INTERNATIONAL BACCALAUREATE

Mathematics: applications and interpretation

MAI

EXERCISES [MAI 1.15]
TRANSFORMATION MATRICES

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A. Paper 1 questions (SHORT)

1. [Maximum mark: 5]

Let $M = \begin{pmatrix} 2 & 5 \\ 1 & 4 \end{pmatrix}$ be a transformation matrix.

(a) The matrix M maps the point (x, y) to the point (x', y'). Write down two liner equations for x' and y' in terms of x and y. [2]

(b) Find images of the following points

(i) O(0,0) (ii) A(1,1) (iii) B(3,5). [3]

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2. [Maximum mark: 7]

Let $M = \begin{pmatrix} 2 & 5 \\ 1 & 4 \end{pmatrix}$ be a transformation matrix.

(a) Find the inverse transformation matrix M^{-1}. [2]

(b) Find image A' of the point $\mathrm{A}(3,-2)$. [2]

(c) Confirm that the image of point A' under M^{-1} is the point A. [2]

(d) Write down the image of the line segment OA, where O is the origin. [1]

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3. [Maximum mark: 8]

Let $M = \begin{pmatrix} 2 & 5 \\ 1 & 4 \end{pmatrix}$ be a transformation matrix, OAB a triangle with vertices $\mathrm{O}(0,0)$, $\mathrm{A}(4,0)$, $\mathrm{B}(0,1)$ and $\mathrm{OA'B'}$ the image of OAB under M.

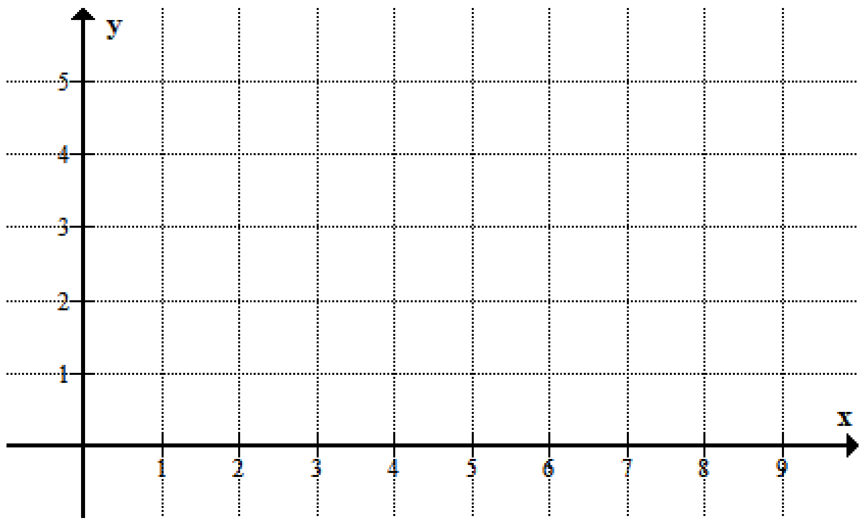
(a) Find the coordinates of the images $\mathrm{A'}$ and $\mathrm{B'}$ [2]

(b) On the diagram below

(i) sketch the triangle OAB. (ii) sketch the image $\mathrm{OA'B'}$. [2]

(c) Find $\det M$. [1]

(d) Find the area of the triangle OAB and **hence** find the area of triangle $\mathrm{OA'B'}$. [3]



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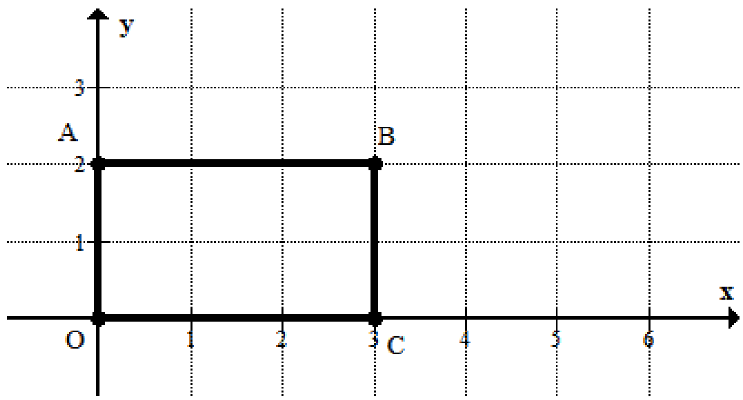
4. [Maximum mark: 12]

The diagram below shows a rectangle OABC of area 6, with vertices

$$\mathrm{O}(0,0),\quad \mathrm{A}(0,2),\quad \mathrm{B}(3,2)\ \text{ and }\ \mathrm{C}(3,0).$$



