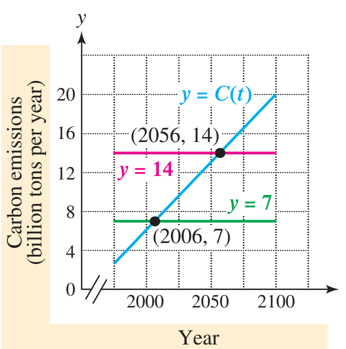
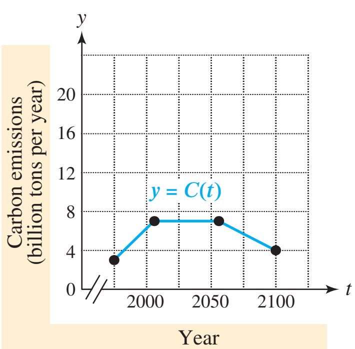
2 Linear Functions and Equations

For over two centuries people have been transferring carbon from below the surface of the earth into the atmosphere. Today, the burning of coal, oil, and natural gas releases 7 billion tons of carbon into the atmosphere each year. If the current rate of growth continues, the amount could double to 14 billion tons by 2056. This increase is modeled by a linear function C in the figure on the left below. The horizontal line $y = 7$ represents the 2006 level of emissions, and the horizontal line $y = 14$ represents a doubling of carbon emissions. Their points of intersection with the graph of C represent when these levels of emission could occur. The figure on the right illustrates what might happen if levels of carbon emission could be held at the 2006 rate of 7 billion tons per year for the next 50 years. In this case, emissions are expected to decline after 50 years. This graph, made up of line segments, is called a *piecewise-linear function*.

The human mind has never invented a labor-saving device greater than algebra.
—J. W. Gibbs



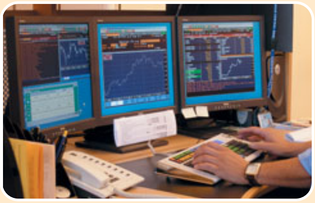


Whatever your point of view, mathematics plays an essential role in understanding the future of carbon emissions; without mathematical support, predictions lack credibility. To model carbon emissions, we need constant, linear, and piecewise-defined functions. All of these important concepts are discussed in this chapter.

Source: R. Socolow and S. Pacala, "A Plan to Keep Carbon in Check," *Scientific American*, September, 2006.

2.1 Linear Functions and Models

- **Understand how functions can be models**
- **Identify a graph or table of a linear function**
- **Model data with a linear function**
- **Evaluate and graph piecewise-defined functions**
- **Evaluate and graph the greatest integer function**
- **Use linear regression to model data (optional)**



Introduction

Throughout history, people have attempted to explain the world around them by creating models. A model is based on observations. It can be a diagram, a graph, an equation, a verbal expression, or some other form of communication. Models are used in diverse areas such as economics, physics, chemistry, astronomy, religion, and mathematics. Regardless of where it is used, a **model** is an *abstraction* with the following two characteristics:

1. A model is able to explain present phenomena. It should not contradict data and information already known to be correct.
2. A model is able to make predictions about data or results. It should use current information to forecast phenomena or create new information.

Mathematical models are used to forecast business trends, design the shapes of cars, estimate ecological trends, control highway traffic, describe epidemics, predict weather, and discover new information when human knowledge is inadequate.

Functions as Models

A function can sometimes be a model. Worldwide, people purchased and downloaded 420 million tracks of music in 2005, and this number increased to 795 million in 2007. (**Source:** International Federation of the Phonographic Industry.) If $t = 0$ corresponds to 2005, $t = 1$ to 2006, and so on, then the linear function D defined by $D(t) = 187.5t + 420$ accurately *models* these *known* data values because

$$D(0) = 187.5(0) + 420 = 420 \quad \text{and} \quad D(2) = 187.5(2) + 420 = 795.$$

We might use function D as a model to *predict* the downloads in 2006 by evaluating $D(1)$.

$$D(1) = 187.5(1) + 420 = 607.5$$

Thus D estimates that **607.5** million tracks of music were downloaded in 2006. Because 2006 is between 2005 and 2007, this calculated value is likely to be more accurate than if we used D to estimate the number of downloads many years into the future or the past. For example, to predict downloads in 2011 we might let $t = 6$ $(2005 + 6 = 2011)$, and to predict downloads in 2001 we might let $t = -4$ $(2005 - 4 = 2001)$.

$$D(6) = 187.5(6) + 420 = 1545$$
$$D(-4) = 187.5(-4) + 420 = -330$$

This model predicts that **1545** million tracks will be downloaded in 2011, which may or may not be correct. However, this model also estimates that **−330** million tracks were downloaded in 2001, which is clearly incorrect.

Estimating values between data points, such as for 2006, is called **interpolation**, and estimating values "outside" of the given data points, such as for 2001 or 2011, is called **extrapolation**. Interpolation tends to be more accurate than extrapolation. Typically, for a model to be accurate, it must have limits on its domain. For example, it might be reasonable to limit the domain of D to $t = 0, 1, 2, 3,$ or 4.

Representations of Linear Functions

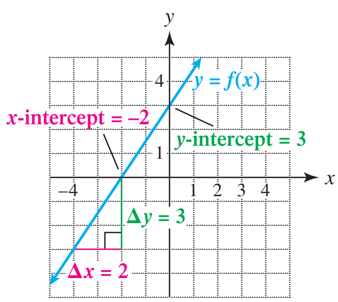


Figure 2.1 Linear Function

Any linear function can be written as $f(x) = ax + b$, where a equals the slope of the graph of f. Also, because $f(\mathbf{0}) = a(\mathbf{0}) + b = \boldsymbol{b}$, the point $(\mathbf{0}, \boldsymbol{b})$ lies on the graph of f and the value of b is the ***y*-intercept** of the graph of f.

Consider the graph of the linear function f shown in Figure 2.1. The graph is a line that intersects each axis once. From the graph we can see that when y increases by 3 units, x increases by 2 units. Thus the change in y is $\Delta y = 3$, the change in x is $\Delta x = 2$, and the slope is $\frac{\Delta y}{\Delta x} = \frac{3}{2}$. The graph f intersects the y-axis at the point $(0, 3)$, and so the y-intercept is 3.

Using this information, we can write a formula, or symbolic representation, of f as

$$f(x) = \underbrace{\frac{3}{2}}_{\text{slope}}x + \underbrace{3}_{y\text{-intercept}}.$$

A function can have at most one y-intercept because $f(0)$ can have at most one value.

The graph of f in Figure 2.1 intersects the x-axis at the point $(-2, 0)$. We say that the ***x*-intercept** on the graph of f is $\mathbf{-2}$. When we evaluate $f(-2)$, we obtain

$$f(\mathbf{-2}) = \frac{3}{2}(\mathbf{-2}) + 3 = \mathbf{0}.$$

An x-intercept corresponds to an input that results in an output of 0. We also say that -2 is a *zero* of the function f, since $f(\mathbf{-2}) = \mathbf{0}$. A **zero** of a function f corresponds to an x-intercept on the graph of f. If the slope of the graph of a linear function f is not 0, then the graph of f has exactly one x-intercept.

A table of values, or numerical representation, for $f(x) = \frac{3}{2}x + 3$ is shown in Table 2.1. Note that for every 1-unit increase in x, $f(x)$ increases by $\frac{3}{2}$, or 1.5, units.

Table 2.1 $f(x) = \frac{3}{2}x + 3$

x	−2	−1	0	1	2
$f(x)$	0	1.5	3	4.5	6

1.5 1.5 1.5 1.5

In general, for each 1-unit increase in x, a linear function g, given by $g(x) = ax + b$, changes by a units. When $a > 0$ the graph of g is increasing for all real numbers x, and when $a < 0$ the graph of g is decreasing for all real numbers x. Linear functions either *always increase* or *always decrease* on their domains, provided $a \neq 0$. If $a = 0$, then g is a constant function.

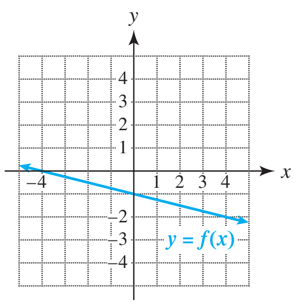


Figure 2.2

EXAMPLE 1 Finding a formula from a graph

Use the graph of a linear function f in Figure 2.2 to complete the following.

(a) Find the slope, y-intercept, and x-intercept.

(b) Write a formula for f.

(c) Find any zeros of f.

SOLUTION

(a) The line **falls 1** unit each time the x-values increase by **4** units. Therefore the slope is $-\frac{1}{4}$. The graph intersects the y-axis at the point $(0, -1)$ and intersects the x-axis at the point $(-4, 0)$. Therefore the y-intercept is -1, and the x-intercept is -4.

(b) Because the slope is $-\frac{1}{4}$ and the y-intercept is -1, it follows that

$$f(x) = -\frac{1}{4}x - 1.$$

(c) Zeros of f correspond to x-intercepts, so the only zero is -4.

Now Try Exercise 13 ◀

Modeling with Linear Functions

Linear functions can be used to model things that change at a constant rate. For example, the distance traveled by a car can be modeled by a linear function *if* the car is traveling at a constant speed.

Modeling with a Linear Function

To model a quantity that is changing at a constant rate with $f(x) = ax + b$, the following formula may be used.

$$f(x) = (\text{constant rate of change})\, x + (\text{initial amount})$$

The constant rate of change corresponds to the slope of the graph of f, and the initial amount corresponds to the y-intercept.

This method is illustrated in the next two examples.

EXAMPLE 2 Writing formulas for functions

Write the formula for a linear function that models each situation. Choose both an appropriate name and an appropriate variable for the function. State what the input variable represents and the domain of the function.

(a) In 2006 the average cost of attending a private college was \$30,000, and it is projected to increase, on average, by \$1750 per year until 2010. (**Source:** CNNMoney.com.)

(b) A car's speed is 50 miles per hour, and it begins to slow down at a constant rate of 10 miles per hour each second.

SOLUTION

(a) **Getting Started** To model cost with a linear function, we need to find two quantities: the initial amount and the rate of change. In this example the initial amount is \$30,000 and the rate of change is \$1750 per year. ▶

Let C be the name of the function and x be the number of years after 2006. Then

$$\begin{aligned} C(x) &= (\textbf{constant rate of change})x + (\textbf{initial amount}) \\ &= \mathbf{1750}x + \mathbf{30{,}000} \end{aligned}$$

models the cost in dollars of attending a private college x years after 2006. Because this projection is valid only until 2010, or for 4 years past 2006, the domain D of function C is

$$D = \{x \mid x = 0, 1, 2, 3, \text{ or } 4\}.$$

Note that x represents a year, so it may be most appropriate to restrict the domain to integer values for x.

(b) Let S be the name of the function and t be the elapsed time in seconds that the car has been slowing down. Then

$$\begin{aligned} S(t) &= (\textbf{constant rate of change})t + (\textbf{initial speed}) \\ &= \mathbf{-10}t + \mathbf{50} \end{aligned}$$

models the speed of the car after an elapsed time of t seconds. Because the car's initial speed is **50** miles per hour and it **slows at 10** miles per hour per second, the car can slow down for at most 5 seconds before it comes to a stop. Thus the domain D of S is

$$D = \{t \mid 0 \le t \le 5\}.$$

Note that t represents time in seconds, so t does not need to be restricted to an integer.

Now Try Exercises 49 and 51 ◀

EXAMPLE 3 Finding a symbolic representation

A 100-gallon tank, initially full of water, is being drained at a rate of 5 gallons per minute.

(a) Write a formula for a linear function f that models the number of gallons of water in the tank after x minutes.

(b) How much water is in the tank after 4 minutes?

(c) Graph f. Identify the x- and y-intercepts and interpret each.

(d) Discuss the domain of f.

SOLUTION

(a) The amount of water in the tank is *decreasing* at 5 gallons per minute, so the constant rate of change is -5. The initial amount of water is 100 gallons.

$$\begin{aligned} f(x) &= (\textbf{constant rate of change})x + (\textbf{initial amount}) \\ &= \mathbf{-5}x + \mathbf{100} \end{aligned}$$

(b) After 4 minutes the tank contains $f(4) = -5(4) + 100 = 80$ gallons.

(c) Since $f(x) = -5x + 100$, the graph has y-intercept 100 and slope -5, as shown in Figure 2.3. The x-intercept is 20, which corresponds to the time in minutes that it takes to empty the tank. The y-intercept corresponds to the gallons of water initially in the tank.

(d) From the graph we see that the domain of f must be restricted to $0 \le x \le 20$. For example, 21 is not in the domain of f because $f(21) = -5(21) + 100 = -5$; the tank cannot hold -5 gallons. Similarly, -1 is not in the domain of f because $f(-1) = -5(-1) + 100 = 105$; the tank holds *at most* 100 gallons.

Now Try Exercise 55 ◀

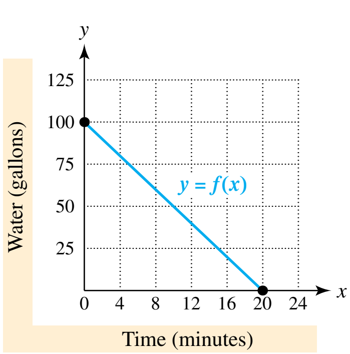


Figure 2.3

If the slopes between consecutive pairs of data points are always the same, the data can be modeled exactly by a linear function. If the slopes between consecutive pairs of data points are nearly the same, then the data can be modeled approximately by a linear function. In the next example we model data approximately.

EXAMPLE 4 Modeling airliner CO_2 emissions

Airliners emit carbon dioxide into the atmosphere when they burn jet fuel. Table 2.2 shows the *average* number y of pounds of carbon dioxide (CO_2) emitted by an airliner for each passenger who flies a distance of x miles.

Table 2.2 Carbon Dioxide Emissions

x (miles)	240	360	680	800
y (pounds)	150	230	435	510

Source: E. Rogers and T. Kostigen, *The Green Book.*

(a) Calculate the slopes of the line segments that connect consecutive data points.
(b) Find a linear function f that models the data.
(c) Graph f and the data. What does the slope of the graph of f indicate?
(d) Calculate $f(1000)$ and interpret the result.

SOLUTION

(a) The slopes of the lines passing through the points (240, 150), (360, 230), (680, 435), and (800, 510) are as follows:

$$m_1 = \frac{230 - 150}{360 - 240} \approx 0.67, \quad m_2 = \frac{435 - 230}{680 - 360} \approx 0.64, \quad \text{and}$$

$$m_3 = \frac{510 - 435}{800 - 680} \approx 0.63.$$

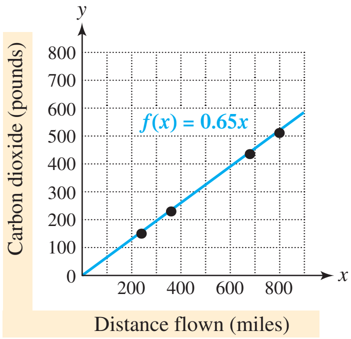


Figure 2.4

(b) Getting Started A linear function can be written as $f(x) = ax + b$. We need to estimate values for a and b. One possibility for a is to find the average of m_1, m_2, and m_3. The value of b equals the y-intercept. ▶

The average of 0.67, 0.64, and 0.63 is 0.65, rounded to the nearest hundredth. Because traveling 0 miles produces 0 pounds of carbon dioxide, let the graph of f pass through (0, 0). Thus the y-intercept is 0 and $f(x) = 0.65x + 0$, where $a = 0.65$ and $b = 0$. Note that answers may vary slightly.

(c) A graph of the four data points and $f(x) = 0.65x$ is shown in Figure 2.4. The slope of 0.65 indicates that, on average, 0.65 pound of carbon dioxide is produced for each mile that a person travels in an airliner.

(d) $f(1000) = 0.65(1000) = 650$; thus 650 pounds of carbon dioxide are emitted into the atmosphere, on average, when a person flies 1000 miles.

Now Try Exercise 61 ◀

Calculator Help

To make a scatterplot, see Appendix A (page AP-3). To plot data and graph an equation in the same viewing rectangle, see Appendix A (page AP-7).

MAKING CONNECTIONS

Slope and Approximately Linear Data Another way to obtain an initial value for a is to calculate the slope between the first and last data point in the table. The value for a can then be adjusted visually by graphing f and the data. In Example 4 this would have resulted in

$$a = \frac{510 - 150}{800 - 240} \approx 0.64,$$

which compares favorably with our decision to let $a = 0.65$.

Piecewise-Defined Functions

When a function f models data, there may not be one formula for $f(x)$ that works. In this case, the function is sometimes defined on pieces of its domain and is therefore called a **piecewise-defined function**. If each piece is linear, the function is a **piecewise-linear function**. An example of a piecewise-defined function is the *Fujita scale,* which classifies tornadoes by intensity. If a tornado has wind speeds between 40 and 72 miles per hour, it is an F1 tornado. Tornadoes with wind speeds greater than 72 miles per hour but not more than 112 miles per hour are F2 tornadoes. The Fujita scale is represented by the following function F, where the input x represents the maximum wind speed of a tornado and the output is the F-scale number from 1 to 5.

$$F(x) = \begin{cases} 1 & \text{if } 40 \le x \le 72 \\ 2 & \text{if } 72 < x \le 112 \\ 3 & \text{if } 112 < x \le 157 \\ 4 & \text{if } 157 < x \le 206 \quad \leftarrow F(180) = 4 \\ 5 & \text{if } 206 < x \le 260 \end{cases}$$

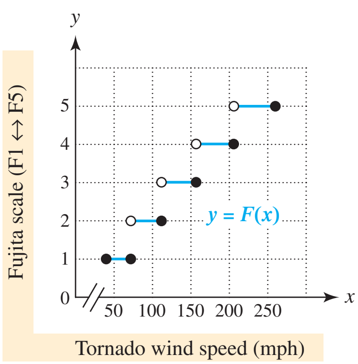


Figure 2.5

For example, if the maximum wind speed is 180 miles per hour, then $F(180) = 4$ because **180** is between **157** and **206**; that is, **157** $<$ **180** $\le$ **206**. Thus a tornado with a maximum wind speed of 180 miles per hour is an F4 tornado.

A graph of $y = F(x)$ is shown in Figure 2.5. It is composed of horizontal line segments. Because each piece is constant, F is sometimes called a **piecewise-constant function** or a **step function**. A solid dot occurs at the point (72, 1) and an open circle occurs at the point (72, 2), because technically a tornado with 72-mile-per-hour winds is an F1 tornado, not an F2 tornado. An open circle indicates that the point is *not* included in the graph of F.

You can draw the graph of a continuous function without picking up your pencil. Because there are breaks in the graph of F, function F is not continuous; rather, it is **discontinuous** at $x = 72, 112, 157$, and 206.

EXAMPLE 5 Evaluating a graphical representation

Figure 2.6 depicts a graph of a piecewise-linear function f. It models the amount of water in thousands of gallons in a swimming pool after x hours have elapsed.

(a) Use the graph to evaluate $f(0)$, $f(25)$, and $f(40)$. Interpret the results.

