = \overrightarrow{MQ}$.
    (c) In a continuous data, the quartile deviation is 10 and the upper quartile is 50, find the lower quartile and coefficient of the quartile deviation.

Group 'C' [11 × 4 = 44]

11. $f : R \to R$ is defined by $f(x) = 2x - 3$ and $g : R \to R$ is another function such that $(fog)(x) = 8x+7$, find the linear function $g(x)$ and $g^{-1}(-3)$.

12. Optimize the objective function $P(x, y) = 4x + 3y$ subject to the constraints $x + y \geq 4, x - 2y \geq 1, x \geq 0, y \geq 0$.

13. For a real valued function $f(x) = \dfrac{3x+1}{2}$,
    (a) What are the values of $f(x)$ at $x = 2.9, 2.99$ and 2.999 ?
    (b) What are the values of $f(x)$ at $x = 3.1, 3.01$ and 3.001 ?
    (c) Find the values of $f(3)$, $\lim\limits_{x\to 3^-} f(x)$ and $\lim\limits_{x\to 3^+} f(x)$.
    (d) Is this function f continuous at $x = 3$? Give reason

14. By using matrix method, solve the following system of equations:

$$x - 2y = 10 \text{ and } 5x + 3y = 11$$

15. The equations of two diameters of a circle which passes through the point (2, 4) are $x - y = 4$ and $2x + 3y + 7 = 0$. What is the equation of the circle? Find it.

16. Find the value of

$$\left(1 + \cos\frac{\pi^c}{8}\right)\left(1 + \cos\frac{3\pi^c}{8}\right)\left(1 + \cos\frac{5\pi^c}{8}\right)\left(1 + \cos\frac{7\pi^c}{8}\right)$$

17. If A, B and C are the angles of a triangle ABC, prove that:

$$\sin(B+C-A)+\sin(C+A-B)+\sin(A+B-C) = 4\sin A.\sin B.\sin C$$

18. From the top of a tower 60 m high, the angles of depression of the top and bottom of a building are observed to be 30° and 60° respectively. Find the height of the building. Also, find the distance between the tower and the building.

19. The equation of a circle C is $x^2 + y^2-4x-6y = 3$. Find the image of the point (4, 5) with respect to the circle C.

20. Find the mean deviation from mean and its coefficient of the given data.

Height (in cm)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of plants	5	7	11	9	8

21. Find the standard deviation and its coefficient of variation from the given data.:

Marks	0 - 4	4 - 8	8 - 12	12 - 16	16 - 20
Number	4	6	10	7	3

Group 'D' [4 × 5 = 20]

22. 22. The production of mobile sets in a factory increases uniformly by a fixed number every year. It produced 6,500 sets in the third year and 8,500 sets in the seventh year.
    (i) Find the production of mobile sets in the first year.
    (ii) Find the number of mobile sets produced in the 10^{th} year.
    (iii) In how many years, will the factory produce altogether 39,000 mobile sets?

23. While playing a game, a rope is tightly stretched by two students. Likewise, another rope is tightly stretched by other two students so that the rope intersects the former rope at an angle of 45°. If both the ropes is represented by the equation $kx^2+7xy+3y^2 = 0$, find the value of k. Also, find the separate equations of the straight ropes in the same position when the value of k is positive.

24. Prove by vector method that the diagonals of a rectangle are equal.

25. The coordinates of vertices of a quadrilateral $ABCD$ are $A(1, 1), B(2, 3), C(4, 2)$ and $D(3, -2)$. Rotate this quadrilateral about origin through 90° in anti-clockwise direction. Reflect this image of quadrilateral about $y = -x$. Write the name of transformation which denotes the combined transformation of above two transformations.

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