Each transformation matrix below, applied on ABCD, results to a new rectangle. Describe each transformation, write down the images of the vertices and find the area of the resulting rectangle (as in the first row)

Matrix	Description of transformation	New vertices	Area
$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$	**horizontal stretch** with a scale factor of 3	O(0,0) A(0,2) B(9,2) C(9,0)	18
$\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$			
$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$			
$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$			

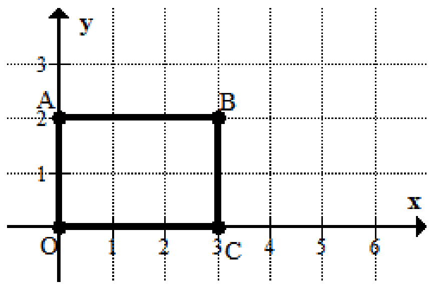
5. [Maximum mark: 7]

(a) Write down the transformation matrices R_{90} and R_{45} which correspond to a clockwise rotation

(i) of angle 90° about the origin. (ii) of angle 45° about the origin. [2]

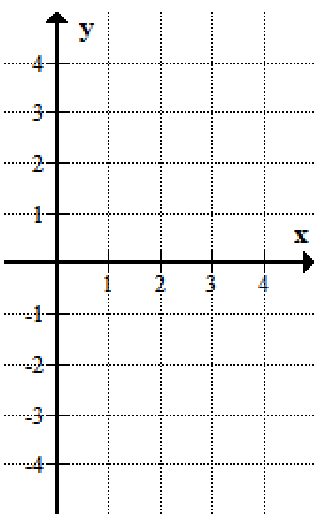
(b) Write down the transformation matrix T corresponding to a reflection in line $y = x$. [2]

The diagram shows a rectangle OABC with vertices $\mathrm{O}(0,0)$, $\mathrm{A}(0,2)$, $\mathrm{B}(3,2)$, $\mathrm{C}(3,0)$.

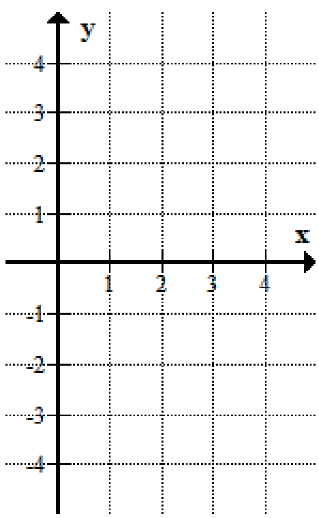


(c) Sketch on the diagrams below the images of the rectangle

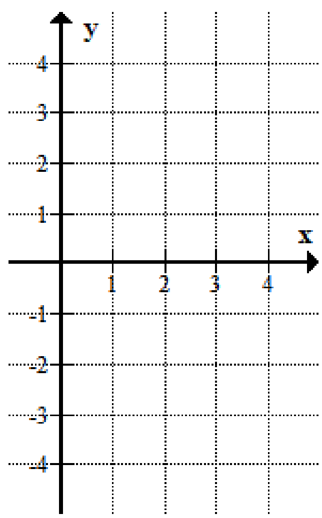
(i) under R_{90}



(ii) under R_{45}



(iii) under T



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6. [Maximum mark: 5]

Let $M = \begin{pmatrix} 2 & 6 \\ 1 & 5 \end{pmatrix}$ be a transformation matrix.

(a) Find $\det M$. [1]

(b) The transformation maps a triangle ABC of area 5 to a triangle $\mathrm{A'B'C'}$. Find the area of the $\mathrm{A'B'C'}$. [2]

(c) The transformation maps a quadrilateral ABCD to a quadrilateral $\mathrm{A'B'C'D'}$ of area 24. Find the area of the ABCD. [2]

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7. [Maximum mark: 6]

Find the transformation matrix which corresponds to

a horizontal stretch with a scale factor of 5;

followed by a reflection in line $y = x$;

followed by a vertical stretch with a scale factor of 2.