(b) Discuss how the amount of water in the pool changed. Is f a continuous function on the interval [0, 50]?

(c) Interpret the slope of each line segment in the graph.

(d) Identify where f is increasing, decreasing, or constant.

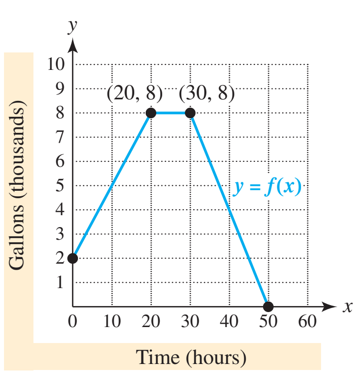


Figure 2.6 Water in a Pool

SOLUTION

(a) From Figure 2.6, when $x = 0$, $y = 2$, so $f(0) = 2$. Initially, there are 2 thousand gallons of water in the pool. The points (25, 8) and (40, 4) lie on the graph of f. Therefore $f(25) = 8$ and $f(40) = 4$. After 25 hours there were 8 thousand gallons in the pool, and after 40 hours there were 4 thousand gallons.

(b) During the first 20 hours, water in the pool increased at a constant rate from 2 thousand to 8 thousand gallons. For the next 10 hours, the water level was constant. Between 30 and 50 hours, the pool was drained at a constant rate. Since there are no breaks in the graph, f is a continuous function on the interval [0, 50].

(c) Slope indicates the rate at which water is entering or leaving the pool. The first line segment connects the points (0, 2) and (20, 8). Its slope is $m = \frac{8 - 2}{20 - 0} = 0.3$. Since the units are thousands of gallons and hours, water is entering the pool during this time at 300 gallons per hour. Between 20 and 30 hours the slope of the line segment is 0. Water is neither entering nor leaving the pool. The slope of the line segment connecting the points (30, 8) and (50, 0) is $m = \frac{0 - 8}{50 - 30} = -0.4$. On this interval water is *leaving* the pool at 400 gallons per hour.

(d) Function f is increasing for $0 \le x \le 20$, decreasing for $30 \le x \le 50$, and constant for $20 \le x \le 30$. These intervals correspond to when the water level in the pool is increasing, decreasing, or remaining constant. ***Now Try Exercise 67***

EXAMPLE 6 Evaluating and graphing a piecewise-defined function

Use $f(x)$ to complete the following.

$$f(x) = \begin{cases} x - 1 & \text{if } -4 \le x < 2 \\ -2x & \text{if } \quad 2 \le x \le 4 \end{cases}$$

(a) What is the domain of f? **(b)** Evaluate $f(-3)$, $f(2)$, $f(4)$, and $f(5)$.
(c) Sketch a graph of f. **(d)** Is f a continuous function on its domain?

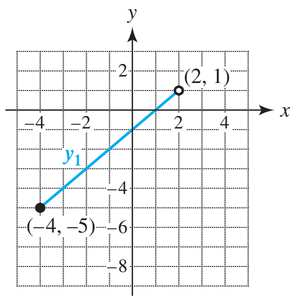


Figure 2.7

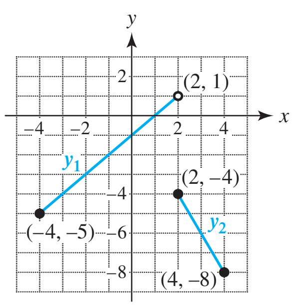


Figure 2.8

SOLUTION

(a) Function f is defined for x-values satisfying either $-4 \le x < 2$ or $2 \le x \le 4$. Thus the domain of f is $D = \{x \mid -4 \le x \le 4\}$, or $[-4, 4]$.

(b) For x-values satisfying $-4 \le x < 2$, $f(x) = x - 1$ and so $f(-3) = -3 - 1 = -4$. Similarly, if $2 \le x \le 4$, then $f(x) = -2x$. Thus $f(2) = -2 \cdot 2 = -4$ and $f(4) = -2 \cdot 4 = -8$. The expression $f(5)$ is undefined because 5 is not in the domain of f.

(c) **Getting Started** Because each piece of $f(x)$ is linear, the graph of $y = f(x)$ consists of two line segments. Therefore we can find the endpoints of each line segment and then sketch the graph. ▶

The first piece is $y_1 = x - 1$ for $-4 \le x < 2$. If $x = -4$, then $y_1 = -5$, and if $x = 2$, then $y_1 = 1$. Thus the endpoints for the first line segment are $(-4, -5)$ and $(2, 1)$. Plot a solid dot at $(-4, -5)$ because $(-4, -5)$ is included, and an open circle at $(2, 1)$, because $(2, 1)$ is *not included* in this piece of the function. Sketch a line segment between these points, as shown in Figure 2.7.

The second piece is $y_2 = -2x$ for $2 \le x \le 4$. If $x = 2$, then $y_2 = -4$, and if $x = 4$, then $y_2 = -8$. Thus the endpoints are $(2, -4)$ and $(4, -8)$. Plot a solid dot for each and connect these points with a line segment, as shown in Figure 2.8.

(d) The function f is not continuous because there is a break in its graph at $x = 2$.

Now Try Exercise 71

The Greatest Integer Function

A common piecewise-defined function used in mathematics is the greatest integer function, denoted $f(x) = [\![x]\!]$. The **greatest integer function** is defined as follows.

$[\![x]\!]$ is the greatest integer less than or equal to x.

Some examples of the evaluation of $[\![x]\!]$ include

$$[\![6.7]\!] = 6, \quad [\![3]\!] = 3, \quad [\![-2.3]\!] = -3, \quad [\![-10]\!] = -10, \quad \text{and} \quad [\![-\pi]\!] = -4.$$

The graph of $y = [\![x]\!]$ is shown in Figure 2.9 on the next page. The greatest integer function is both a piecewise-constant function and a step function.

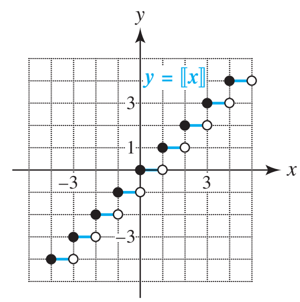


Figure 2.9 The Greatest Integer Function

$$[\![x]\!] = \begin{cases} \quad\vdots & \\ -2 & \text{if } -2 \le x < -1 \\ -1 & \text{if } -1 \le x < 0 \\ 0 & \text{if } \ \ 0 \le x < 1 \\ 1 & \text{if } \ \ 1 \le x < 2 \\ 2 & \text{if } \ \ 2 \le x < 3 \\ \quad\vdots & \end{cases}$$

Calculator Help

To access the greatest integer function or to set a calculator in dot mode, see Appendix A (pages AP-7 and AP-8).

In some applications, fractional parts are either not allowed or ignored. Framing lumber for houses is measured in 2-foot multiples, and mileage charges for rental cars may be calculated to the mile.

Suppose a car rental company charges \$31.50 per day plus \$0.25 for each mile driven, where fractions of a mile are ignored. The function given by $f(x) = 0.25[\![x]\!] + 31.50$ calculates the cost of driving x miles in one day. For example, the cost of driving 100.4 miles is

$$f(100.4) = 0.25[\![100.4]\!] + 31.50 = 0.25(100) + 31.50 = \$56.50.$$

On some calculators and computers, the greatest integer function is denoted int(X). A graph of $Y_1 = 0.25*\text{int}(X) + 31.5$ is shown in Figure 2.10.

[0, 10, 1] by [31, 35, 1]

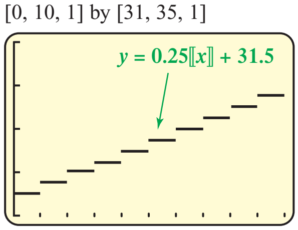


Figure 2.10 Dot Mode

[0, 10, 1] by [31, 35, 1]

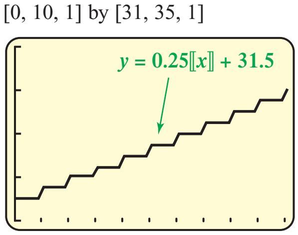


Figure 2.11 Connected Mode

MAKING CONNECTIONS

Connected and Dot Modes Graphing calculators often connect points to make a graph look continuous. However, if a graph has breaks in it, a graphing calculator may connect points where there should be breaks. In *dot mode*, points are plotted but not connected. Figure 2.11 is the same graph shown in Figure 2.10, except that it is plotted in *connected mode*. Note that connected mode generates an inaccurate graph of this step function.

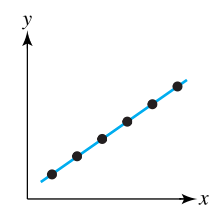
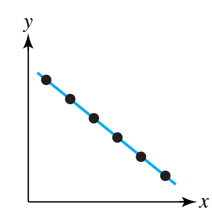
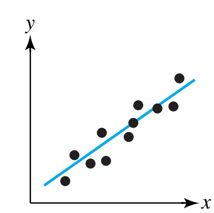
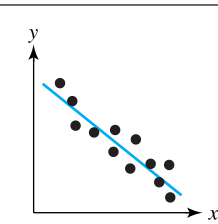
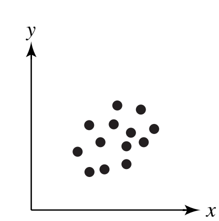
Linear Regression (Optional)

We have used linear functions to model data involving the variables x and y. Unknown values for y were predicted at given values of x. Problems where one variable is used to predict the behavior of a second variable are called **regression** problems. If a linear function or line is used to approximate the data, then the technique is referred to as **linear regression**.

We have already solved problems by selecting a line that *visually* fits the data in a scatterplot. However, this technique has some disadvantages. First, it does not produce a unique line. Different people may arrive at different lines to fit the same data. Second, the line is not determined automatically by a calculator or computer. A person must view the data and adjust the line until it "fits." By contrast, a statistical method used to determine a unique linear function or line is based on **least squares**.

Correlation Coefficient Most graphing calculators have the capability to calculate the least-squares regression line automatically after the data points have been entered. When determining the least-squares line, calculators often compute a real number r, called the **correlation coefficient**, where $-1 \leq r \leq 1$. When r is positive and near 1, low x-values correspond to low y-values and high x-values correspond to high y-values. For example, there is a positive correlation between years of education x and income y. More years of education correlate with higher income. When r is near -1, the reverse is true. Low x-values correspond to high y-values and high x-values correspond to low y-values. An example is the relation between latitude and average yearly temperature. As latitude increases (moving toward either the north or the south pole), the average yearly temperature decreases. Therefore there will be a negative correlation between latitude and average yearly temperature. If $r \approx 0$, then there is little or no correlation between the data points. In this case, a linear function does not provide a suitable model. A summary of these concepts is shown in Table 2.3.

Table 2.3 Correlation Coefficient r ($-1 \leq r \leq 1$)

Value of r	Comments	Sample Scatterplot
$r = 1$	There is an exact linear fit. The line passes through all data points and has a positive slope.	
$r = -1$	There is an exact linear fit. The line passes through all data points and has a negative slope.	
$0 < r < 1$	There is a positive correlation. As the x-values increase, so do the y-values. The fit is not exact.	
$-1 < r < 0$	There is a negative correlation. As the x-values increase, the y-values decrease. The fit is not exact.	
$r = 0$	There is no correlation. The data has no tendency toward being linear. A regression line predicts poorly.	

MAKING CONNECTIONS

Correlation and Causation When geese begin to fly north, summer is coming and the weather becomes warmer. Geese flying north correlates with warmer weather. However, geese flying north clearly does not *cause* warmer weather. It is important to remember that correlation does not always indicate the cause.

Calculator Help
To find a line of least-squares fit, see Appendix A (page AP-8).

In the next example we use a graphing calculator to find the line of least-squares fit that models three data points.

EXAMPLE 7 Determining a line of least-squares fit

Find the line of least-squares fit for the data points (1, 1), (2, 3), and (3, 4). What is the correlation coefficient? Plot the data and graph the line.

SOLUTION Begin by entering the three data points into the STAT EDIT menu. Refer to Figures 2.12–2.15. Select the LinReg(ax + b) option from the STAT CALC menu. From the home screen we can see that the line (linear function) of least-squares is given by the formula $y = \frac{3}{2}x - \frac{1}{3}$. The correlation coefficient is $r \approx 0.98$. Since $r \neq 1$, the line does not provide an *exact* model of the data.

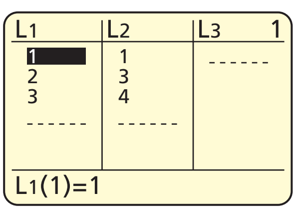


Figure 2.12

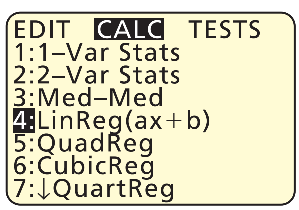


Figure 2.13

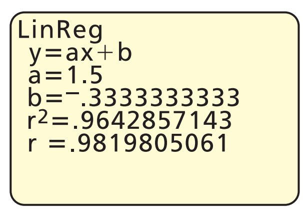


Figure 2.14

[0, 5, 1] by [0, 5, 1]

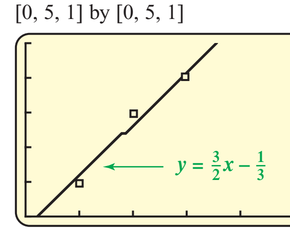


Figure 2.15

Now Try Exercise 85

An Application In the next example we use regression to find a linear function that models numbers of airline passengers at some of the largest U.S. airports.

EXAMPLE 8 Predicting airline passengers

Table 2.4 lists the numbers in millions of airline passengers at some of the largest airports in the United States during 2002 and 2006.

(a) Graph the data by using the 2002 data for x-values and the corresponding 2006 data for y-values. Predict whether the correlation coefficient will be positive or negative.

(b) Use a calculator to find the linear function f based on least-squares regression that models the data. Graph $y = f(x)$ and the data in the same viewing rectangle.

(c) In 2002 Newark International Airport had 29.0 million passengers. Assuming that this airport followed a trend similar to that of the five airports listed in Table 2.4, use your linear function to estimate the number of passengers at Newark International in 2006. Compare this result to the actual value of 36.7 million passengers.

Table 2.4 Airline Passengers (millions)

Airport	2002	2006
Atlanta (Hartsfield)	76.9	84.4
Chicago (O'Hare)	66.5	77.0
Los Angeles (LAX)	56.2	61.0
Dallas/Fort Worth	52.8	60.2
Denver	35.7	47.3

Source: Airports Association Council International.

SOLUTION

(a) A scatterplot of the data is shown in Figure 2.16. Because increasing x-values correspond to increasing y-values, the correlation coefficient will be positive.

(b) Because $y = f(x)$, the formula for a linear function f that models the data is given by $f(x) = 0.9384x + 11.9098$, where coefficients have been rounded to four decimal places. See Figure 2.17. Graphs of f and the data are shown in Figure 2.18.

(c) We can use $f(x)$ to predict y when $x = 29.0$.

$$y = f(29.0) = 0.9384(29.0) + 11.9098 \approx 39.1 \text{ million}$$

This value is more than the actual value of 36.7 million.

Calculator Help

To copy the regression equation directly into Y_1, see Appendix A (page AP-13).

[30, 90, 10] by [30, 90, 10]

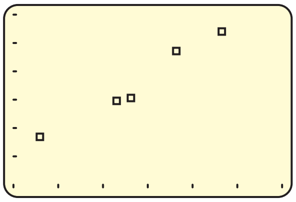


Figure 2.16

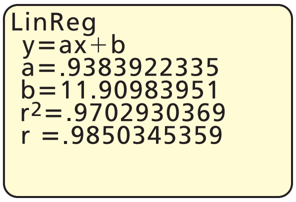


Figure 2.17

[30, 90, 10] by [30, 90, 10]

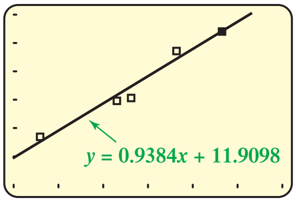


Figure 2.18

Now Try Exercise 93

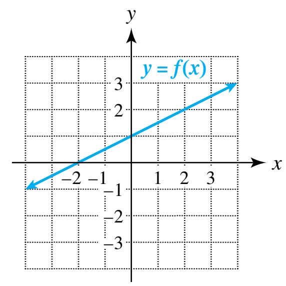
2.1 Putting It All Together

The following table summarizes important concepts.

Concept	Description
Models	A good model describes and explains current data. It should also make predictions and forecast phenomena.
Linear model	If a quantity experiences a constant rate of change, then it can be modeled by a linear function in the form $f(x) = ax + b$. $f(x) = (\textbf{constant rate of change})x + (\textbf{initial value})$

continued on next page

continued from previous page

Concept	Description
Graph of a linear function	The graph of a linear function is a line. If $f(x) = ax + b$, then the slope equals a and the y-intercept equals b. The following graph has slope $\frac{1}{2}$, y-intercept 1, and formula $f(x) = \frac{1}{2}x + 1$. The zero of f is the x-intercept, -2. 
Piecewise-defined function	A function is piecewise-defined if it has different formulas on different intervals of its domain. Many times the domain is restricted. $$f(x) = \begin{cases} 2x - 3 & \text{if } -3 \le x < 1 \\ x + 5 & \text{if } \quad 1 \le x \le 5 \end{cases}$$ When $x = 2$ then $f(x) = x + 5$, so $f(2) = 2 + 5 = 7$. The domain of f is $[-3, 5]$.
Correlation coefficient r	The values of r satisfy $-1 \le r \le 1$, where a line fits the data better if r is near -1 or 1. A value near 0 indicates a poor fit.
Least-squares regression line	The line of least-squares fit for the points (1, 3), (2, 5), and (3, 6) is $y = \frac{3}{2}x + \frac{5}{3}$ and $r \approx 0.98$. Try verifying this with a calculator.

2.1

Functions as Models

1. ***U.S. Vehicle Production*** In 2000 there were 12.8 million vehicles produced in the United States, and in 2004 there were 12.0 million. The formula $V(t) = -0.2t + 12.8$ models these data exactly, where $t = 0$ corresponds to 2000, $t = 1$ to 2001, and so on.
   (a) Verify that $V(t)$ gives the exact values in millions for 2000 and 2004.
   (b) Use $V(t)$ to estimate the number of vehicles manufactured in 2002 and 2006. Do these estimates involve interpolation or extrapolation?
   (c) The actual value for 2002 was 12.3 million and for 2006 was 11.3 million. Discuss the accuracy of your results from part (b).

2. ***U.S. Advertising Expenditures*** In 2002 \$237 billion was spent on advertising in the United States, and in 2004 this amount was \$264 billion. The formula $A(t) = 13.5t + 237$ models these data exactly, where $t = 0$ corresponds to 2002, $t = 1$ to 2003, and so on.
   (a) Verify that $A(t)$ gives the exact values in billions of dollars for 2002 and 2004.
   (b) Use $A(t)$ to estimate the advertising expenditures in 2000 and 2003. Do these estimates involve interpolation or extrapolation?
   (c) The actual value for 2000 was \$244 billion and for 2003 was \$245 billion. Discuss the accuracy of your results from part (b).

