

[MAA 2.7] ASYMPTOTES

SOLUTIONS

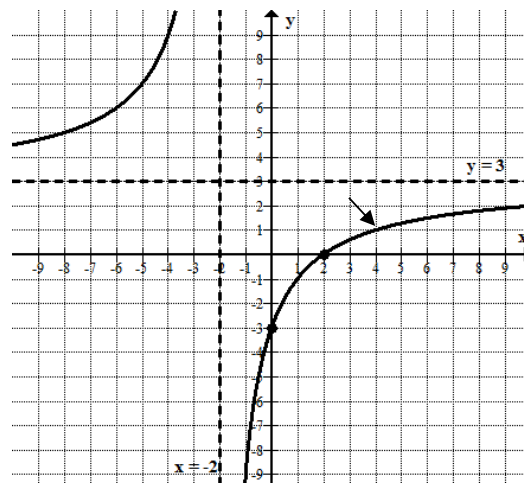
Compiled by: Christos Nikolaidis

O. Practice questions

1.

<i>Function</i>	<i>Vertical asymptote</i>	<i>Horizontal</i>
$f(x) = \frac{2x+7}{x-5}$	$x = 5$	$y = 2$
$f(x) = \frac{x+7}{2x-3}$	$x = \frac{3}{2}$	$y = \frac{1}{2}$
$f(x) = \frac{-4x+1}{2x-6}$	$x = 3$	$y = -2$
$f(x) = \frac{2}{x-5}$	$x = 5$	$y = 0$
$f(x) = \frac{2}{x-5} + 1$	$x = 5$	$y = 1$
$f(x) = \frac{2}{x-5} - 1$	$x = 5$	$y = -1$
$f(x) = \frac{2x+7}{x-5} + 1$	$x = 5$	$y = 3$
$f(x) = \frac{-4x+1}{2x-6} + 3$	$x = 3$	$y = 1$
$f(x) = \frac{5-x}{5+x}$	$x = -5$	$y = -1$
$f(x) = \frac{3x+134}{2x+5}$	$x = -5/2$	$y = 3/2$

2. (a) H.A.: $y = 3$, V.A.: $x = -2$
 (b) x -intercept: $x = 2$, y -intercept: $y = -3$
 (c)



- (d) $x \neq -2$, $y \neq 3$
 (e) $\frac{3x-6}{x+2} = 1 \Leftrightarrow 3x-6 = x+2 \Leftrightarrow x = 4$ so $(4, 1)$

3. (a) H.A.: $y=0$, V.A.: $x=2$
 (b) there is no x -intercept, y -intercept: $y = -\frac{3}{2}$
 (d) $x \neq 2$, $y \neq 0$
4. (a) $a=3$, $b=-6$, $c=2$
 (b) $f(4)=1$, $f^{-1}(1)=4$.
 (c) $-2 < x \leq 4$

5. (a)

$y = f(x) + 2$	$y = 3$	$y = f(x+2)$	$y = 1$
$y = f(x) - 2$	$y = -1$	$y = f(x-2)$	$y = 1$
$y = 2f(x)$	$y = 2$	$y = f(2x)$	$y = 1$
$y = f(x)/2$	$y = 1/2$	$y = f(x/2)$	$y = 1$
$y = -f(x)$	$y = -1$	$y = f(-x)$	$y = 1$

- (b)

$y = f(x) + 2$	$x = 2$	$y = f(x+2)$	$x = 0$
$y = f(x) - 2$	$x = 2$	$y = f(x-2)$	$x = 4$
$y = 2f(x)$	$x = 2$	$y = f(2x)$	$x = 1$
$y = f(x)/2$	$x = 2$	$y = f(x/2)$	$x = 4$
$y = -f(x)$	$x = 2$	$y = f(-x)$	$x = -2$

6. (a) $f(x) = 4 + \frac{5}{x+3} = \frac{4(x+3)}{x+3} + \frac{5}{x+3} = \frac{4x+17}{x+3}$
 (b) $g(x) = \frac{x+10}{x+3} = \frac{x+3+7}{x+3} = 1 + \frac{7}{x+3}$
 (c) $h(x) = \frac{2x+10}{x+3} = \frac{2(x+3)+4}{x+3} = 2 + \frac{4}{x+3}$

- (d)

	f	g	h
Vertical asymptote	$x = -3$	$x = -3$	$x = -3$
Horizontal asymptote	$y = 4$	$y = 1$	$y = 2$

7. (a) H.A.: $y=3$, V.A.: $x=2$
 (b) $\frac{1}{x} \rightarrow \frac{4}{x} \rightarrow \frac{4}{x-2} \rightarrow \frac{4}{x-2} + 3$

Vertical stretch with s.f. 4

Horizontal translation 2 units to the right

Vertical translation 3 units up

8. (a) H.A.: $y=3$, V.A.: $x=2$
- (b) $h(x) = \frac{3x+1}{x-2} = \frac{3(x-2)+7}{x-2} = 3 + \frac{7}{x-2}$

$$\frac{1}{x} \rightarrow \frac{7}{x} \rightarrow \frac{7}{x-2} \rightarrow \frac{7}{x-2} + 3$$