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8. [Maximum mark: 6]

(a) Find the transformation matrix A which gives a reflection in line $y = \sqrt{3}x$; [4]

(b) Find the image of point P(0,2) under A. [2]

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9. [Maximum mark: 7]

(a) Find the transformation matrix M which corresponds to

a reflection in line $y = \frac{\sqrt{3}}{3}x$;

followed by a clockwise rotation of angle 30° about the origin. [5]

(b) The resulting transformation matrix M corresponds to a single reflection in line $y = (\tan\theta)x$. Find the value of θ. [2]

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10. [Maximum mark: 6]

The affine transformation T has the form $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(a) Find the image of the line segment PQ where $\mathrm{P}(0,1)$ and $\mathrm{Q}(1,2)$. [4]

(b) On the same diagram sketch the line segments PQ and $\mathrm{P'Q'}$. [2]

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11. [Maximum mark: 4]

An affine transformation has the form $AX + B$, where A is a 2×2 matrix and B is a 2×1 matrix (i.e. a vector). Write down the matrices A and B in each of the following cases

(a) The affine transformation corresponds to a horizontal translation 1 unit to the right and a vertical translation 2 units up. [2]

(b) The affine transformation corresponds to a vertical stretch with a scale factor of 4, followed by a translation by the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$. [2]

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12. [Maximum mark: 7]

Diagram 1 below shows a triangle OPQ with vertices $O(0,0)$, $P(-1,2)$ and $Q(3,-3)$.

Diagram 2 below shows a triangle $OP'Q'$ with vertices O (0.0), $P'(2,0)$) and $Q'(0,3)$.

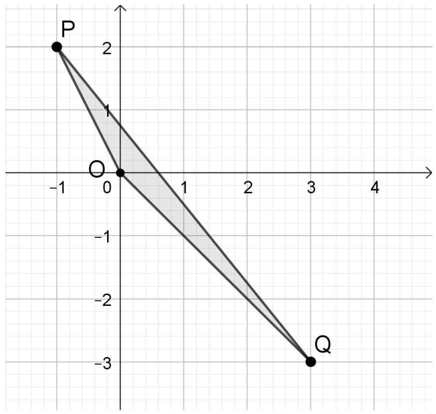


diagram 1

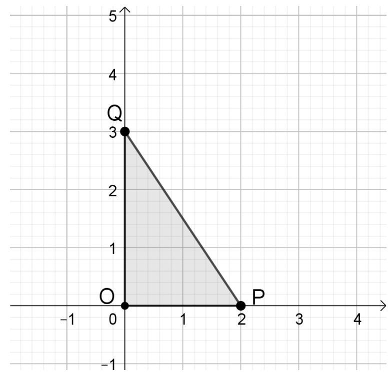


diagram 2

The transformation matrix $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ maps P to P' and Q to Q'.

(a) Find a,b,c and d. [5]

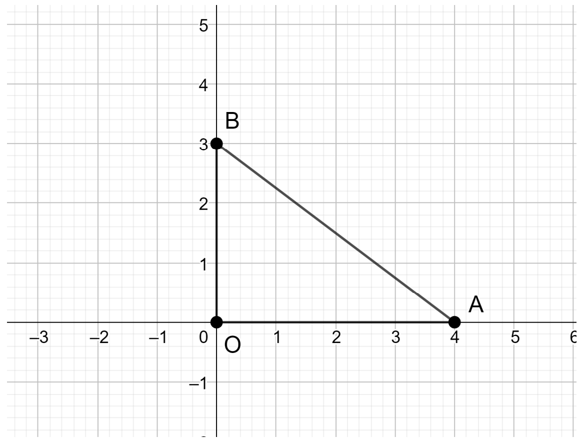
The image of the triangle OPQ under A is the triangle $OP'Q'$.

(b) Find $\det A$ and **hence** find the area of triangle OPQ. [2]

B. Paper 2 questions (LONG)

13. [Maximum mark: 16]

The diagram below shows a triangle OAB with vertices O(0.0), A(4,0) and B(0,3)



(a) Describe the transformation matrix $H = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ and find the images of the points O, A and B. [3]

(b) Write down the transformation matrix V that stretches the triangle OAB vertically by a scale factor of 3. [1]

(c) The transformation matrix R rotates the triangle OAB clockwise by 90°.

(i) Write down the matrix R.

(ii) Sketch the image of the triangle OAB under the matrix transformation R. [4]

(d) Let $P = VRH$, the product of the three matrices described above.

(i) Find P

(ii) Describe the corresponding sequence of transformations in the correct order.

(iii) Sketch the image of the triangle OAB under the transformation matrix P.

(iv) The same result as P can be achieved by the product TR where T is a single 2×2 transformation matrix. Find T. [8]

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14. [Maximum mark: 16]

Complete the table below.

Matrix	**Description**
$\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$	Horizontal stretch with a scale factor of 5
	Vertical stretch with a scale factor of 7
$\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$	
	Clockwise rotation by an angle 60°
$\begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$	
	Reflection in line $y = \sqrt{3}x$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Reflection in line …………
	Reflection in line $y = 2x$
	Clockwise rotation by an angle 20°

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