Exercises 3–6: A function f is given. Determine whether f models the data exactly or approximately.

3. $f(x) = 5x - 2$

x	1	2	3	4
y	3	8	13	18

4. $f(x) = 1 - 0.2x$

x	5	10	15	20
y	0	−1	−2	−4

5. $f(x) = 3.7 - 1.5x$

x	−6	0	1
y	12.7	3.7	2.1

6. $f(x) = 13.3x - 6.1$

x	1	2	5
y	7.2	20.5	60.4

Exercises 7–10: Find the formula for a linear function f that models the data in the table exactly.

7.

x	−2	0	4
$f(x)$	4	3	1

8.

x	−6	0	3
$f(x)$	−5	−1	1

9.

x	1	2	3
$f(x)$	7	9	11

10.

x	15	30	45
$f(x)$	40	30	20

Exercises 11 and 12: Write a symbolic representation (formula) for a function f that computes the following.

11. (a) The number of pounds in x ounces

(b) The number of dimes in x dollars

(c) The monthly electric bill in dollars if x kilowatt-hours are used at 6 cents per kilowatt-hour and there is a fee of \$6.50

(d) The cost of skiing x times with a \$500 season pass

12. (a) The distance traveled by a car moving at 50 miles per hour for x hours

(b) The total number of hours in day x

(c) The distance in miles between a runner and home after x hours if the runner starts 1 mile from home and jogs *away* from home at 6 miles per hour

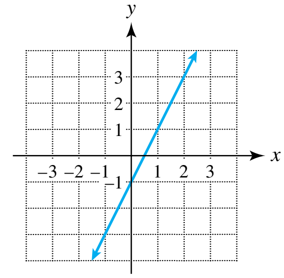
(d) A car's speed in feet per second after x seconds if its tires are 2 feet in diameter and rotating 14 times per second

Graphs of Linear Functions

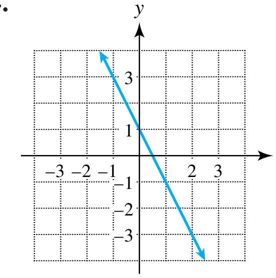
Exercises 13–18: The graph of a linear function f is shown.

(a) Identify the slope, y-intercept, and x-intercept.
(b) Write a formula for f.
(c) Estimate the zero of f.
(d) Is f increasing or decreasing on its domain?

13.



14.



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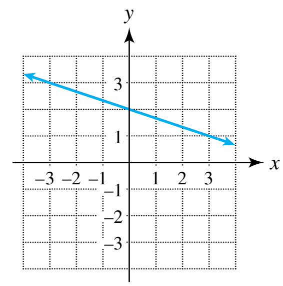
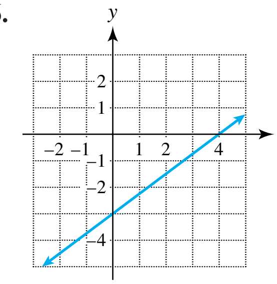
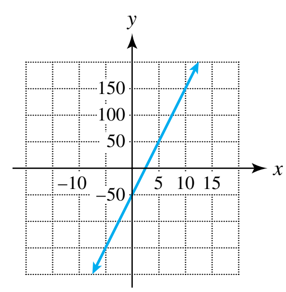


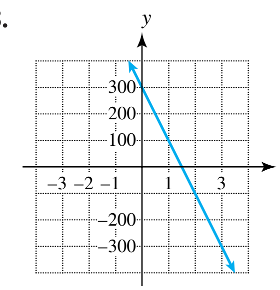
16.



17.



18.



Exercises 19–32: Graph the linear function by hand. Identify the slope and y-intercept.

19. $f(x) = 3x + 2$

20. $f(x) = -\frac{3}{2}x$

21. $f(x) = \frac{1}{2}x - 2$

22. $f(x) = 3 - x$

23. $g(x) = -2$

24. $g(x) = 20 - 10x$

25. $f(x) = 4 - \frac{1}{2}x$

26. $f(x) = 2x - 3$

27. $g(x) = \frac{1}{2}x$

28. $g(x) = 3$

29. $g(x) = 5 - 5x$

30. $g(x) = \frac{3}{4}x - 2$

31. $f(x) = 20x - 10$

32. $f(x) = -30x + 20$

Exercises 33–38: Write a formula for a linear function f whose graph satisfies the conditions.

33. Slope $-\frac{3}{4}$, y-intercept $\frac{1}{3}$

34. Slope -122, y-intercept 805

35. Slope 15, passing through the origin

36. Slope 1.68, passing through (0, 1.23)

37. Slope 0.5, passing through (1, 4.5)

38. Slope -2, passing through $(-1, 5)$

Exercises 39–44: ***Average Rate of Change*** *Find the average rate of change of f from -2 to 2. What is the average rate of change of f from x_1 to x_2, where $x_1 \neq x_2$?*

39. $f(x) = 10$

40. $f(x) = -5$

41. $f(x) = -\frac{1}{4}x$

42. $f(x) = \frac{5}{3}x$

43. $f(x) = 4 - 3x$

44. $f(x) = 5x + 1$

Modeling with Linear Functions

Exercises 45–48: Match the situation with the graph (a–d) that models it best, where x-values represent time.

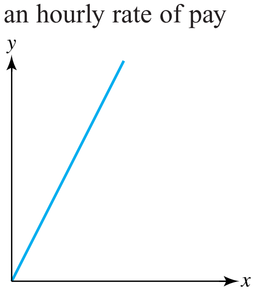
45. Height of the Empire State Building from 1990 to 2000

46. Average cost of a new car from 1980 to 2000

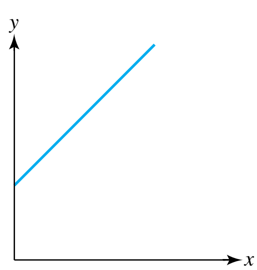
47. Distance between a runner in a race and the finish line

48. Amount of money earned after x hours when working at an hourly rate of pay

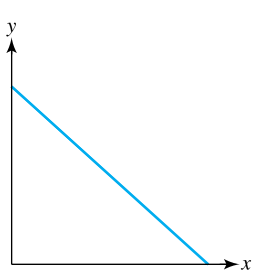
a.



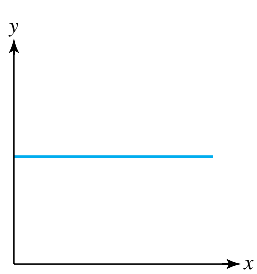
b.



c.



d.



Exercises 49–54: Write a formula for a linear function that models the situation. Choose both an appropriate name and an appropriate variable for the function. State what the input variable represents and the domain of the function. Assume that the domain is an interval of the real numbers.

49. ***U.S. Homes with Internet*** In 2006 about 68% of U.S. homes had Internet access. This percentage was expected to increase, on average, by 1.5 percentage points per year for the next 4 years. (**Source:** 2007 Digital Future Report.)

50. ***U.S. Cell Phones*** In 2005 there were about 208 million U.S. cell phone subscribers. This number was expected to increase, on average, by 20 million per year for the next 3 years. (**Source:** CTIA Industry Survey.)

51. ***Velocity of a Falling Object*** A stone is dropped from a water tower and its velocity increases at a rate of 32 feet per second. The stone hits the ground with a velocity of 96 feet per second.

52. ***Speed of a Car*** A car is traveling at 30 miles per hour, and then it begins to slow down at a constant rate of 6 miles per hour every 4 seconds.

53. ***Population Density*** In 1900 the average number of people per square mile in the United States was 21.5, and it increased, on average, by 5.81 people every 10 years until 2000. (**Source:** Bureau of the Census.)

54. ***Injury Rate*** In 1992 the number of injury cases recorded in private industry per 100 full-time workers was 8.3, and it decreased, on average, by 0.32 injury every year until 2001. (**Source:** Bureau of Labor Statistics.)

55. ***Draining a Water Tank*** A 300-gallon tank is initially full of water and is being drained at a rate of 10 gallons per minute.

(a) Write a formula for a function W that gives the number of gallons of water in the tank after t minutes.

(b) How much water is in the tank after 7 minutes?

(c) Graph W and identify and interpret the intercepts.

(d) Find the domain of W.

56. ***Filling a Tank*** A 500-gallon tank initially contains 200 gallons of fuel oil. A pump is filling the tank at a rate of 6 gallons per minute.

(a) Write a formula for a linear function f that models the number of gallons of fuel oil in the tank after x minutes.

(b) Graph f. What is an appropriate domain for f?

(c) Identify the y-intercept and interpret it.

(d) Does the x-intercept of the graph of f have any physical meaning in this problem? Explain.

57. ***HIV Infections*** In 2006 there were 40 million people worldwide who had been infected with HIV. At that time the infection rate was 4.3 million people per year. (**Source:** United Nations AIDS and World Health Organization.)

(a) Write a formula for a linear function f that models the total number of people in millions who were infected with HIV x years after 2006.

(b) Estimate the number of people who may have been infected by the year 2012.

58. ***Birth Rate*** In 1990 the number of births per 1000 people in the United States was 16.7 and decreasing at 0.21 birth per 1000 people each year. (**Source:** National Center for Health Statistics.)

(a) Write a formula for a linear function f that models the birth rate x years after 1990.

(b) Estimate the birth rate in 2003 and compare the estimate to the actual value of 14.

59. ***Ice Deposits*** A roof has a 0.5-inch layer of ice on it from a previous storm. Another ice storm begins to deposit ice at a rate of 0.25 inch per hour.

(a) Find a formula for a linear function f that models the thickness of the ice on the roof x hours after the second ice storm started.

(b) How thick is the ice after 2.5 hours?

60. ***Rainfall*** Suppose that during a storm rain is falling at a rate of 1 inch per hour. The water coming from a circular roof with a radius of 20 feet is running down a downspout that can accommodate 400 gallons of water per hour. See the figure.

(a) Determine the number of cubic inches of water falling on the roof in 1 hour.

(b) One gallon equals about 231 cubic inches. Write a formula for a function g that computes the gallons of water landing on the roof in x hours.

(c) How many gallons of water land on the roof during a 2.5-hour rain storm?

(d) Will one downspout be sufficient to handle this type of rainfall? How many downspouts should there be?

Exercises 61 and 62: ***Modeling Fuel Consumption*** *The table shows the distance y in miles traveled by a vehicle using x gallons of gasoline.*

(a) Calculate the slopes of the line segments that connect consecutive points.
(b) Find a linear function that models the data.
(c) Graph f and the data together. What does the slope indicate?
(d) Evaluate f(30) and interpret the result.

61.

x (gallons)	5	10	15	20
y (miles)	84	169	255	338

62.

x (gallons)	5	10	15	20
y (miles)	194	392	580	781

Piecewise-Defined Functions

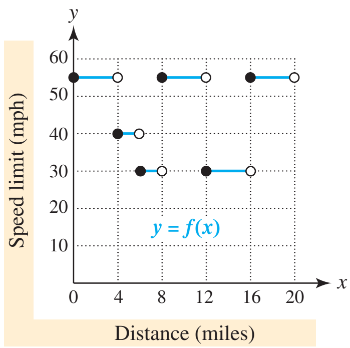
63. ***Speed Limits*** The graph of $y = f(x)$ on the next page gives the speed limit y along a rural highway x miles from its starting point.

(a) What are the maximum and minimum speed limits along this stretch of highway?

(b) Estimate the miles of highway with a speed limit of 55 miles per hour.

(c) Evaluate $f(4)$, $f(12)$, and $f(18)$.

(d) At what x-values is the graph discontinuous? Interpret each discontinuity.



64. ***ATM*** The graph of $y = f(x)$ depicts the amount of money y in dollars in an automatic teller machine (ATM) after x minutes.

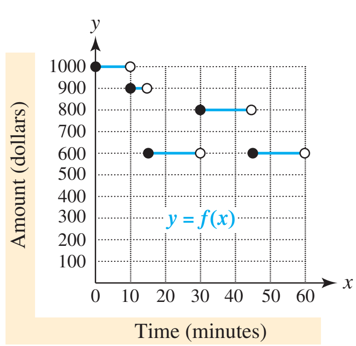
(a) Determine the initial and final amounts of money in the ATM.

(b) Evaluate $f(10)$ and $f(50)$. Is f continuous?

(c) How many *withdrawals* occurred?

(d) When did the largest withdrawal occur? How much was it?

(e) How much was deposited into the machine?



65. ***First-Class Mail*** In March 2008, the retail flat rate in dollars for first-class mail weighing up to 5 ounces could be computed by the piecewise-constant function P, where x is the number of ounces.

$$P(x) = \begin{cases} 0.80 & \text{if } 0 < x \leq 1 \\ 0.97 & \text{if } 1 < x \leq 2 \\ 1.14 & \text{if } 2 < x \leq 3 \\ 1.31 & \text{if } 3 < x \leq 4 \\ 1.48 & \text{if } 4 < x \leq 5 \end{cases}$$

(a) Evaluate $P(1.5)$ and $P(3)$. Interpret your results.

(b) Sketch a graph of P. What is the domain of P?

(c) Where is P discontinuous on its domain?

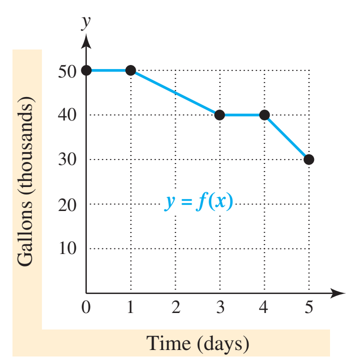
66. ***Swimming Pool Levels*** The graph of $y = f(x)$ shows the amount of water y in thousands of gallons remaining in a swimming pool after x days.

(a) Estimate the initial and final amounts of water in the pool.

(b) When did the amount of water in the pool remain constant?

(c) Approximate $f(2)$ and $f(4)$.

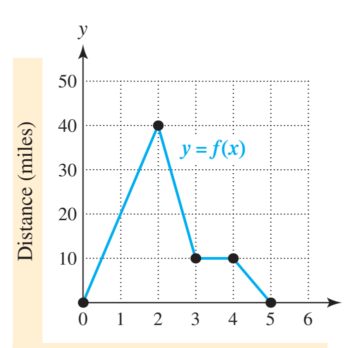
(d) At what rate was water being drained from the pool when $1 \leq x \leq 3$?



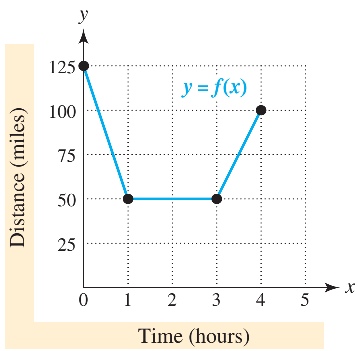
Exercises 67 and 68: An individual is driving a car along a straight road. The graph shows the driver's distance from home after x hours.

(a) Use the graph to evaluate $f(1.5)$ and $f(4)$.
(b) Interpret the slope of each line segment.
(c) Describe the motion of the car.
(d) Identify where f is increasing, decreasing, or constant.

67.



68.



Exercises 69–74: Complete the following for $f(x)$.

(a) Determine the domain of f.
(b) Evaluate $f(-2)$, $f(0)$, and $f(3)$.
(c) Graph f.
(d) Is f continuous on its domain?

69. $f(x) = \begin{cases} 2 & \text{if } -5 \le x \le -1 \\ x + 3 & \text{if } -1 < x \le 5 \end{cases}$

70. $f(x) = \begin{cases} 2x + 1 & \text{if } -3 \le x < 0 \\ x - 1 & \text{if } 0 \le x \le 3 \end{cases}$

71. $f(x) = \begin{cases} 3x & \text{if } -1 \le x < 1 \\ x + 1 & \text{if } 1 \le x \le 2 \end{cases}$

72. $f(x) = \begin{cases} -2 & \text{if } -6 \le x < -2 \\ 0 & \text{if } -2 \le x < 0 \\ 3x & \text{if } 0 \le x \le 4 \end{cases}$

73. $f(x) = \begin{cases} x & \text{if } -3 \le x \le -1 \\ 1 & \text{if } -1 < x < 1 \\ 2 - x & \text{if } 1 \le x \le 3 \end{cases}$

74. $f(x) = \begin{cases} 3 & \text{if } -4 \le x \le -1 \\ x - 2 & \text{if } -1 < x \le 2 \\ 0.5x & \text{if } 2 < x \le 4 \end{cases}$

Exercises 75 and 76: Graph f.

75. $f(x) = \begin{cases} -\frac{1}{2}x + 1 & \text{if } -4 \le x \le -2 \\ 1 - 2x & \text{if } -2 < x \le 1 \\ \frac{2}{3}x + \frac{4}{3} & \text{if } 1 < x \le 4 \end{cases}$

76. $f(x) = \begin{cases} \frac{3}{2} - \frac{1}{2}x & \text{if } -3 \le x < -1 \\ -2x & \text{if } -1 \le x \le 2 \\ \frac{1}{2}x - 5 & \text{if } 2 < x \le 3 \end{cases}$

77. Use $f(x)$ to complete the following:

$$f(x) = \begin{cases} 3x - 1 & \text{if } -5 \le x < 1 \\ 4 & \text{if } 1 \le x \le 3 \\ 6 - x & \text{if } 3 < x \le 5 \end{cases}$$

(a) Evaluate f at $x = -3, 1, 2,$ and 5.

(b) On what interval is f constant?

(c) Sketch a graph of f. Is f continuous on its domain?

78. Use $g(x)$ to complete the following.

$$g(x) = \begin{cases} -2x - 6 & \text{if } -8 \le x \le -2 \\ x & \text{if } -2 < x < 2 \\ 0.5x + 1 & \text{if } 2 \le x \le 8 \end{cases}$$

(a) Evaluate g at $x = -8, -2, 2,$ and 8.

(b) For what x-values is g increasing?

(c) Sketch a graph of g. Is g continuous on its domain?

Greatest Integer Function

Exercises 79–82: Complete the following.

(a) Use dot mode to graph the function f in the standard viewing rectangle.
(b) Evaluate $f(-3.1)$ and $f(1.7)$.

79. $f(x) = [\![2x - 1]\!]$ **80.** $f(x) = [\![x + 1]\!]$

81. $f(x) = 2[\![x]\!] + 1$ **82.** $f(x) = [\![-x]\!]$

83. ***Lumber Costs*** The lumber used to frame walls of houses is frequently sold in multiples of 2 feet. If the length of a board is not exactly a multiple of 2 feet, there is often no charge for the additional length. For example, if a board measures at least 8 feet but less than 10 feet, then the consumer is charged for only 8 feet.

(a) Suppose that the cost of lumber is \$0.80 for every 2 feet. Find a formula for a function f that computes the cost of a board x feet long for $6 \le x \le 18$.

(b) Graph f.

(c) Determine the costs of boards with lengths of 8.5 feet and 15.2 feet.