Vertical stretch with s.f. 7

Horizontal translation 2 units to the right

Vertical translation 3 units up

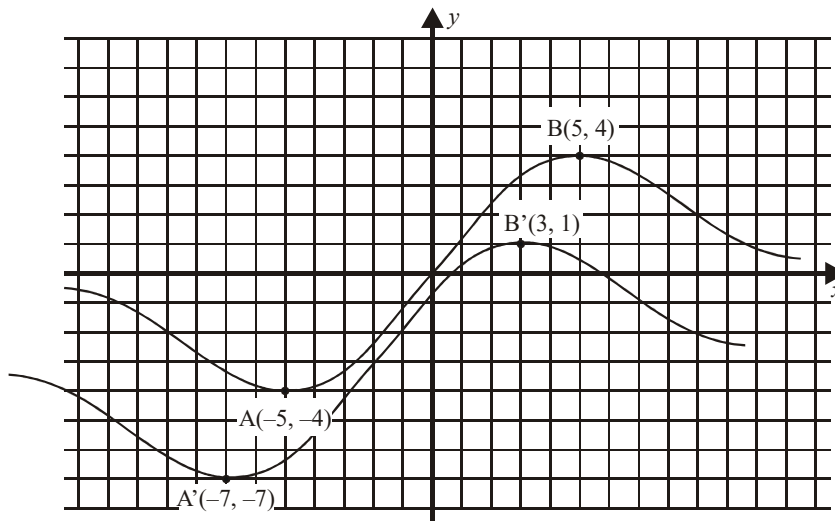
9. (a) $y=3$
- (b) $x=2, x=5$
- (c)

(1) $\lim_{x \rightarrow +\infty} f(x) = 1$	(4) $\lim_{x \rightarrow -\infty} f(x) = 1$
(2) $\lim_{x \rightarrow 2^-} f(x) = -\infty$	(5) $\lim_{x \rightarrow 2^+} f(x) = +\infty$
(3) $\lim_{x \rightarrow 5^-} f(x) = -\infty$	(6) $\lim_{x \rightarrow 5^+} f(x) = +\infty$

- (d) (i) (1) and (4) (ii) (2) and (5) for $x=2$ and (3) and (6) for $x=5$

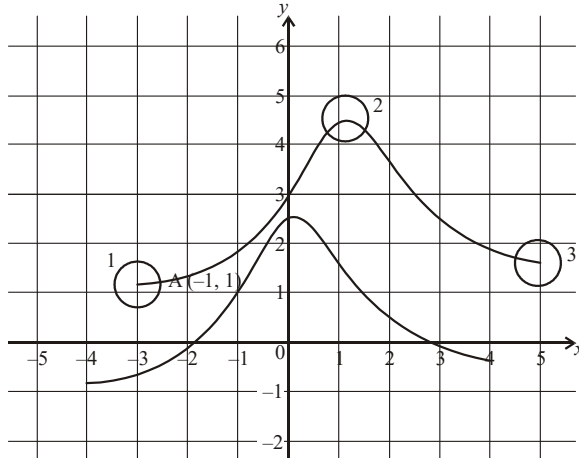
A. Past paper questions (SHORT)

10. (a) $x \neq 2, y \neq 3$
- (b) H.A.: $y=3$, V.A.: $x=2$
- (c) x -intercept $(-1/6, 0)$, y -intercept $(0, -1/4)$
11. (a) $x \neq 2, y \neq 0$
- (b) H.A.: $y=0$, V.A.: $x=2$
- (c) there is no x -intercept, y -intercept $(0, -3/2)$
12. (a)



- (b) (i) Asymptote: $y=0$, (ii) Asymptote: $y=-3$, (iii) Asymptote: $y=-3$

13. (a)

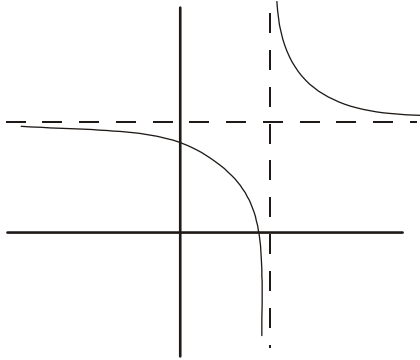


Notes: left and right end points in circles 1,3, max in circle 2

- (b) $y = 1$ (must be an equation)
 (c) $(0, 3)$
 (d) (i) Asymptote: $y = -1$, (ii) Asymptote: $y = -2$

14. (a) (i) $p = 2$
 (ii) $10 = \frac{q}{3-2}$ (or equivalent), so $q = 10$
 (b) Reflection, in x -axis

