



Rules of Differentiation- Implicit Differentiation

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I. For each of the following equations, find $\frac{dy}{dx}$ using implicit differentiation.

1) $y^3 + y^2 - 7y - x^2 = 9$ $y' = \frac{2x}{3y^2 + 2y - 7}$

2) $x^3 + y^3 = 27$ $y' = \frac{-x^2}{y^2}$

3) $x^3 - 2xy + y^2 = 8$ $y' = \frac{3x^2}{2x + y}$

4) $2x^3 - y^2 = 8x + y$ $y' = \frac{(24 + 1)}{2(3x^2 + 1)}$

5) $x^2y + 3y^2x = -3$ $y' = \frac{-y(3y + 2x)}{x(x + 6y)}$

6) $2x^3y^3 - 5x = 2y$ $y' = \frac{-6x^2 + 5}{6x^3y^2 - 2}$

7) $e^{\sin(2y)} = 2x^2$ $y' = 2xe^{-\sin(2y)} \sec(2y)$

8) $3x^2 + \sin(y) = 5y^2$ $y' = 10y - 6x + \cos(y)$

II. Use implicit differentiation to find $\frac{dy}{dx}$, and find the value of the derivative at the indicated point.

9) $4y^3 - 9y = 5x$ at $(0, \frac{3}{2})$ $y' = \frac{5}{18}$

10) $\sqrt{x} + \sqrt{y} = 5$ at $(4, 9)$ $y' = -\frac{5}{12}$

Implicit Differentiation

1) $y^3 + y^2 - 7y - x^2 = 9$

$$(3y^2 \cdot y') + (2y \cdot y') - (7 \cdot y') - 2x = 0$$

$$(3y^2 \cdot y') + (2y \cdot y') - (7 \cdot y') = 2x$$

$$y' = \frac{2x}{3y^2 + 2y - 7} = \frac{-2(-6)(86)}{3(3 \cdot 86)^2 + 2(3 \cdot 86) - 7}$$

$$y' = -0.2056$$

6) $2x^3y^3 - 5x = 2y$

$$(6x^2 \cdot y^3) + (3y^2 \cdot y' \cdot 2x^3) - 5 = 2y'$$

$$(3y^2 \cdot y' \cdot 2x^3) + 2y' = 6x^2y^3 + 5$$

$$y' = (3y^2 \cdot 2x^3 - 2) = (-6x^2y^3) + 5$$

$$y' = \frac{-6x^2y^3 + 5}{6x^3y^2 - 2}$$

$$y' = \frac{-6x^2y^3 + 5}{6x^3y^2 - 2}$$

2) $x^3 + y^3 = 27$

$$3x^2 + 3y^2 \cdot y' = 0$$

$$3y^2 \cdot y' = -3x^2$$

$$y' = \frac{-3x^2}{3y^2} = \frac{-x^2}{y^2}$$

7) $e^{\sin(2y)} = 2x^2$

$$2y' \cos(2y) e^{\sin(2y)} = 4x$$

$$2e^{\sin(2y)} \cos(2y) y' = 4x$$

$$y' = \frac{2xe^{-\sin(2y)} \sec(2y)}{2}$$

8) $3x^2 + \sin(y) = 5y^2$

$$6x + y' \cos(y) = 10y$$

$$y' \cos(y) = 10y - 6x$$

$$y' = \frac{10y - 6x + \sec(y)}{\cos(y)}$$

5) $x^3 - 2xy + y^2 = 8$

$$3x^2 - (2x \cdot y') + (-2 \cdot y) + 2y = 0$$

$$-2xy' = -3x^2 + 2y - 2y$$

$$y' = \frac{-3x^2}{-2xy} = \frac{3x^2}{2xy}$$

4) $2x^3 - y^2 = 8x + y$

$$6x^2 - (2y \cdot y') = 8 + y'$$

$$6x^2 - 8 = (2y \cdot y') + y'$$

$$2(3x^2 - 4) = (2y + 1)y'$$

$$y' = \frac{(2y + 1)}{2(3x^2 - 4)}$$

$$y' = \frac{(2y + 1)}{2(3x^2 - 4)}$$

9) $4y^3 - 9y = 5x$ at $(0, \frac{3}{2})$

$$(12y^2 \cdot y') - 9y' = 5$$

$$y'(12y^2 - 9) = 5$$

$$y' = \frac{5}{12y^2 - 9} = \frac{5}{12(\frac{3}{2})^2 - 9}$$

$$y' = \frac{5}{18}$$

5) $x^2y + 3y^2x = -3$

$$(2x \cdot y) + (x^2 \cdot y') + (6y \cdot y' \cdot x) + (3y^2) = 0$$

$$(x^2 \cdot y') + (6y \cdot y' \cdot x) = -3y^2 - 2xy$$

$$y'(x^2 + 6yx) = -3y^2 - 2xy$$

$$y' = \frac{-3y^2 - 2xy}{(x^2 + 6yx)} = \frac{-y(3y + 2x)}{x(x + 6y)}$$

$$y' = \frac{-y(3y + 2x)}{x(x + 6y)}$$

10) $\sqrt{x} + \sqrt{y} = 5$ at $(4, 9)$

$$(x)^{1/2} + (y)^{1/2} = 5$$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2} \cdot y' = 0$$

$$y' = -\frac{1}{2}x^{-1/2} - \frac{1}{2}y^{-1/2}$$

$$y' = -\frac{1}{2}(4)^{-1/2} - \frac{1}{2}(9)^{-1/2}$$

$$y' = -\frac{1}{4} - \frac{1}{6} = -\frac{3}{12} - \frac{2}{12}$$

$$y' = -\frac{5}{12}$$