84. ***Cost of Carpet*** Each foot of carpet purchased from a 12-foot-wide roll costs \$36. If a fraction of a foot is purchased, a customer does not pay for the extra amount. For example, if a customer wants 14 feet of carpet and the salesperson cuts off 14 feet 4 inches, the customer does not pay for the extra 4 inches.

(a) How much does 9 feet 8 inches of carpet from this roll cost?

(b) Using the greatest integer function, write a formula for the price P of x feet of carpet.

Linear Regression

Exercises 85 and 86: Find the line of least-squares fit for the given data points. What is the correlation coefficient? Plot the data and graph the line.

85. $(-2, 2), (1, 0), (3, -2)$ **86.** $(-1, -1), (1, 4), (2, 6)$

Exercises 87–90: Complete the following.

(a) Conjecture whether the correlation coefficient r for the data will be positive, negative, or zero.

(b) Use a calculator to find the equation of the least-squares regression line and the value of r.

(c) Use the regression line to predict y when $x = 2.4$.

87.

x	-1	0	1	2	3
y	-5.7	-2.6	1.1	3.9	7.3

88.

x	-4	-2	0	2	4
y	1.2	2.8	5.3	6.7	9.1

89.

x	1	3	5	7	10
y	5.8	-2.4	-10.7	-17.8	-29.3

90.

x	-4	-3	-1	3	5
y	37.2	33.7	27.5	16.4	9.8

91. ***Distant Galaxies*** In the late 1920s the famous observational astronomer Edwin P. Hubble (1889–1953) determined both the distance to several galaxies and the velocity at which they were receding from Earth. Four galaxies with their distances in light-years and velocities in miles per second are listed in the table at the top of the next column.

Galaxy	Distance	Velocity
Virgo	50	990
Ursa Minor	650	9,300
Corona Borealis	950	15,000
Bootes	1700	25,000

Source: A. Sharov and I. Novikov, *Edwin Hubble: The Discoverer of the Big Bang Universe.*

(a) Let x be distance and y be velocity. Plot the data points in $[-100, 1800, 100]$ by $[-1000, 28000, 1000]$.

(b) Find the least-squares regression line.

(c) If the galaxy Hydra is receding at a speed of 37,000 miles per second, estimate its distance.

92. ***Cell Phones*** One of the early problems with cell phones was the delay involved with placing a call when the system was busy. One study analyzed this delay. The table shows that as the number of calls increased by P percent, the average delay time D to put through a call also increased.

P (%)	0	20	40	60	80	100
D (minutes)	1	1.6	2.4	3.2	3.8	4.4

Source: A Mehrotra, *Cellular Radio: Analog and Digital Systems.*

(a) Let P correspond to x-values and D to y-values. Find the least-squares regression line that models these data. Plot the data and the regression line.

(b) Estimate the delay for a 50% increase in the number of calls.

93. ***Passenger Travel*** The table shows the number of miles (in trillions) traveled by passengers of all types for various years, where $x = 0$ corresponds to 1970, $x = 10$ to 1980, and so on.

Year (1970 $\leftrightarrow$ 0)	0	10	20	30	35
Miles (trillions)	2.2	2.8	3.7	4.7	5.1

Source: Department of Transportation.

(a) Make a scatterplot of the data. Predict whether the correlation coefficient will be positive or negative.

(b) Use regression to find a formula $f(x) = ax + b$ so that f models the data.

(c) Graph f and the data. Interpret the slope.

(d) Predict the number of passenger miles in 2010.

94. ***High School Enrollment*** The table lists the number of students (in millions) attending U.S. public school (grades 9–12) for selected years, where $x = 0$ corresponds to 2000, $x = 1$ to 2001, and so on.

x (year)	0	3	5	7
y (students)	13.5	14.3	14.8	15.1

Source: National Center for Education Statistics.

(a) Use regression to find a formula $f(x) = ax + b$ so that f models the data.

(b) Graph f and the data. Interpret the slope.

(c) Estimate enrollment in 2002 and compare the estimate to the actual value of 14.1 million.

Writing about Mathematics

95. How can you recognize a symbolic representation (formula) of a linear function? How can you recognize a graph or table of values of a linear function?

96. A student graphs $f(x) = x^2 - x$ in the viewing rectangle [2, 2.1, 0.01] by [1.9, 2.3, 0.1]. Using the graph, the student decides that f is a linear function. How could you convince the student otherwise?

97. Explain how average rate of change relates to a linear function.

98. Find a real data set on the Internet that can be modeled by a linear function. Find the linear modeling function. Is your model exact or approximate? Explain.

99. Explain how you determine whether a linear function is increasing, decreasing, or constant. Give an example of each.

100. Explain what a piecewise-defined function is and why it is used. Sketch a graph of a continuous piecewise-linear function f that increases, decreases, and is constant. Let the domain of f be $-4 \leq x \leq 4$.

EXTENDED AND DISCOVERY EXERCISES

1. ***Height and Shoe Size*** In this exercise you will determine if there is a relationship between height and shoe size.

(a) Have classmates write their sex, shoe size, and height in inches on a slip of paper. When you have enough information, complete the following table—one for adult males and one for adult females.

Height (inches)					
Shoe size					

(b) Make a scatterplot of each table, with height on the x-axis and shoe size on the y-axis. Is there any relationship between height and shoe size? Explain.

(c) Try to find a linear function that models each data set.

Exercises 2 and 3: ***Linear Approximation*** *Graph the function f in the standard viewing rectangle.*

(a) Choose any curved portion of the graph of f and repeatedly zoom in. Describe how the graph appears. Repeat this process on different portions of the graph.

(b) Under what circumstances could a linear function be used to accurately model a nonlinear graph?

2. $f(x) = 4x - x^3$

3. $f(x) = x^4 - 5x^2$

2.2 Equations of Lines

- **Write the point-slope and slope-intercept forms**
- **Find the intercepts of a line**
- **Write equations for horizontal, vertical, parallel, and perpendicular lines**
- **Model data with lines and linear functions (optional)**
- **Use direct variation to solve problems**

Introduction

Apple Corporation sold approximately 4.4 million iPods in fiscal 2004 and 46.4 million iPods in fiscal 2006, making the iPod the fastest selling music player in history. (**Source:** Apple Corporation.) Can we use this information to make estimates about future sales? Mathematics is often used to analyze data and to make predictions. One of the simplest ways to make estimates is to use linear functions and lines. This section discusses how to use data points to find equations of lines. See Example 4.

Forms for Equations of Lines

Point-Slope Form Suppose that a nonvertical line with slope m passes through the point (x_1, y_1). If (x, y) is any point on this nonvertical line with $x \neq x_1$, then the change in y is $\Delta y = y - y_1$, the change in x is $\Delta x = x - x_1$, and the slope equals $m = \frac{\Delta y}{\Delta x} = \frac{y - y_1}{x - x_1}$, as illustrated in Figure 2.19.

With this slope formula, the equation of the line can be found.

$$m = \frac{y - y_1}{x - x_1} \qquad \text{Slope formula}$$

$$y - y_1 = m(x - x_1) \qquad \text{Cross multiply.}$$

$$y = m(x - x_1) + y_1 \qquad \text{Add } y_1 \text{ to each side.}$$

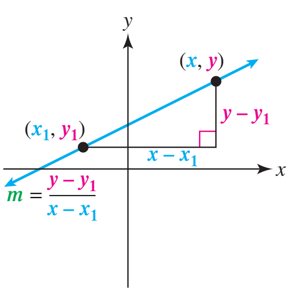


Figure 2.19

The equation $y - y_1 = m(x - x_1)$ is traditionally called the *point-slope form* of the equation of a line. Since we think of y as being a function of x, written $y = f(x)$, the equivalent form $y = m(x - x_1) + y_1$ will also be referred to as the point-slope form. The point-slope form is not unique, as any point on the line can be used for (x_1, y_1). However, these point-slope forms are *equivalent*—their graphs are identical.

Point-Slope Form

The line with slope m passing through the point (x_1, y_1) has an equation

$$y = m(x - x_1) + y_1, \quad \text{or} \quad y - y_1 = m(x - x_1),$$

the **point-slope form** of the equation of a line.

In the next example we find the equation of a line given two points.

EXAMPLE 1 Determining a point-slope form

Find an equation of the line passing through the points $(-2, -3)$ and $(1, 3)$. Plot the points and graph the line by hand.

SOLUTION Begin by finding the slope of the line.

$$m = \frac{3 - (-3)}{1 - (-2)} = \frac{6}{3} = 2$$

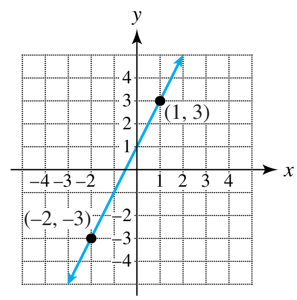


Figure 2.20

Substituting $(x_1, y_1) = (\mathbf{1}, \mathbf{3})$ and $m = \mathbf{2}$ into the point-slope form results in

$$y = \mathbf{2}(x - \mathbf{1}) + \mathbf{3}. \qquad y = m(x - x_1) + y_1$$

If we use the point $(-2, -3)$, the point-slope form is

$$y = 2(x + 2) - 3. \qquad \text{Note that } (x - (-2)) = (x + 2).$$

This line and the two points are shown in Figure 2.20. ***Now Try Exercise 1***

Slope-Intercept Form The two point-slope forms found in Example 1 are equivalent.

$y = 2(x - 1) + 3$	$y = 2(x + 2) - 3$	Point-slope form
$y = 2x - 2 + 3$	$y = 2x + 4 - 3$	Distributive property
$\mathbf{y = 2x + 1}$	$\mathbf{y = 2x + 1}$	Simplify.

Algebra Review

To review the distributive property, see Chapter R (page R-15).

Both point-slope forms simplify to the same equation.

The form $y = mx + b$ is called the *slope-intercept form*. Unlike the point-slope form, it is *unique*. The real number m represents the slope and the real number b represents the y-intercept, as illustrated in Figure 2.21.

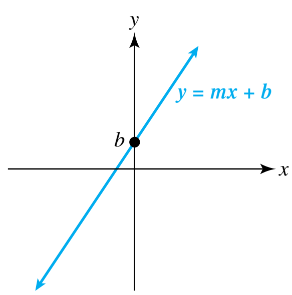


Figure 2.21

> **Slope-Intercept Form**
>
> The line with slope m and y-intercept b is given by
>
> $$y = mx + b,$$
>
> the **slope-intercept form** of the equation of a line.

EXAMPLE 2 Finding equations of lines

Find the point-slope form for the line that satisfies the conditions. Then convert this equation into slope-intercept form.

(a) Slope $-\frac{1}{2}$, passing through the point $(-3, -7)$

(b) x-intercept -4, y-intercept 2

SOLUTION

(a) Let $m = -\frac{1}{2}$ and $(x_1, y_1) = (-3, -7)$ in the point-slope form.

$$y = m(x - x_1) + y_1 \qquad \text{Point-slope form}$$

$$y = -\frac{1}{2}(x + 3) - 7 \qquad \text{Substitute.}$$

The slope-intercept form can be found by simplifying.

$$y = -\frac{1}{2}(x + 3) - 7 \qquad \text{Point-slope form}$$

$$y = -\frac{1}{2}x - \frac{3}{2} - 7 \qquad \text{Distributive property}$$

$$y = -\frac{1}{2}x - \frac{17}{2} \qquad \text{Slope-intercept form}$$

(b) The line passes through the points $(-4, 0)$ and $(0, 2)$. Its slope is

$$m = \frac{2 - 0}{0 - (-4)} = \frac{1}{2}.$$

Thus a point-slope form for the line is $y = \frac{1}{2}(x + 4) + 0$, where the point $(-4, 0)$ is used for (x_1, y_1). The slope-intercept form is $y = \frac{1}{2}x + 2$. ***Now Try Exercises 5 and 9***

The next example demonstrates how to find the slope-intercept form of a line without first finding the point-slope form.

EXAMPLE 3 Finding slope-intercept form

Find the slope-intercept form of the line passing through the points $(-2, 1)$ and $(2, 3)$.

SOLUTION

Getting Started We need to determine m and b in the slope-intercept form, $y = mx + b$. First find the slope m. Then substitute *either* point into the equation and determine b.

$$m = \frac{3 - 1}{2 - (-2)} = \frac{2}{4} = \frac{1}{2}$$

Thus $y = \frac{1}{2}x + b$. To find b, we substitute $(2, 3)$ in this equation.

$$3 = \frac{1}{2}(2) + b \qquad \text{Let } x = 2 \text{ and } y = 3.$$

$$3 = 1 + b \qquad \text{Multiply.}$$

$$2 = b \qquad \text{Determine } b.$$

Thus $y = \frac{1}{2}x + 2$. ***Now Try Exercise 21***

An Application

In the next example we model the data about iPods discussed in the introduction to this section.

EXAMPLE 4 Estimating iPod sales

Apple Corporation sold approximately 4.4 million iPods in fiscal 2004 and 46.4 million iPods in fiscal 2006.

(a) Find the point-slope form of the line passing through $(2004, 4.4)$ and $(2006, 46.4)$. Interpret the slope of the line as a rate of change.

(b) Sketch a graph of the data and the line connecting these points.

(c) Estimate sales in 2005 and compare the estimate to the true value of 23 million. Did your estimate involve interpolation or extrapolation?

(d) Estimate sales in 2003 and 2008. Discuss the accuracy of your answers.

SOLUTION

Getting Started First find the slope m of the line connecting the data points, and then substitute this value for m and either of the two data points in the point-slope form. We can use this equation to estimate sales by substituting the required year for x in the equation.

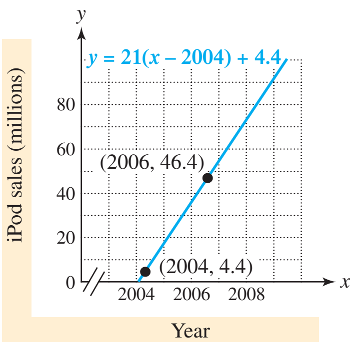


Figure 2.22 iPod Sales

(a) The slope of the line passing through (**2004**, **4.4**) and (**2006**, **46.4**) is

$$m = \frac{46.4 - 4.4}{2006 - 2004} = 21.$$

Thus sales of iPods increased, on average, by **21** million iPods per year from **2004** to **2006**. If we substitute **21** for m and (**2004**, **4.4**) for (x_1, y_1), the point-slope form is

$$y = 21(x - 2004) + 4.4.$$

(b) The requested line passing through the data points is shown in Figure 2.22.

(c) If $x = 2005$, then $y = 21(2005 - 2004) + 4.4 = 25.4$ million. This value is slightly high; its calculation involves interpolation because 2005 lies between 2004 and 2006.

(d) We can use the equation to estimate 2003 and 2008 sales as follows.

$$y = 21(2003 - 2004) + 4.4 = -16.6 \text{ million} \qquad \text{Let } x = 2003.$$

$$y = 21(2008 - 2004) + 4.4 = 88.4 \text{ million} \qquad \text{Let } x = 2008.$$

Both estimates involve extrapolation, because 2003 and 2008 are not between 2004 and 2006. The 2003 value is clearly incorrect because sales cannot be negative. The 2008 value appears to be more reasonable. ***Now Try Exercise 81***

Finding Intercepts

The point-slope form and the slope-intercept form are not the only forms for the equation of a line. An equation of a line is in **standard form** when it is written as

$$ax + by = c,$$

where a, b, and c are constants. By using standard form, we can write the equation of any line, including vertical lines (which are discussed later in this section). Examples of equations of lines in standard form include

$$2x - 3y = -6, \qquad \underset{(a = 0)}{y = \frac{1}{4}}, \qquad \underset{(b = 0)}{x = -3}, \qquad \text{and} \qquad -3x + y = \frac{1}{2}.$$

Standard form is a convenient form for finding the x- and y-intercepts of a line. Once the intercepts have been found, we can graph the line. For example, to find the x-intercept for the line determined by $3x + 4y = 12$, we let $y = 0$ and solve for x to obtain

$$3x + 4(0) = 12, \quad \text{or} \quad x = 4.$$

The x-intercept is 4. To find the y-intercept, we let $x = 0$ and solve for y to obtain

$$3(0) + 4y = 12, \quad \text{or} \quad y = 3.$$

The y-intercept is 3. Thus the graph of $3x + 4y = 12$ passes through the points (4, 0) and (0, 3). Knowing these two points allows us to graph the line. This technique can be used to find intercepts on the graph of any equation, not just lines written in standard form.

Finding Intercepts

To find any x-intercepts, let $y = 0$ in the equation and solve for x.

To find any y-intercepts, let $x = 0$ in the equation and solve for y.

NOTE To solve $ax = b$, divide each side by a to obtain $x = \frac{b}{a}$. Thus $5x = 20$ implies that $x = \frac{20}{5} = 4$. Linear equations are solved in general in the next section.

EXAMPLE 5 Finding intercepts

Locate the x- and y-intercepts for the line whose equation is $4x + 3y = 6$. Use the intercepts to graph the equation.

SOLUTION To locate the x-intercept, let $y = 0$ in the equation.

$$4x + 3(0) = 6 \qquad \text{Let } y = 0.$$
$$x = 1.5 \qquad \text{Divide by 4.}$$

The x-intercept is 1.5. Similarly, to find the y-intercept, substitute $x = 0$ into the equation.

$$4(0) + 3y = 6 \qquad \text{Let } x = 0.$$
$$y = 2 \qquad \text{Divide by 3.}$$

The y-intercept is 2. Therefore the line passes through the points (1.5, 0) and (0, 2), as shown in Figure 2.23. *Now Try Exercise 57*

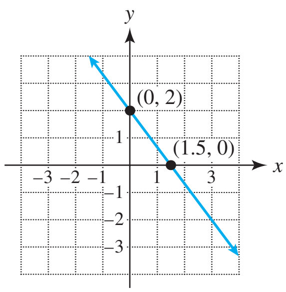


Figure 2.23

Horizontal, Vertical, Parallel, and Perpendicular Lines

Horizontal and Vertical Lines

The graph of a constant function f, defined by the formula $f(x) = b$, is a horizontal line having slope 0 and y-intercept b.

A vertical line cannot be represented by a function because distinct points on a vertical line have the same x-coordinate. In fact, this is the distinguishing feature of points on a vertical line—they all have the same x-coordinate. The vertical line shown in Figure 2.24 is $x = 3$. The equation of a vertical line with x-intercept k is given by $x = k$, as shown in Figure 2.25. Horizontal lines have slope 0, and vertical lines have an undefined slope.

CLASS DISCUSSION

Why do you think that a vertical line sometimes is said to have "infinite slope"? What are some problems with taking this phrase too literally?