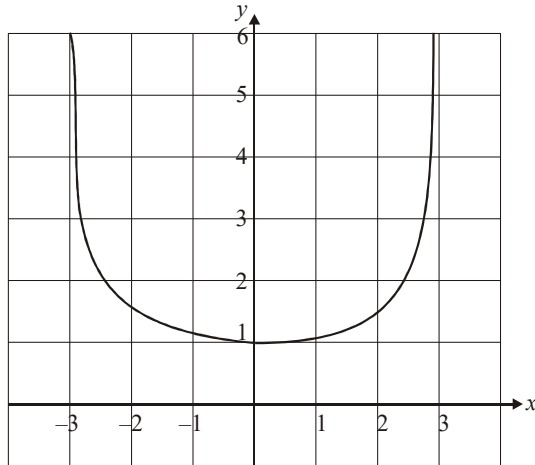
15. (a)



- (b) (i) $y = 3, x = \frac{5}{2}$ (must be equations)
 (ii) $x = \frac{14}{6}$ $\left(\frac{7}{3}$ or 2.33, also accept $\left(\frac{14}{6}, 0\right)\right)$
 (iii) $y = \frac{14}{6}$ ($y = 2.8$) $\left(\text{accept}\left(0, \frac{14}{5}\right)\text{ or } (0, 2.8)\right)$

16. (i) $p = 2$
 (ii) $q = 1$

17. (a)



Note: the y-intercept at 1.

(b) $x = 3, x = -3$

(c) $y \geq 1$

B. Exam style questions (LONG)

18. (a) (i) $f(x) = \frac{2x+1}{x-3} = 2 + \frac{7}{x-3}$ or $f(x) = \frac{2x+1}{x-3} = \frac{2 + \frac{1}{x}}{1 - \frac{3}{x}}$

Therefore as $|x| \rightarrow \infty f(x) \rightarrow 2 \Rightarrow y = 2$ is an asymptote

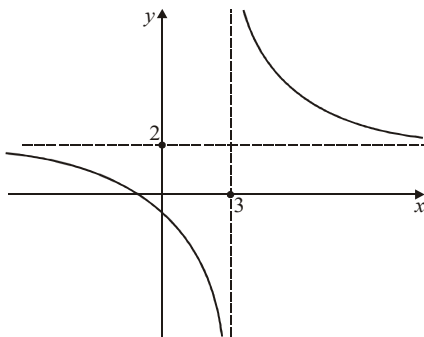
OR $\lim_{x \rightarrow \infty} \frac{2x+1}{x-3} = 2 \Rightarrow y = 2$ is an asymptote

(ii) Asymptote at $x = 3$

(iii) $P(3, 2)$

(b) $f(x) = 0 \Rightarrow x = -\frac{1}{2} \left(-\frac{1}{2}, 0\right)$ $x = 0 \Rightarrow f(x) = -\frac{1}{3} \left(0, -\frac{1}{3}\right)$

(c)



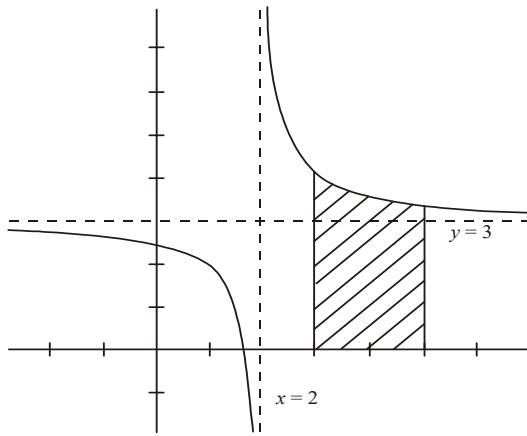
19. (a) solve for x in terms of y : $x = \frac{y+5}{3}$

$$f^{-1}(x) = \frac{x+5}{3}$$

(b) $(g^{-1} \circ f)(x) = (3x-5) + 2 = 3x-3$

(c) $\frac{x+3}{3} = 3x-3$ ($x+3 = 9x-9$) so $x = \frac{12}{8}$

(d) (i) $h(x) = \frac{3x-5}{x-2}$



(ii) (Vertical asymptote) $x = 2$, (Horizontal asymptote) $y = 3$

(e) (i) $\frac{3x-5}{x-2} = y \Leftrightarrow 3x-5 = yx-2y \Leftrightarrow x(y-3) = 2y-5 \Leftrightarrow x = \frac{2y-5}{y-3}$

$$h^{-1}(x) = \frac{2x-5}{x-3}$$

(ii) (Vertical asymptote) $x = 3$, (Horizontal asymptote) $y = 2$

20. (a) $x_1 = -0.790$ and $x_1 = 1.79$

(b) (i) $x \neq -0.790$ $x \neq 1.79$ (ii) $y \in \mathbb{R}$

(c) (i) $a = -0.790$ (ii) $b = 1.79$

(d) When x is large,

the value of $g(x)$ becomes much larger than the value of $2x^3$.

As a consequence, the value of $\frac{2x^3}{g(x)}$ approaches 0.

Thus $f(x)$ approaches 1.

(e) (i) At A, $x = -1$ (ii) At B, $x = 1$