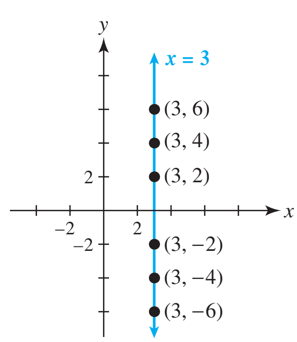


Figure 2.24

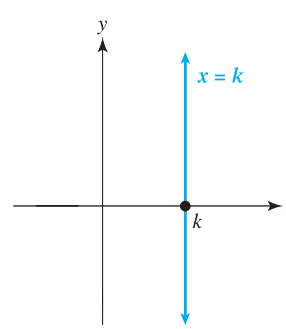


Figure 2.25

Equations of Horizontal and Vertical Lines

An equation of the horizontal line with y-intercept b is $y = b$. An equation of the vertical line with x-intercept k is $x = k$.

EXAMPLE 6 Finding equations of horizontal and vertical lines

Find equations of vertical and horizontal lines passing through the point (8, 5).

SOLUTION The x-coordinate of the point $(8, 5)$ is 8. The vertical line $x = 8$ passes through every point in the xy-plane with an x-coordinate of 8, including the point $(8, 5)$. Similarly, the horizontal line $y = 5$ passes through every point with a y-coordinate of 5, including $(8, 5)$.

Now Try Exercises 49 and 51

Parallel Lines Slope is an important concept when determining whether two lines are parallel or perpendicular. Two nonvertical parallel lines have equal slopes.

Parallel Lines

Two lines with slopes m_1 and m_2, neither of which is vertical, are parallel if and only if their slopes are equal; that is, $m_1 = m_2$.

NOTE The phrase "if and only if" is used when two statements are mathematically equivalent. If two nonvertical lines are parallel, then it is true that $m_1 = m_2$. Conversely, if two nonvertical lines have equal slopes, then they are parallel. Either condition implies the other.

EXAMPLE 7 Finding parallel lines

Find the slope-intercept form of a line parallel to $y = -2x + 5$, passing through $(4, 3)$.

SOLUTION The line $y = -2x + 5$ has slope -2, so any parallel line also has slope $m = -2$. The line passing through $(4, 3)$ with slope -2 is determined as follows.

$$y = -2(x - 4) + 3 \qquad \text{Point-slope form}$$

$$y = -2x + 8 + 3 \qquad \text{Distributive property}$$

$$y = -2x + 11 \qquad \text{Slope-intercept form}$$

Now Try Exercise 35

Perpendicular Lines Two lines with nonzero slopes are perpendicular if and only if the product of their slopes is equal to -1.

Perpendicular Lines

Two lines with nonzero slopes m_1 and m_2 are perpendicular if and only if their slopes have product -1; that is, $m_1 m_2 = -1$.

For perpendicular lines, m_1 and m_2 are *negative reciprocals*. That is, $m_1 = -\frac{1}{m_2}$ and $m_2 = -\frac{1}{m_1}$. Table 2.5 shows examples of values for m_1 and m_2 that result in perpendicular lines because $m_1 m_2 = -1$.

Table 2.5 Slopes of Perpendicular Lines

m_1	$\frac{1}{2}$	$\frac{6}{5}$	5	-1	$-\frac{2}{3}$
m_2	-2	$-\frac{5}{6}$	$-\frac{1}{5}$	1	$\frac{3}{2}$
$m_1 m_2$	-1	-1	-1	-1	-1

EXAMPLE 8 Finding perpendicular lines

Find the slope-intercept form of the line perpendicular to $y = -\frac{2}{3}x + 2$, passing through the point $(-2, 1)$. Graph the lines.

SOLUTION The line $y = -\frac{2}{3}x + 2$ has slope $-\frac{2}{3}$. The negative reciprocal of $m_1 = -\frac{2}{3}$ is $m_2 = \frac{3}{2}$. The slope-intercept form of a line having slope $\frac{3}{2}$ and passing through $(-2, 1)$ can be found as follows.

$$
\begin{aligned}
y &= m(x - x_1) + y_1 && \text{Point-slope form} \\
y &= \frac{3}{2}(x + 2) + 1 && \text{Let } m = \tfrac{3}{2},\ x_1 = -2, \text{ and } y_1 = 1. \\
y &= \frac{3}{2}x + 3 + 1 && \text{Distributive property} \\
y &= \frac{3}{2}x + 4 && \text{Slope-intercept form}
\end{aligned}
$$

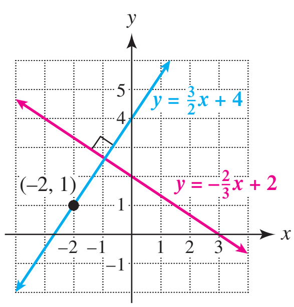


Figure 2.26 Perpendicular Lines

Figure 2.26 shows graphs of these perpendicular lines. ***Now Try Exercise 41***

NOTE If a graphing calculator is used to graph these lines, a square viewing rectangle must be used for the lines to appear perpendicular.

EXAMPLE 9 Determining a rectangle

In Figure 2.27 a rectangle is outlined by four lines denoted y_1, y_2, y_3, and y_4. Find the equation of each line.

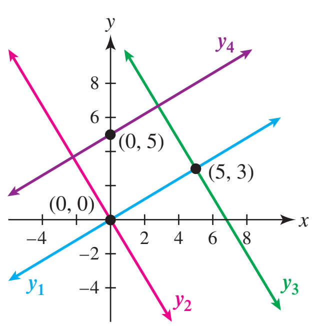


Figure 2.27

SOLUTION

***Line y_1*:** This line passes through the points $(0, 0)$ and $(5, 3)$, so $m = \frac{3}{5}$ and the y-intercept is 0. Its equation is $y_1 = \frac{3}{5}x$.

***Line y_2*:** This line passes through the point $(0, 0)$ and is perpendicular to y_1, so its slope is given by $m = -\frac{5}{3}$ and the y-intercept is 0. Its equation is $y_2 = -\frac{5}{3}x$.

***Line y_3*:** This line passes through the point $(5, 3)$ and is parallel to y_2, so its slope is given by $m = -\frac{5}{3}$. In a point-slope form, its equation is $y_3 = -\frac{5}{3}(x - 5) + 3$, which is equivalent to $y_3 = -\frac{5}{3}x + \frac{34}{3}$.

Line y_4*:** This line passes through the point $(0, 5)$ and is parallel to y_1, so its slope is given by $m = \frac{3}{5}$. Its equation is $y_4 = \frac{3}{5}x + 5$. ***Now Try Exercise 97

Calculator Help
To set a square viewing rectangle, see Appendix A (page AP-6).

CLASS DISCUSSION

Check the results from Example 9 by graphing the four equations in the same viewing rectangle. How does your graph compare with Figure 2.27? Why is it important to use a square viewing rectangle?

Modeling Data (Optional)

Point-slope form can sometimes be useful when modeling real data. In the next example we model the rise in the cost of tuition and fees at private colleges and universities.

EXAMPLE 10 Modeling data

Table 2.6 lists the average tuition and fees at private colleges for selected years.

Calculator Help

To make a scatterplot, see Appendix A (page AP-3). To plot data and graph an equation, see Appendix A (page AP-7).

Table 2.6 Tuition and Fees at Private Colleges

Year	1980	1985	1990	1995	2000	2005
Cost	\$3617	\$6121	\$9340	\$12,432	\$16,233	\$21,235

Source: The College Board.

(a) Make a scatterplot of the data.

(b) Find a linear function, given by $f(x) = m(x - x_1) + y_1$, that models the data. Interpret the slope m.

(c) Use f to estimate tuition and fees in 1998. Compare the estimate to the actual value of \$14,709. Did your answer involve interpolation or extrapolation?

SOLUTION

(a) See Figure 2.28.

[1978, 2007, 5] by [0, 25000, 5000]

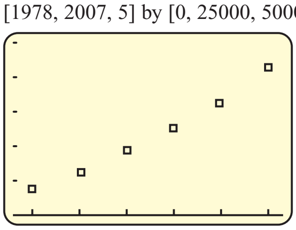


Figure 2.28

(b) The data table contains several points that could be used for (x_1, y_1). For example, we could choose the first data point, (1980, 3617), and then write

$$f(x) = m(x - 1980) + 3617.$$

To estimate a slope m we could choose two points that appear to lie on a line that models the data. For example, if we choose the first data point, (1980, 3617), and the fifth data point, (2000, 16233), then the slope m is

$$m = \frac{16,233 - 3617}{2000 - 1980} = 630.8.$$

This slope indicates that tuition and fees have risen, on average, \$630.80 per year.

Figure 2.29 shows the graphs of $f(x) = 630.8(x - 1980) + 3617$ and the data. It is important to realize that *answers may vary* when modeling real data because if you choose different points, the resulting equation for $f(x)$ will be different. Also, you may choose to adjust the slope or use linear regression to obtain a better fit to the data.

[1978, 2007, 5] by [0, 25000, 5000]

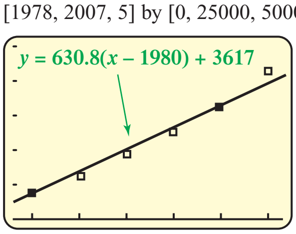


Figure 2.29

(c) To estimate the tuition and fees in 1998, evaluate $f(1998)$.

$$f(1998) = 630.8(1998 - 1980) + 3617 = \$14,971.40$$

This value differs from the actual value by less than \$300 and involves interpolation.

Now Try Exercise 85

MAKING CONNECTIONS

Modeling and Forms of Equations In Example 10 we modeled college tuition and fees by using the formula

$$f(x) = 630.8(x - 1980) + 3617.$$

This point-slope form readily reveals that tuition and fees cost \$3617 in 1980 and have risen, on average, \$630.80 per year. In slope-intercept form, this formula becomes

$$f(x) = 630.8x - 1,245,367.$$

Although the slope is apparent in slope-intercept form, it is less obvious that the actual value of tuition in 1980 was \$3617. Which form is more convenient often depends on the problem being solved.

Direct Variation

When a change in one quantity causes a proportional change in another quantity, the two quantities are said to *vary directly* or to *be directly proportional*. For example, if we work for \$8 per hour, our pay is proportional to the number of hours that we work. Doubling the hours doubles the pay, tripling the hours triples the pay, and so on.

Direct Variation

Let x and y denote two quantities. Then y is **directly proportional** to x, or y **varies directly** with x, if there exists a nonzero number k such that

$$y = kx.$$

The number k is called the **constant of proportionality** or the **constant of variation**.

If a person earns \$57.75 working for 7 hours, the constant of proportionality k is the hourly pay rate. If y represents the pay in dollars and x the hours worked, then k is found by substituting values for x and y into the equation $y = kx$ and solving for k. That is,

$$57.75 = k(7), \quad \text{or} \quad k = \frac{57.75}{7} = 8.25,$$

so the hourly pay rate is \$8.25 and, in general, $y = 8.25x$.

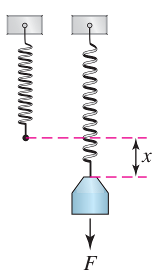


Figure 2.30 A Spring Being Stretched

An Application Hooke's law states that the distance that an elastic spring stretches beyond its natural length is *directly proportional* to the amount of weight hung on the spring, as illustrated in Figure 2.30. This law is valid whether the spring is stretched or compressed. The constant of proportionality is called the **spring constant**. Thus if a weight or force F is applied and the spring stretches a distance x beyond its natural length, then the equation $F = kx$ models this situation, where k is the spring constant.

EXAMPLE 11 Working with Hooke's Law

A 12-pound weight is hung on a spring, and it stretches 2 inches.

(a) Find the spring constant.

(b) Determine how far the spring will stretch when a 19-pound weight is hung on it.

SOLUTION

(a) Let $F = kx$, given that $F = 12$ pounds and $x = 2$ inches. Thus

$$12 = k(2), \quad \text{or} \quad k = 6,$$

and the spring constant equals 6.

(b) Thus $F = 19$ and $F = 6x$ implies that $19 = 6x$, or $x = \frac{19}{6} \approx 3.17$ inches.

Now Try Exercise 113

The following four-step method can often be used to solve variation problems.

Solving a Variation Problem

When solving a variation problem, the following steps can be used.

STEP 1: Write the general equation for the type of variation problem that you are solving.

STEP 2: Substitute given values in this equation so the constant of variation k is the only unknown value in the equation. Solve for k.

STEP 3: Substitute the value of k in the general equation in Step 1.

STEP 4: Use this equation to find the requested quantity.

EXAMPLE 12 Solving a direct variation problem

Let T vary directly with x, and suppose that $T = 33$ when $x = 5$. Find T when $x = 31$.

SOLUTION

STEP 1: The equation for direct variation is $T = kx$.

STEP 2: Substitute 33 for T and 5 for x. Then solve for k.

$$T = kx \qquad \text{Direct variation equation}$$

$$33 = k(5) \qquad \text{Let } T = 33 \text{ and } x = 5.$$

$$\frac{33}{5} = k \qquad \text{Divide each side by 5.}$$

STEP 3: Thus $T = \frac{33}{5}x$, or $T = 6.6x$.

STEP 4: When $x = 31$, we have $T = 6.6(31) = 204.6$.

Now Try Exercise 101

Suppose that for each point (x, y) in a data set the ratios $\frac{y}{x}$ are all equal to some constant k. That is, $\frac{y}{x} = k$ for each data point. Then $y = kx$, and so y varies directly with x and the constant of variation is k. In addition, the data points (x, y) all lie on the line $y = kx$, which has slope k and passes through the origin. These concepts are used in the next example.

EXAMPLE 13 Modeling memory requirements

Table 2.7 lists the megabytes (MB) x needed to record y seconds of music.

Table 2.7 Recording Digital Music

x (MB)	0.23	0.49	1.16	1.27
y (sec)	10.7	22.8	55.2	60.2

Source: Gateway 2000 System CD.

(a) Compute the ratios $\frac{y}{x}$ for the four data points. Does y vary directly with x? If it does, what is the constant of variation k?

(b) Estimate the seconds of music that can be stored on 5 megabytes.

(c) Graph the data in Table 2.7 and the line $y = kx$.

SOLUTION

(a) The four ratios $\frac{y}{x}$ from Table 2.7 are

$$\frac{10.7}{0.23} \approx 46.5, \quad \frac{22.8}{0.49} \approx 46.5, \quad \frac{55.2}{1.16} \approx 47.6, \quad \text{and} \quad \frac{60.2}{1.27} \approx 47.4.$$

[0, 1.5, 0.5] by [0, 70, 10]

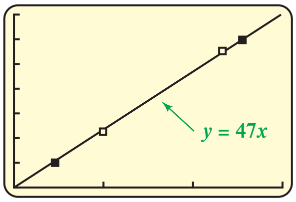


Figure 2.31

Because the ratios are nearly equal and these are real data, it is reasonable to say that y is directly proportional to x. The constant of proportionality is about **47**, the average of the four ratios. This means that $y = 47x$ and we can store about **47** seconds of music per megabyte.

(b) Let $x = 5$ in the equation $y = 47x$, to obtain $y = 47(5) = 235$ seconds.

(c) Graphs of the data and the line $y = 47x$ are shown in Figure 2.31.

Now Try Exercise 115 ◀

2.2 Putting It All Together

The following table summarizes some important topics.

Concept	Comments	Examples
Point-slope form $y = m(x - x_1) + y_1$ or $y - y_1 = m(x - x_1)$	Used to find the equation of a line, given two points or one point and the slope	Given two points (5, 1) and (4, 3), first compute $m = \frac{3 - 1}{4 - 5} = -2$. An equation of this line is $y = -2(x - 5) + 1$.
Slope-intercept form $y = mx + b$	A unique equation for a line, determined by the slope m and the y-intercept b	An equation of the line with slope 5 and y-intercept -4 is $y = 5x - 4$.
Direct variation	The variable y is directly proportional to x or varies directly with x if $y = kx$ for some nonzero constant k. Constant k is the constant of proportionality or the constant of variation.	If the sales tax rate is 7%, the sales tax y on a purchase of x dollars is calculated by $y = 0.07x$, where $k = 0.07$. The sales tax on a purchase of \$125 is $y = 0.07(125) = \$8.75$.

The following table summarizes the important concepts concerning special types of lines.

Concept	Equation(s)	Examples
Horizontal line	$y = b$, where b is a constant	A horizontal line with y-intercept 7 has the equation $y = 7$.
Vertical line	$x = k$, where k is a constant	A vertical line with x-intercept -8 has the equation $x = -8$.
Parallel lines	$y = m_1x + b_1$ and $y = m_2x + b_2$, where $m_1 = m_2$	The lines given by $y = -3x - 1$ and $y = -3x + 5$ are parallel because they both have slope -3.
Perpendicular lines	$y = m_1x + b_1$ and $y = m_2x + b_2$, where $m_1m_2 = -1$	The lines $y = 2x - 5$ and $y = -\frac{1}{2}x + 2$ are perpendicular because $m_1m_2 = 2\left(-\frac{1}{2}\right) = -1$.

2.2 Exercises

Equations of Lines

Exercises 1–4: Find the point-slope form of the line passing through the given points. Use the first point as (x_1, y_1). Plot the points and graph the line by hand.

1. $(1, 2), (3, -2)$

2. $(-2, 3), (1, 0)$

3. $(-3, -1), (1, 2)$

4. $(-1, 2), (-2, -3)$

Exercises 5–10: Find a point-slope form of the line satisfying the conditions. Use the first point given for (x_1, y_1). Then convert the equation to slope-intercept form.

5. Slope -2.4, passing through $(4, 5)$

6. Slope 1.7, passing through $(-8, 10)$

7. Passing through $(1, -2)$ and $(-9, 3)$

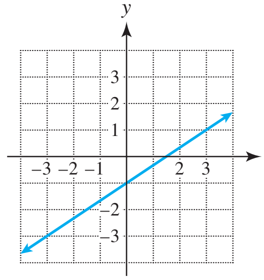
8. Passing through $(-6, 10)$ and $(5, -12)$

9. x-intercept 4, y-intercept -3

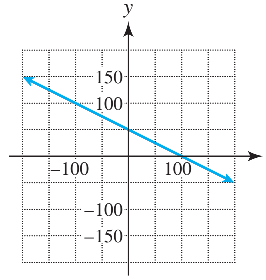
10. x-intercept -2, y-intercept 5

Exercises 11–14: Find the slope-intercept form for the line in the figure.

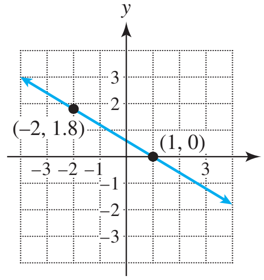
11.



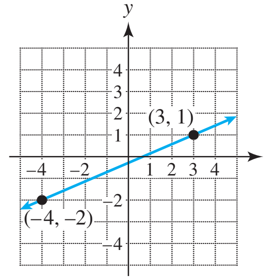
12.



13.



14.



Exercises 15–20: ***Concepts*** *Match the equation to its graph (a–f) shown in the next column.*

15. $y = m(x - x_1) + y_1, m > 0$

16. $y = m(x - x_1) + y_1, m < 0$

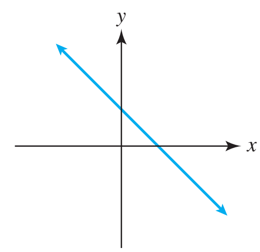
17. $y = mx, m > 0$

18. $y = mx + b, m < 0$ and $b > 0$

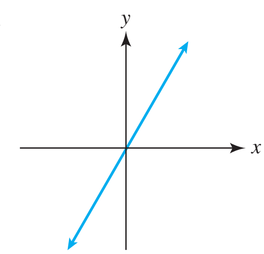
19. $x = k, k > 0$

20. $y = b, b < 0$

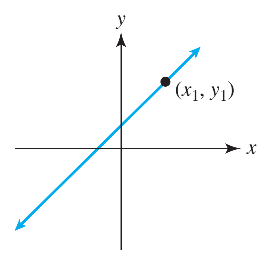
a.



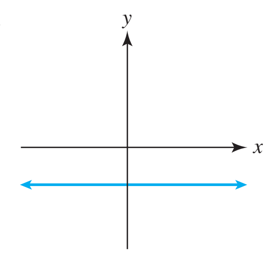
b.



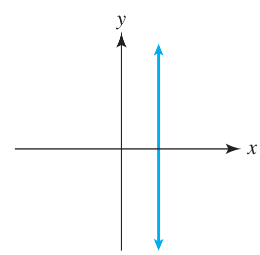
c.



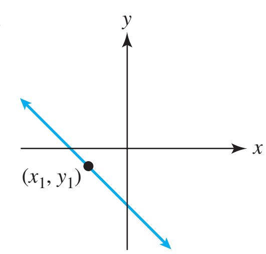
d.



e.



f.



Exercises 21–48: Find the slope-intercept form for the line satisfying the conditions.

21. Passing through $(-1, -4)$ and $(1, 2)$

22. Passing through $(-1, 6)$ and $(2, -3)$

23. Passing through $(4, 5)$ and $(1, -3)$

24. Passing through $(8, -2)$ and $(-2, 3)$

25. y-intercept 5, slope -7.8

26. y-intercept -155, slope 5.6

27. y-intercept 45, x-intercept 90

28. x-intercept -6, y-intercept -8

29. Slope -3, passing through $(0, 5)$

30. Slope $\frac{1}{3}$, passing through $\left(\frac{1}{2}, -2\right)$

31. Passing through $(0, -6)$ and $(4, 0)$

32. Passing through $\left(\frac{3}{4}, -\frac{1}{4}\right)$ and $\left(\frac{5}{4}, \frac{7}{4}\right)$

33. Passing through $\left(\frac{1}{2}, \frac{3}{4}\right)$ and $\left(\frac{1}{5}, \frac{2}{3}\right)$

34. Passing through $\left(-\frac{7}{3}, \frac{5}{3}\right)$ and $\left(\frac{5}{6}, -\frac{7}{6}\right)$

35. Parallel to $y = 4x + 16$, passing through $(-4, -7)$

36. Parallel to the line $y = -\frac{3}{4}(x - 100) - 99$, passing through $(1, 3)$

37. Perpendicular to the line $y = -\frac{2}{3}(x - 1980) + 5$, passing through $(1980, 10)$

38. Perpendicular to $y = 6x - 10$, passing through $(15, -7)$

39. Parallel to $y = \frac{2}{3}x + 3$, passing through $(0, -2.1)$

40. Parallel to $y = -4x - \frac{1}{4}$, passing through $(2, -5)$

41. Perpendicular to $y = -2x$, passing through $(-2, 5)$

42. Perpendicular to $y = -\frac{6}{7}x + \frac{3}{7}$, passing through $(3, 8)$

43. Perpendicular to $x + y = 4$, passing through $(15, -5)$

44. Parallel to $2x - 3y = -6$, passing through $(4, -9)$

45. Passing through $(5, 7)$ and parallel to the line passing through $(1, 3)$ and $(-3, 1)$

46. Passing through $(1990, 4)$ and parallel to the line passing through $(1980, 3)$ and $(2000, 8)$

47. Passing through $(-2, 4)$ and perpendicular to the line passing through $\left(-5, \frac{1}{2}\right)$ and $\left(-3, \frac{2}{3}\right)$

48. Passing through $\left(\frac{3}{4}, \frac{1}{4}\right)$ and perpendicular to the line passing through $(-3, -5)$ and $(-4, 0)$

Exercises 49–56: Find an equation of the line satisfying the conditions.

49. Vertical, passing through $(-5, 6)$

50. Vertical, passing through $(1.95, 10.7)$

51. Horizontal, passing through $(-5, 6)$

52. Horizontal, passing through $(1.95, 10.7)$

53. Perpendicular to $y = 15$, passing through $(4, -9)$

54. Perpendicular to $x = 15$, passing through $(1.6, -9.5)$

55. Parallel to $x = 4.5$, passing through $(19, 5.5)$

56. Parallel to $y = -2.5$, passing through $(1985, 67)$

Finding Intercepts

Exercises 57–68: Determine the x- and y-intercepts on the graph of the equation. Graph the equation.

57. $4x - 5y = 20$

58. $-3x - 5y = 15$

59. $x - y = 7$

60. $15x - y = 30$

61. $6x - 7y = -42$

62. $5x + 2y = -20$

63. $y - 3x = 7$

64. $4x - 3y = 6$

65. $0.2x + 0.4y = 0.8$

66. $\frac{2}{3}y - x = 1$

67. $y = 8x - 5$

68. $y = -1.5x + 15$

*Exercises 69–72: The **intercept form of a line** is $\frac{x}{a} + \frac{y}{b} = 1$. Determine the x- and y-intercepts on the graph of the equation. Draw a conclusion about what the constants a and b represent in this form.*

69. $\dfrac{x}{5} + \dfrac{y}{7} = 1$

70. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

71. $\dfrac{2x}{3} + \dfrac{4y}{5} = 1$

72. $\dfrac{5x}{6} - \dfrac{y}{2} = 1$

Exercises 73 and 74: (Refer to Exercises 69–72.) Write the intercept form for the line with the given intercepts.

73. x-intercept 5, y-intercept 9

74. x-intercept $\frac{2}{3}$, y-intercept $-\frac{5}{4}$

Interpolation and Extrapolation

Exercises 75–78: The table lists data that are exactly linear.

(a) Find the slope-intercept form of the line that passes through these data points.

(b) Predict y when x = −2.7 and 6.3. Decide if these calculations involve interpolation or extrapolation.

75.

x	-3	-2	-1	0	1
y	-7.7	-6.2	-4.7	-3.2	-1.7

76.

x	-2	-1	0	1	2
y	10.2	8.5	6.8	5.1	3.4

77.

x	5	23	32	55	61
y	94.7	56.9	38	-10.3	-22.9

78.

x	-11	-8	-7	-3	2
y	-16.1	-10.4	-8.5	-0.9	8.6

79. ***Air Safety Inspectors*** The number of air safety inspectors for selected years is shown in the table.

Year	1998	1999	2000
Inspectors	3305	3185	3089

Source: Federal Aviation Administration.

(a) Find a linear function f that models these data. Is f exact or approximate?

(b) Use f to estimate the number of inspectors in 2005. Compare your answer to the actual value of 3450. Did your estimate involve interpolation or extrapolation?

(c) Explain the difficulty with trying to model these data with a linear function.

80. ***Deaths on School Grounds*** Deaths on school grounds nationwide for school years ending in year x are shown in the table.

x (year)	1998	1999	2000
y (deaths)	43	26	9

Source: FBI.

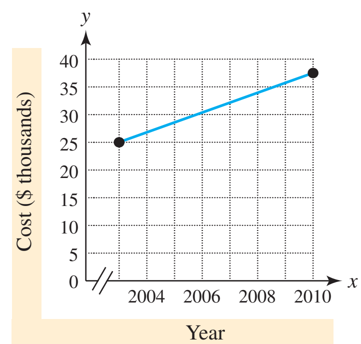
(a) Find a linear function f that models these data. Is f exact or approximate?

(b) Use f to estimate the number of deaths on school grounds in 2003. Compare your answer to the actual value of 49. Did your estimate involve interpolation or extrapolation?

(c) Explain the difficulty with trying to model these data with a linear function.

Applications

81. ***Projected Cost of College*** In 2003 the average annual cost of attending a private college or university, including tuition, fees, room, and board, was \$25,000. This cost is projected to rise to \$37,000 in 2010, as illustrated in the figure. (**Source:** Cerulli Associates.)

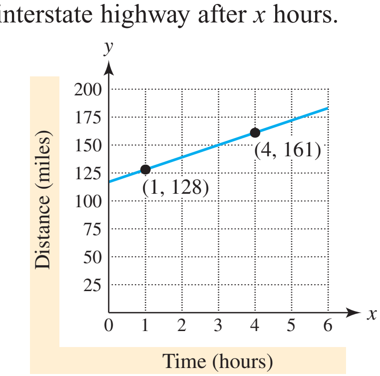


(a) Find a point-slope form of the line passing through the points (2003, 25000) and (2010, 37000). Interpret the slope.

(b) Use the equation to estimate the cost of attending a private college in 2007. Did your estimate involve interpolation or extrapolation?

(c) Find the slope-intercept form of this line.

82. ***Distance*** A person is riding a bicycle along a straight highway. The graph shows the rider's distance y in miles from an interstate highway after x hours.



(a) How fast is the bicyclist traveling?

(b) Find the slope-intercept form of the line.

(c) How far was the bicyclist from the interstate highway initially?

(d) How far was the bicyclist from the interstate highway after 1 hour and 15 minutes?

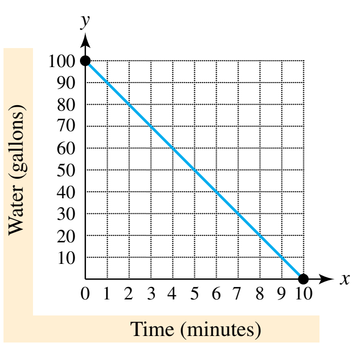
83. ***Music on the Internet*** In 2002 sales of premium online music totaled \$1.6 billion. In 2005 this revenue reached \$3.6 billion. (**Source:** Jupiter Research.)

(a) Find a point-slope form of the line passing through (2002, 1.6) and (2005, 3.6). Interpret the slope.

(b) Use the equation to estimate projected sales in 2008. Did you use interpolation or extrapolation?

(c) Find the slope-intercept form of this line.

84. ***Water in a Tank*** The graph shows the amount of water in a 100-gallon tank after x minutes have elapsed.



(a) Is water entering or leaving the tank? How much water is in the tank after 3 minutes?

(b) Find the x- and y-intercepts. Interpret each intercept.

(c) Find the slope-intercept form of the equation of the line. Interpret the slope.

(d) Estimate the x-coordinate of the point $(x, 50)$ that lies on the line.

85. ***E-mail Spam*** The table lists the average number of worldwide spam messages *daily* in billions for selected years.

Year	1999	2000	2001	2002	2003	2004
Messages	1.0	2.3	4.0	5.6	7.3	8.8

Source: IDC.

(a) Make a scatterplot of the data.

(b) Find a linear function f that models these data. (Answers may vary.) Interpret the slope m.

(c) Use your function to estimate the average number of worldwide spam messages daily during 2007.

86. ***Tuition and Fees*** The table in the next column lists average tuition and fees at public 4-year colleges.

Year	1980	1985	1990
Tuition and fees	\$804	\$1318	\$1908

Year	1995	2000	2005
Tuition and fees	\$2811	\$3487	\$5491

Source: The College Board.

(a) Make a scatterplot of the data.

(b) Find a linear function that models the data. Interpret the slope m.

(c) Use this function to estimate tuition in 1992. Compare it to the actual value of \$2334.

(d) Of the six data values, which one would you remove to make the data "more linear"? Explain.

87. ***Toyota Vehicles Sold*** The table lists the U.S. sales of Toyota vehicles in millions.

Year	1998	2000	2002	2004
Vehicles	1.4	1.6	1.8	2.0

Source: Autodata.

(a) Make a scatterplot of the data.

(b) Find $f(x) = m(x - x_1) + y_1$ so that $f(x)$ models these data. Interpret the slope m.

(c) Is $f(x)$ an exact or approximate model for the data listed in the table?

88. ***Farm Pollution*** In 1988 the number of farm pollution incidents reported in England and Wales was 4000. This number had increased at a rate of 280 per year since 1979. (**Source:** C. Mason, *Biology of Freshwater Pollution*.)

(a) Find an equation $y = m(x - x_1) + y_1$ that models these data, where y represents the number of pollution incidents during the year x.

(b) Estimate the number of incidents in 1975.

89. ***Cost of Driving*** The cost of driving a car includes both fixed costs and mileage costs. Assume that insurance and car payments cost \$350 per month and gasoline, oil, and routine maintenance cost \$0.29 per mile.

(a) Find a linear function f that gives the annual cost of driving this car x miles.

(b) What does the y-intercept on the graph of f represent?

90. ***Average Wages*** The average hourly wage (adjusted to 1982 dollars) was \$8.46 in 1970 and \$8.18 in 2005. (**Source:** Department of Commerce.)

(a) Find an equation of a line that passes through the points (1970, 8.46) and (2005, 8.18).

(b) Interpret the slope.

(c) Approximate the hourly wage in 2000. Compare the estimate to the actual value of \$8.04.

Exercises 91 and 92: ***Modeling Real Data*** *The table contains data that can be modeled by a linear function f.*

(a) Make a scatterplot of the data. (Do not try to plot the undetermined point in the table.)

(b) Find a formula for f. Graph f together with the data.

(c) Interpret the slope m.

(d) Use f to approximate the undetermined value.

91. Asian-American population in millions

Year	1996	1999	2002	2005	2008
Population	9.7	10.9	12.0	13.4	?

Source: Bureau of the Census.

92. Population of the western states in millions

Year	1950	1970	1990	2010
Population	20.2	34.8	52.8	?

Source: Bureau of the Census.

Perspectives and Viewing Rectangles

93. Graph $y = \frac{1}{1024}x + 1$ in [0, 3, 1] by [−2, 2, 1].

(a) Is the graph a horizontal line?

(b) Why does the calculator screen appear as it does?

94. Graph $y = 1000x + 1000$ in the standard window.

(a) Is the graph a vertical line?

(b) Explain why the calculator screen appears as it does.

95. ***Square Viewing Rectangle*** Graph the lines $y = 2x$ and $y = -\frac{1}{2}x$ in the standard viewing rectangle.

(a) Do the lines appear to be perpendicular?

(b) Graph the lines in the following viewing rectangles.

i. [−15, 15, 1] by [−10, 10, 1]

ii. [−10, 10, 1] by [−3, 3, 1]

iii. [−3, 3, 1] by [−2, 2, 1]

Do the lines appear to be perpendicular in any of these viewing rectangles?

(c) Determine the viewing rectangles where perpendicular lines will appear perpendicular. (Answers may vary.)

96. ***Square Viewing Rectangle*** Continuing with Exercise 95, make a conjecture about which viewing rectangles result in the graph of a circle with radius 5 and center at the origin appearing circular.

i. [−9, 9, 1] by [−6, 6, 1]

ii. [−5, 5, 1] by [−10, 10, 1]

iii. [−5, 5, 1] by [−5, 5, 1]

iv. [−18, 18, 1] by [−12, 12, 1]

Test your conjecture by graphing this circle in each viewing rectangle. (*Hint:* Graph $y_1 = \sqrt{25 - x^2}$ and $y_2 = -\sqrt{25 - x^2}$ to create the circle.)

Graphing a Rectangle

Exercises 97–100: (Refer to Example 9.) A rectangle is determined by the stated conditions. Find the slope-intercept form of the four lines that outline the rectangle.

97. Vertices (0, 0), (2, 2), and (1, 3)

98. Vertices (1, 1), (5, 1), and (5, 5)

99. Vertices (4, 0), (0, 4), (0, −4), and (−4, 0)

100. Vertices (1, 1) and (2, 3); the point (3.5, 1) lies on a side of the rectangle.

Direct Variation

Exercises 101–104: Let y be directly proportional to x. Complete the following.

101. Find y when $x = 5$, if $y = 7$ when $x = 14$.

102. Find y when $x = 2.5$, if $y = 13$ when $x = 10$.

103. Find y when $x = \frac{1}{2}$, if $y = \frac{3}{2}$ when $x = \frac{2}{3}$.

104. Find y when $x = 1.3$, if $y = 7.2$ when $x = 5.2$.

Exercises 105–108: Find the constant of proportionality k and the undetermined value in the table if y is directly proportional to x. Support your answer by graphing the equation $y = kx$ and the data points.

105.

x	3	5	6	8
y	7.5	12.5	15	?

106.

x	1.2	4.3	5.7	?
y	3.96	14.19	18.81	23.43

107. Sales tax y on a purchase of x dollars

x	\$25	\$55	?
y	\$1.50	\$3.30	\$5.10

108. Cost y of buying x compact discs having the same price

x	3	4	5
y	\$41.97	\$55.96	?

109. ***Cost of Tuition*** The cost of tuition is directly proportional to the number of credits taken. If 11 credits cost \$720.50, find the cost of taking 16 credits. What is the constant of proportionality?

110. ***Strength of a Beam*** The maximum load that a horizontal beam can carry is directly proportional to its width. If a beam 1.5 inches wide can support a load of 250 pounds, find the load that a beam of the same type can support if its width is 3.5 inches.

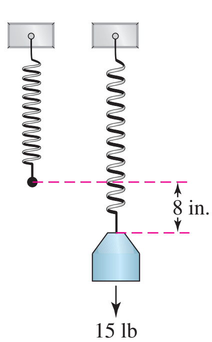
111. ***Antarctic Ozone Layer*** Stratospheric ozone occurs in the atmosphere between altitudes of 12 and 18 miles. Ozone in the stratosphere is frequently measured in Dobson units, where 300 Dobson units corresponds to an ozone layer 3 millimeters thick. In 1991 the reported minimum in the Antarctic *ozone hole* was about 110 Dobson units. (**Source:** R. Huffman, *Atmospheric Ultraviolet Remote Sensing*.)

(a) The thickness y of the ozone layer is directly proportional to the number of Dobson units x. Find the constant of proportionality k.

(b) How thick was the ozone layer in 1991?

112. ***Weight on Mars*** The weight of an object on Earth is directly proportional to the weight of an object on Mars. If a 25-pound object on Earth weighs 10 pounds on Mars, how much would a 195-pound astronaut weigh on Mars?

113. ***Hooke's Law*** Suppose a 15-pound weight stretches a spring 8 inches, as shown in the figure.



(a) Find the spring constant.

(b) How far will a 25-pound weight stretch this spring?

114. ***Hooke's Law*** If an 80-pound force compresses a spring 3 inches, how much force must be applied to compress the spring 7 inches?

115. ***Force of Friction*** The table lists the force F needed to push a cargo box weighing x pounds on a wood floor.

x (lb)	150	180	210	320
F (lb)	26	31	36	54

(a) Compute the ratio $\frac{F}{x}$ for each data pair in the table. Interpret these ratios.

(b) Approximate a constant of proportionality k satisfying $F = kx$. (k is the *coefficient of friction*.)

(c) Graph the data and the equation together.

(d) Estimate the force needed to push a 275-pound cargo box on the floor.

116. ***Electrical Resistance*** The electrical resistance of a wire varies directly with its length. If a 255-foot wire has a resistance of 1.2 ohms, find the resistance of 135 feet of the same type of wire. Interpret the constant of proportionality in this situation.

Writing about Mathematics

117. Compare the slope-intercept form with the point-slope form. Give examples of each.

118. Give an example of two quantities in real life that vary directly. Explain your answer. Use an equation to describe the relationship between the two quantities.

119. The graph of $f(x) = 3 - 2x$ passes through $(-1, 5)$ and $(3, -3)$. Evaluate $f(1)$ and find the midpoint of the two points. Compare and explain your results.

120. Explain how you would find the equation of a line passing through two points. Give an example.

EXTENDED AND DISCOVERY EXERCISES

*Exercises 1 and 2: **Estimating Populations** Biologists sometimes use direct variation to estimate the number of fish in small lakes. They start by tagging a small number of fish and then releasing them. They assume that over a period of time, the tagged fish distribute themselves evenly throughout the lake. Later, they collect a second sample. The total* *number of fish and the number of tagged fish in the second sample are counted. To determine the total population of fish in the lake, biologists assume that the proportion of tagged fish in the second sample is equal to the proportion of tagged fish in the entire lake. This technique can also be used to count other types of animals, such as birds, when they are not migrating.*

1. Eighty-five fish are tagged and released into a pond. A later sample of 94 fish from the pond contains 13 tagged fish. Estimate the number of fish in the pond.

2. Sixty-three blackbirds are tagged and released. Later it is estimated that out of a sample of 32 blackbirds, only 8 are tagged. Estimate the population of blackbirds in the area.

CHECKING BASIC CONCEPTS FOR SECTIONS 2.1 AND 2.2

1. Graph $f(x) = 4 - 2x$ by hand. Identify the slope, the x-intercept, and the y-intercept.

2. The death rate from heart disease for people ages 15 through 24 is 2.7 per 100,000 people.

(a) Write a function f that models the number of deaths in a population of x million people 15 to 24 years old.

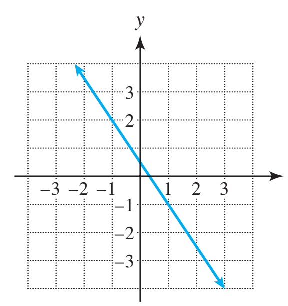
(b) There are about 39 million people in the United States who are 15 to 24 years old. Estimate the number of deaths from heart disease in this age group.

3. A driver of a car is initially 50 miles south of home, driving 60 miles per hour south. Write a function f that models the distance between the driver and home.

4. Find an equation of the line passing through the points $(-3, 4)$ and $(5, -2)$. Give equations of lines that are parallel and perpendicular to this line.

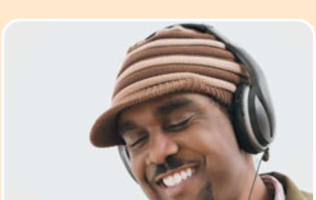
5. Find equations of horizontal and vertical lines that pass through the point $(-4, 7)$.

6. Write the slope-intercept form of the line.



7. Find the x- and y-intercepts of the graph of the equation $-3x + 2y = -18$.

- **Learn about equations and recognize a linear equation**
- **Solve linear equations symbolically**
- **Solve linear equations graphically and numerically**
- **Solve problems involving percentages**
- **Apply problem-solving strategies**



Introduction

In Example 4 of Section 2.2, we modeled iPod sales y in millions during year x with the equation of a line, or *linear function*, given by

$$f(x) = 21(x - 2004) + 4.4.$$

To predict the year when iPod sales might be **130** million, we could set the formula for $f(x)$ equal to **130** and solve the *linear equation*

$$130 = 21(x - 2004) + 4.4$$

for x. This section discusses linear equations and their solutions. See Example 4 in this section.

Equations

An **equation** is a statement that two mathematical expressions are equal. Equations always contain an equals sign. Some examples of equations include

$$x + 15 = 9x - 1, \quad x^2 - 2x + 1 = 2x, \quad z + 5 = 0,$$
$$xy + x^2 = y^3 + x, \quad \text{and} \quad 1 + 2 = 3.$$

The first three equations have one variable, the fourth equation has two variables, and the fifth equation contains only constants. For now, our discussion concentrates on equations with one variable.

To **solve** an equation means to find all values for the variable that make the equation a true statement. Such values are called **solutions**. The set of all solutions is the **solution set**. The solutions to the equation $x^2 - 1 = 0$ are 1 or -1, written as $x = \pm 1$. Either value for x **satisfies** the equation. The solution set is $\{-1, 1\}$. Two equations are **equivalent** if they have the same solution set. For example, the equations $x + 2 = 5$ and $x = 3$ are equivalent.

If an equation has no solutions, then its solution set is empty and the equation is called a **contradiction**. The equation $x + 2 = x$ has no solutions and is a contradiction. However, if every (meaningful) value for the variable is a solution, then the equation is an **identity**. The equation $x + x = 2x$ is an identity because every value for x makes the equation true. Any equation that is satisfied by some, but not all, values of the variable is a **conditional equation**. The equation $x^2 - 1 = 0$ is a conditional equation. Only the values -1 and 1 for x make this equation a true statement.

Like functions, equations can be either *linear* or *nonlinear*. A linear equation is one of the simplest types of equations.

Linear Equation in One Variable

A **linear equation** in one variable is an equation that can be written in the form

$$ax + b = 0,$$

where a and b are constants with $a \neq 0$.

If an equation is not linear, then we say that it is a **nonlinear equation**. The following are examples of linear equations. In each case, rules of algebra could be used to write the equation in the form $ax + b = 0$ with $a \neq 0$. A linear equation has *exactly one* solution: $-\frac{b}{a}$.

$$x - 12 = 0, \quad 2x - 4 = -x, \quad 2(1 - 4x) = 16, \quad x - 5 + 3(x - 1) = 0$$

Symbolic Solutions

Linear equations can be solved symbolically, and the solution is *always exact.* To solve a linear equation symbolically, we usually apply the *properties of equality* to the given equation and transform it into an equivalent equation that is simpler.

Properties of Equality

Addition Property of Equality

If a, b, and c are real numbers, then

$$a = b \quad \text{is equivalent to} \quad a + c = b + c.$$

Multiplication Property of Equality

If a, b, and c are real numbers with $c \neq 0$, then

$$a = b \quad \text{is equivalent to} \quad ac = bc.$$

Loosely speaking, the addition property states that "if equals are added to equals, the results are equal." For example, if $x + 5 = 15$, then we can add -5 to each side of the equation, or equivalently subtract 5 from each side, to obtain the following.

$$\begin{aligned} x + 5 &= 15 && \text{Given equation} \\ x + 5 - 5 &= 15 - 5 && \text{Subtract 5 from each side.} \\ x &= 10 && \text{Simplify.} \end{aligned}$$

Similarly, the multiplication property states that "if equals are multiplied by nonzero equals, the results are equal." For example, if $5x = 20$, then we can multiply each side by $\frac{1}{5}$, or equivalently divide each side by 5, to obtain the following.

$$\begin{aligned} 5x &= 20 && \text{Given equation} \\ \frac{5x}{5} &= \frac{20}{5} && \text{Divide each side by 5.} \\ x &= 4 && \text{Simplify.} \end{aligned}$$

These two properties along with the distributive property are applied in the next two examples.

EXAMPLE 1 Solving a linear equation symbolically

Solve the equation $3(x - 4) = 2x - 1$. Check your answer.

SOLUTION

Getting Started First we apply the distributive property: $a(b - c) = ab - ac$. Thus

$$3(x - 4) = 3 \cdot x - 3 \cdot 4 = 3x - 12. ▶$$

Algebra Review
To review the distributive properties, see Chapter R (page R-15).

Solving the given equation results in the following.

$$\begin{aligned} 3(x - 4) &= 2x - 1 && \text{Given equation} \\ 3x - 12 &= 2x - 1 && \text{Distributive property} \\ 3x - 2x - 12 + 12 &= 2x - 2x - 1 + 12 && \text{Subtract 2x and add 12.} \\ 3x - 2x &= 12 - 1 && \text{Simplify.} \\ x &= 11 && \text{Simplify.} \end{aligned}$$

The solution is 11. We can check our answer as follows.

$$\begin{aligned} 3(x - 4) &= 2x - 1 && \text{Given equation} \\ 3(11 - 4) &\stackrel{?}{=} 2 \cdot 11 - 1 && \text{Let } x = 11. \\ 21 &= 21 && \text{The answer checks.} \end{aligned}$$

Now Try Exercise 23 ◀

EXAMPLE 2 Solving a linear equation symbolically

Solve $3(2x - 5) = 10 - (x + 5)$. Check your answer.

SOLUTION

Getting Started In this problem subtraction must be distributed over the quantity $(x + 5)$. Thus

$$10 - (x + 5) = 10 - 1(x + 5) = 10 - x - 5. ▶$$

Solving the given equation results in the following.

$$\begin{aligned} 3(2x - 5) &= 10 - (x + 5) && \text{Given equation} \\ 6x - 15 &= 10 - x - 5 && \text{Distributive property} \\ 6x - 15 &= 5 - x && \text{Simplify.} \\ 7x - 15 &= 5 && \text{Add } x \text{ to each side.} \\ 7x &= 20 && \text{Add 15 to each side.} \\ x &= \frac{20}{7} && \text{Divide each side by 7.} \end{aligned}$$

The solution is $\frac{20}{7}$. To check this answer, let $x = \frac{20}{7}$ and simplify.

$$\begin{aligned} 3(2x - 5) &= 10 - (x + 5) && \text{Given equation} \\ 3\left(2 \cdot \frac{20}{7} - 5\right) &\stackrel{?}{=} 10 - \left(\frac{20}{7} + 5\right) && \text{Let } x = \tfrac{20}{7}. \\ \frac{15}{7} &= \frac{15}{7} && \text{The answer checks.} \end{aligned}$$

Now Try Exercise 27

Fractions and Decimals When fractions or decimals appear in an equation, we sometimes can make our work simpler by multiplying each side of the equation by the least common denominator (LCD) or a common denominator of all fractions in the equation. This method is illustrated in the next example.

EXAMPLE 3 Eliminating fractions and decimals

Solve each linear equation.

(a) $\frac{t - 2}{4} - \frac{1}{3}t = 5 - \frac{1}{12}(3 - t)$ **(b)** $0.03(z - 3) - 0.5(2z + 1) = 0.23$

Algebra Review
To review least common multiples and least common denominators, see Chapter R (page R-32).

SOLUTION

(a) To eliminate fractions, multiply each side (or term in the equation) by the LCD, 12.

$$\begin{aligned} \frac{t - 2}{4} - \frac{1}{3}t &= 5 - \frac{1}{12}(3 - t) && \text{Given equation} \\ \frac{12(t - 2)}{4} - \frac{12}{3}t &= 12(5) - \frac{12}{12}(3 - t) && \text{Multiply each side (term) by 12.} \\ 3(t - 2) - 4t &= 60 - (3 - t) && \text{Simplify.} \\ 3t - 6 - 4t &= 60 - 3 + t && \text{Distributive property} \\ -t - 6 &= 57 + t && \text{Combine like terms on each side.} \\ -2t &= 63 && \text{Add } -t \text{ and 6 to each side.} \\ t &= -\frac{63}{2} && \text{Divide each side by } -2. \end{aligned}$$

The solution is $-\frac{63}{2}$.

(b) To eliminate decimals, multiply each side (or term in the equation) by 100.

$$
\begin{aligned}
0.03(z - 3) - 0.5(2z + 1) &= 0.23 && \text{Given equation} \\
3(z - 3) - 50(2z + 1) &= 23 && \text{Multiply each side (term) by 100.} \\
3z - 9 - 100z - 50 &= 23 && \text{Distributive property} \\
-97z - 59 &= 23 && \text{Combine like terms.} \\
-97z &= 82 && \text{Add 59 to each side.} \\
z &= -\frac{82}{97} && \text{Divide each side by } -97.
\end{aligned}
$$

The solution is $-\frac{82}{97}$.

Now Try Exercises 33 and 37

An Application In the next example we solve the equation presented in the introduction to this section.

EXAMPLE 4 Estimating iPod sales

The linear function defined by $f(x) = 21(x - 2004) + 4.4$ estimates iPod sales (in millions of units) during fiscal year x. Use $f(x)$ to estimate when iPod sales could reach 130 million.

SOLUTION

We need to find x so that $f(x) = 130$.

$$
\begin{aligned}
21(x - 2004) + 4.4 &= 130 && \text{Equation to be solved} \\
21(x - 2004) &= 125.6 && \text{Subtract 4.4 from each side.} \\
21x - 42{,}084 &= 125.6 && \text{Distributive property} \\
21x &= 42{,}209.6 && \text{Add 42,084 to each side.} \\
x &= \frac{42{,}209.6}{21} && \text{Divide each side by 21.} \\
x &\approx 2010 && \text{Approximate.}
\end{aligned}
$$

This model predicts that iPod sales could reach **130** million units in fiscal **2010**.

Now Try Exercise 91

Contradictions, Identities, and Conditional Equations The next example illustrates how an equation can have no solutions (contradiction), one solution (conditional equation), or infinitely many solutions (identity).

EXAMPLE 5 Identifying contradictions, identities, and conditional equations

Identify each equation as a contradiction, identity, or conditional equation.

(a) $7 + 6x = 2(3x + 1)$ **(b)** $2x - 5 = 3 - (1 + 2x)$

(c) $2(5 - x) - 25 = 3(x - 5) - 5x$

SOLUTION

(a)

$$
\begin{aligned}
7 + 6x &= 2(3x + 1) && \text{Given equation} \\
7 + 6x &= 6x + 2 && \text{Distributive property} \\
7 &= 2 && \text{Subtract } 6x \text{ from each side.}
\end{aligned}
$$

The statement $7 = 2$ is false and there are *no* solutions. The equation is a contradiction.

(b)

$$\begin{aligned} 2x - 5 &= 3 - (1 + 2x) && \text{Given equation} \\ 2x - 5 &= 3 - 1 - 2x && \text{Distributive property} \\ 4x &= 7 && \text{Add } 2x \text{ and 5 to each side.} \\ x &= \frac{7}{4} && \text{Divide each side by 4.} \end{aligned}$$

There is one solution: $\frac{7}{4}$. This is a conditional equation.

(c)

$$\begin{aligned} 2(5 - x) - 25 &= 3(x - 5) - 5x && \text{Given equation} \\ 10 - 2x - 25 &= 3x - 15 - 5x && \text{Distributive property} \\ -2x - 15 &= -2x - 15 && \text{Simplify each side.} \\ 0 &= 0 && \text{Add } 2x \text{ and 15 to each side.} \end{aligned}$$

The statement $0 = 0$ is true and the solution set includes *all real numbers*. The equation is an identity.

Now Try Exercises 39, 41, and 43

Graphical and Numerical Solutions

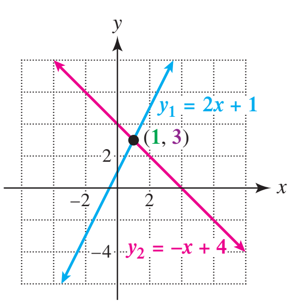


Figure 2.32

Graphical Solutions

The equation $f(x) = g(x)$ results whenever the formulas for two functions f and g are set equal to each other. A solution to this equation corresponds to the x-coordinate of a point where the graphs of f and g intersect. This technique is called the *intersection-of-graphs method*. If the graphs of f and g are lines with different slopes, then their graphs intersect once. For example, if $f(x) = 2x + 1$ and $g(x) = -x + 4$, then the equation $f(x) = g(x)$ becomes $2x + 1 = -x + 4$. To apply the intersection-of-graphs method, we graph $y_1 = 2x + 1$ and $y_2 = -x + 4$, as shown in Figure 2.32.

Their graphs intersect at the point $(1, 3)$. Because the variable in the given equation $2x + 1 = -x + 4$ is x, the solution is 1, the x-coordinate of the point of intersection. When $x = 1$, the functions f and g both assume the value 3—that is, $f(1) = 2(1) + 1 = 3$ and $g(1) = -1 + 4 = 3$. The value 3 is the y-coordinate of the point of intersection $(1, 3)$.

The intersection-of-graphs method is summarized below.

Intersection-of-Graphs Method

The **intersection-of-graphs method** can be used to solve an equation graphically.

STEP 1: Set y_1 equal to the left side of the equation, and set y_2 equal to the right side of the equation.

STEP 2: Graph y_1 and y_2.

STEP 3: Locate any points of intersection. The x-coordinates of these points correspond to solutions to the equation.

EXAMPLE 6 Solving an equation graphically and symbolically

Solve $2x - 1 = \frac{1}{2}x + 2$ graphically and symbolically.

SOLUTION

Graphical Solution Graph $y_1 = 2x - 1$ and $y_2 = \frac{1}{2}x + 2$. Their graphs intersect at the point $(2, 3)$, as shown in Figure 2.33, so the solution is 2. Figure 2.34 shows these graphs as created by a graphing calculator.

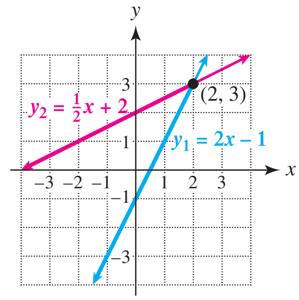


Figure 2.33 Intersection of Graphs

[−6, 8, 2] by [−6, 8, 2]

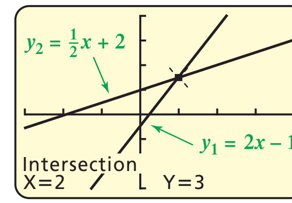


Figure 2.34

Symbolic Solution

$2x - 1 = \frac{1}{2}x + 2$	Given equation
$2x = \frac{1}{2}x + 3$	Add 1 to each side.
$\frac{3}{2}x = 3$	Subtract $\frac{1}{2}x$ from each side.
$\frac{2}{3} \cdot \frac{3}{2}x = \frac{2}{3} \cdot 3$	Multiply each side by $\frac{2}{3}$.
$x = 2$	Multiply fractions.

The solution is 2 and agrees with the graphical solution. ***Now Try Exercise 53*** ◀

EXAMPLE 7 Applying the intersection-of-graphs method

During the 1990s, compact discs were a new technology that replaced cassette tapes. The percentage share of music sales (in dollars) held by compact discs from 1987 to 1998 could be modeled by $f(x) = 5.91x + 13.7$. During the same time period the percentage share of music sales held by cassette tapes could be modeled by $g(x) = -4.71x + 64.7$. In these formulas $x = 0$ corresponds to 1987, $x = 1$ to 1988, and so on. Use the intersection-of-graphs method to estimate the year when the percentage share of CDs equaled the percentage share of cassettes. (**Source:** Recording Industry Association of America.)

SOLUTION We must solve the linear equation $f(x) = g(x)$, or equivalently,

$$5.91x + 13.7 = -4.71x + 64.7.$$

Graph $Y_1 = 5.91X + 13.7$ and $Y_2 = -4.71X + 64.7$, as in Figure 2.35. In Figure 2.36 their graphs intersect near the point (4.8, 42.1). Since $x = 0$ corresponds to 1987 and $1987 + 4.8 \approx 1992$, it follows that in 1992 sales of CDs and cassette tapes were approximately equal. Each had about 42.1% of the sales in 1992.

Calculator Help

To find the point of intersection in Figure 2.36, see Appendix A (page AP-8).

[0, 12, 2] by [0, 100, 10]

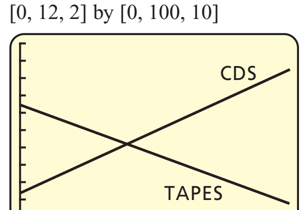


Figure 2.35

[0, 12, 2] by [0, 100, 10]

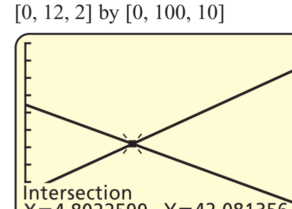


Figure 2.36

Now Try Exercise 93 ◀

Numerical Solutions Equations can be solved symbolically and graphically, as demonstrated in the previous examples. Sometimes it is also possible to find a numerical solution to an equation by using a table of values. When the solution is an integer or a convenient fraction, we can usually find it by using the table feature on a graphing calculator. However, when the solution is a fraction with a repeating decimal or an irrational number, a numerical method gives only an approximate solution. In Example 10 of Section 2.2, we constructed the formula

$$f(x) = 630.8(x - 1980) + 3617$$

to model tuition and fees at private colleges and universities during year x. To determine when tuition and fees were about \$15,600, we could solve the equation

$$630.8(x - 1980) + 3617 = 15,600$$

for x by making a table of values for $f(x)$, as shown in Figure 2.37. By scrolling through the x-values, we can see that tuition and fees at private schools were about \$15,600 in 1999. This equation could also be solved symbolically and graphically.

$f(1999) \approx 15,600$

X	Y1
1996	13710
1997	14341
1998	14971
1999	15602
2000	16233
2001	16864
2002	17495

X=1999

Figure 2.37

Calculator Help

To make a table of values, see Appendix A (page AP-6).

NOTE Regardless of whether we use a symbolic, graphical, or numerical method to solve an equation, we should find the same solution set. However, our answers may differ slightly because of rounding.

EXAMPLE 8 Solving an equation numerically

Solve $\sqrt{3}(2x - \pi) + \frac{1}{3}x = 0$ numerically to the nearest tenth.

SOLUTION Enter $Y_1 = \sqrt{(3)}(2X - \pi) + X/3$ and make a table for y_1, incrementing by 1, as shown in Figure 2.38. This table shows that when $x = 1, y_1 < 0$, and when $x = 2, y_1 > 0$. Thus the solution is located in the interval $1 < x < 2$. To obtain a more accurate answer, make a table for y_1 starting at 1 and incrementing by 0.1, as shown in Figure 2.39. Now we see that the solution lies in the interval $1.4 < x < 1.5$. In Figure 2.40, which starts at 1.4 and increments by 0.01, we see that the solution lies in the interval $1.43 < x < 1.44$. To the nearest tenth, the solution is 1.4. *Now Try Exercise 71*

X	Y1	
⁻3	⁻16.83	
⁻2	⁻13.04	
⁻1	⁻9.239	
0	⁻5.441	
1	⁻1.644	$y_1 < 0$
2	2.1535	$y_1 > 0$
3	5.9509	

X=2

Figure 2.38

X	Y1	
1	⁻1.644	
1.1	⁻1.264	
1.2	⁻.8845	
1.3	⁻.5047	
1.4	⁻.125	$y_1 < 0$
1.5	.25475	$y_1 > 0$
1.6	.6345	

X=1.5

Figure 2.39

X	Y1	
1.4	⁻.125	
1.41	⁻.087	
1.42	⁻.049	
1.43	⁻.0111	$y_1 < 0$
1.44	.02691	$y_1 > 0$
1.45	.06488	
1.46	.10286	

X=1.44

Figure 2.40

MAKING CONNECTIONS

Symbolic, Graphical, and Numerical Solutions Linear equations can be solved symbolically, graphically, and numerically. Symbolic solutions to linear equations are *always exact*, whereas graphical and numerical solutions are *sometimes approximate*. The following example illustrates how to solve the equation $2x - 1 = 3$ with each method.

Symbolic Solution

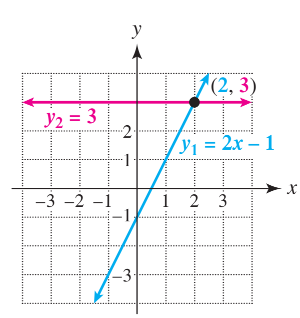
$$2x - 1 = 3$$
$$2x = 4$$
$$x = 2$$

Check:

$$2(2) - 1 \stackrel{?}{=} 3$$
$$3 = 3$$

It checks.

Graphical Solution



The solution is 2.

Numerical Solution

x	0	1	2	3
$2x - 1$	−1	1	3	5

Because $2x - 1$ equals 3 when $x = 2$, the solution to $2x - 1 = 3$ is 2.

Percentages Applications involving percentages often result in linear equations because percentages can be computed by linear functions. A function for taking P percent of x is given by $f(x) = \frac{P}{100}x$, where $\frac{P}{100}$ is the decimal form for P percent. For example, to calculate 35% of x, let $f(x) = 0.35x$. Then 35% of \$150 is $f(150) = 0.35(150) = 52.5$, or \$52.50.

EXAMPLE 9 Solving an application involving percentages

A survey found that 76% of bicycle riders do not wear helmets. (**Source:** Opinion Research Corporation for Glaxo Wellcome, Inc.)

(a) Find a formula $f(x)$ for a function that computes the number of people who do not wear helmets among x bicycle riders.

(b) There are approximately 38.7 million riders of all ages who do not wear helmets. Find the total number of bicycle riders.

SOLUTION

(a) A function f that computes 76% of x is given by $f(x) = 0.76x$.

(b) We must find the x-value for which $f(x) = 38.7$ million, or solve the equation $0.76x = 38.7$. Solving gives $x = \frac{38.7}{0.76} \approx 50.9$ million bike riders.

Now Try Exercise 99 ◀

Solving for a Variable

The circumference C of a circle is given by $C = 2\pi r$, where r is the radius. This equation is *solved* for C. That is, given r, we can easily calculate C. For example, if $r = 4$, then $C = 2\pi(4)$, or $C = 8\pi$. However, if we are given C, then it is more work to calculate r. Solving the equation for r makes it simpler to calculate r.

$$C = 2\pi r \quad \text{Given equation}$$

$$\frac{C}{2\pi} = r \quad \text{Divide each side by } 2\pi.$$

The equation $r = \frac{C}{2\pi}$ is solved for r.

EXAMPLE 10 Solving for a variable

The area of a trapezoid with bases a and b and height h is given by $A = \frac{1}{2}h(a + b)$. Solve this equation for b.

SOLUTION

Getting Started If we multiply each side by 2 and divide each side by h, the right side of the equation becomes $a + b$. Subtracting a from each side isolates b. ▶

$$A = \frac{1}{2}h(a + \mathbf{b}) \quad \text{Given equation}$$

$$2A = h(a + \mathbf{b}) \quad \text{Multiply each side by 2.}$$

$$\frac{2A}{h} = a + \mathbf{b} \quad \text{Divide each side by } h.$$

$$\frac{2A}{h} - a = \mathbf{b} \quad \text{Subtract } a \text{ from each side.}$$

The equation $b = \frac{2A}{h} - a$ is solved for b.

Now Try Exercise 85 ◀

Problem-Solving Strategies

To become more proficient at solving problems, we need to establish a procedure to guide our thinking. The following steps may be helpful in solving application problems.

Solving Application Problems

STEP 1: Read the problem and make sure you understand it. Assign a variable to what you are being asked to find. If necessary, write other quantities in terms of this variable.

STEP 2: Write an equation that relates the quantities described in the problem. You may need to sketch a diagram and refer to known formulas.

STEP 3: Solve the equation and determine the solution.

STEP 4: Look back and check your solution. Does it seem reasonable?

These steps are applied in the next four examples.

EXAMPLE 11 Working together

A large pump can empty a tank of gasoline in 5 hours, and a smaller pump can empty the same tank in 9 hours. If both pumps are used to empty the tank, how long will it take?

SOLUTION

STEP 1: We are asked to find the time it takes for *both* pumps to empty the tank. Let this time be t.

t: Time to empty the tank

STEP 2: In 1 hour the large pump will empty $\frac{1}{5}$ of the tank and the smaller pump will empty $\frac{1}{9}$ of the tank. The fraction of the tank that they will empty together in 1 hour is given by $\frac{1}{5} + \frac{1}{9}$. In 2 hours the large pump will empty $\frac{2}{5}$ of the tank and the smaller pump will empty $\frac{2}{9}$ of the tank. The fraction of the tank that they will empty together in 2 hours is $\frac{2}{5} + \frac{2}{9}$. Similarly, in t hours the fraction of the tank that the two pumps can empty is $\frac{t}{5} + \frac{t}{9}$. Since the tank is empty when this fraction reaches 1, we must solve the following equation.

$$\frac{t}{5} + \frac{t}{9} = 1$$

STEP 3: Multiply by the LCD, 45, to eliminate fractions.

$$\frac{45t}{5} + \frac{45t}{9} = 45 \qquad \text{Multiply by LCD.}$$

$$9t + 5t = 45 \qquad \text{Simplify.}$$

$$14t = 45 \qquad \text{Add like terms.}$$

$$t = \frac{45}{14} \approx 3.21 \qquad \text{Divide by 14 and approximate.}$$

Working together, the two pumps can empty the tank in about 3.21 hours.

STEP 4: This sounds reasonable. Working together the two pumps should be able to empty the tank faster than the large pump working alone, but not twice as fast. Note that $\frac{3.21}{5} + \frac{3.21}{9} \approx 1.$

Now Try Exercise 101

EXAMPLE 12 Solving an application involving motion

In 1 hour an athlete traveled 10.1 miles by running first at 8 miles per hour and then at 11 miles per hour. How long did the athlete run at each speed?

SOLUTION

STEP 1: We are asked to find the time spent running at each speed. If we let x represent the time in hours spent running at 8 miles per hour, then $1 - x$ represents the time spent running at 11 miles per hour because the total running time was 1 hour.

x: Time spent running at 8 miles per hour

$1 - x$: Time spent running at 11 miles per hour

STEP 2: Distance d equals rate r times time t: that is, $d = rt$. In this example we have two rates (or speeds) and two times. The total distance must sum to 10.1 miles.

$$d = r_1t_1 + r_2t_2 \qquad \text{General equation}$$

$$10.1 = 8x + 11(1 - x) \qquad \text{Substitute.}$$

STEP 3: We can solve this equation symbolically.

$$10.1 = 8x + 11 - 11x \qquad \text{Distributive property}$$

$$10.1 = 11 - 3x \qquad \text{Combine like terms.}$$

$$3x = 0.9 \qquad \text{Add } 3x\text{; subtract } 10.1.$$

$$x = 0.3 \qquad \text{Divide by 3.}$$

The athlete runs 0.3 hour (18 minutes) at 8 miles per hour and 0.7 hour (42 minutes) at 11 miles per hour.

STEP 4: We can check this solution as follows.

$$8(0.3) + 11(0.7) = 10.1 \qquad \text{It checks.}$$

This sounds reasonable. The runner's average speed was 10.1 miles per hour so the runner must have run longer at 11 miles per hour than at 8 miles per hour.

Now Try Exercise 103

Similar triangles are often used in applications involving geometry. Similar triangles are used to solve the next application.

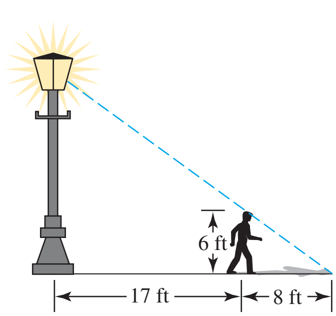


Figure 2.41

EXAMPLE 13 Solving an application involving similar triangles

A person 6 feet tall stands 17 feet from the base of a streetlight, as illustrated in Figure 2.41. If the person's shadow is 8 feet, estimate the height of the streetlight.

SOLUTION

STEP 1: We are asked to find the height of the streetlight in Figure 2.41. Let x represent this height.

x: Height of the streetlight

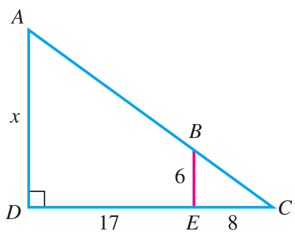


Figure 2.42

Geometry Review
To review similar triangles, see Chapter R (page R-5).

STEP 2: In Figure 2.42, triangle ACD is similar to triangle BCE. Thus ratios of corresponding sides are equal.

$$\frac{AD}{BE} = \frac{DC}{EC}$$

$$\frac{x}{6} = \frac{17 + 8}{8}$$

STEP 3: We can solve this equation symbolically.

$$\frac{x}{6} = \frac{25}{8} \quad \text{Simplify.}$$

$$x = \frac{6 \cdot 25}{8} \quad \text{Multiply by 6.}$$

$$x = 18.75 \quad \text{Simplify.}$$

The height of the streetlight is 18.75 feet.

STEP 4: One way to check this answer is to form a different proportion. Note that x is to $17 + 8$ in triangle ADC as 6 is to 8 in triangle BEC. If $x = 18.75$, then $\frac{18.75}{25} = \frac{6}{8}$, which is true, and our answer checks.

Now Try Exercise 107

EXAMPLE 14 Mixing acid in chemistry

Pure water is being added to 153 milliliters of a 30% solution of hydrochloric acid. How much water should be added to dilute the solution to a 13% mixture?

SOLUTION

STEP 1: We are asked to find the amount of water that should be added to 153 milliliters of 30% acid to make a 13% solution. Let this amount of water be equal to x. See Figure 2.43.

x: Amount of pure water to be added

$x + 153$: Final volume of 13% solution

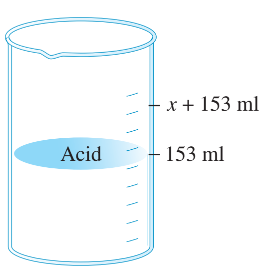


Figure 2.43

STEP 2: Since only water is added, the total amount of acid in the solution after the water is added must equal the amount of acid before the water is added. The volume of pure acid after the water is added equals 13% of $x + 153$ milliliters, and the volume of pure acid before the water is added equals 30% of 153 milliliters. We must solve the following equation.

$$0.13(x + 153) = 0.30(153)$$

STEP 3: Begin by dividing each side by 0.13.

$$x + 153 = \frac{0.30(153)}{0.13} \quad \text{Divide by 0.13.}$$

$$x = \frac{0.30(153)}{0.13} - 153 \quad \text{Subtract 153.}$$

$$x \approx 200.08 \quad \text{Approximate.}$$

We should add about 200 milliliters of pure water.

STEP 4: Initially the solution contains $0.30(153) = 45.9$ milliliters of pure acid. If we add 200 milliliters of water to the 153 milliliters, the final solution is 353 milliliters, which includes 45.9 milliliters of pure acid. Its concentration is $\frac{45.9}{353} \approx 0.13$, or about 13%.

Now Try Exercise 111

2.3 Putting It All Together

A general four-step procedure for solving applications is found on page 122. The following table summarizes some of the important concepts in this section.

Concept	Explanation	Examples
Linear equation	A linear equation can be written as $ax + b = 0,\ a \neq 0$.	$4x + 5 = 0$ $3x - 1 = x + 2$
Addition property	$a = b$ is equivalent to $a + c = b + c$.	$x - 7 = 25$ $x - 7 + 7 = 25 + 7$ $x = 32$
Multiplication property	$a = b$ is equivalent to $ac = bc,\ c \neq 0$.	$\frac{1}{2}x = 4$ $2 \cdot \frac{1}{2}x = 4 \cdot 2$ $x = 8$
Distributive property	$a(b + c) = ab + ac$ $a(b - c) = ab - ac$	$2(5 + x) = 10 + 2x$ $-(2 - x) = -1(2 - x) = -2 + x$
Identity	An equation that is true for all (meaningful) values of the variable	$3(x - 2) = 3x - 6$ $2x + 3x = (2 + 3)x = 5x$
Contradiction	An equation that has no solutions	$x + 5 = x$ $2x - 2x = 5$
Conditional equation	An equation that is satisfied by some, but not all, of the values of the variable	$2x - 1 = 5$ Given equation $2x = 6$ Add 1. $x = 3$ Divide by 2.
Percentages	P percent of x equals $\frac{P}{100}x$, where $\frac{P}{100}$ is the decimal form for P percent.	35% of x is calculated by $f(x) = \frac{35}{100}x$, or $f(x) = 0.35x$.

continued on next page

continued from previous page

The following example illustrates how to solve $5x - 1 = 3$ symbolically, graphically, and numerically.

Symbolic Solution

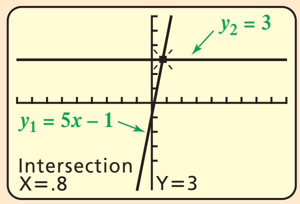
$$5x - 1 = 3$$
$$5x = 4$$
$$x = \frac{4}{5}$$

Check:

$$5\left(\frac{4}{5}\right) - 1 \stackrel{?}{=} 3$$
$$4 - 1 \stackrel{?}{=} 3$$
$$3 = 3$$

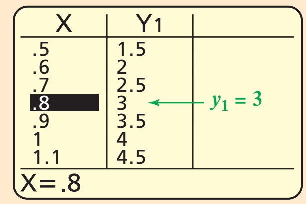
Graphical Solution

$[-9, 9, 1]$ by $[-6, 6, 1]$



The graphs of $y_1 = 5x - 1$ and $y_2 = 3$ intersect at $(0.8, 3)$, so the solution is 0.8.

Numerical Solution



In the table, $y_1 = 3$ when $x = 0.8$.

2.3 Exercises

Concepts about Linear Equations

1. How many solutions are there to $ax + b = 0$ with $a \neq 0$?
2. How many times does the graph of $y = ax + b$ with $a \neq 0$ intersect the x-axis?
3. Apply the distributive property to $4 - (5 - 4x)$.
4. What property is used to solve $15x = 5$?
5. If $f(x) = ax + b$ with $a \neq 0$, how are the zero of f and the x-intercept of the graph of f related?
6. Distinguish between a contradiction and an identity.

Identifying Linear and Nonlinear Equations

Exercises 7–12: Determine whether the equation is linear or nonlinear by trying to write it in the form $ax + b = 0$.

7. $3x - 1.5 = 7$
8. $100 - 23x = 20x$
9. $2\sqrt{x} + 2 = 1$
10. $4x^3 - 7 = 0$
11. $7x - 5 = 3(x - 8)$
12. $2(x - 3) = 4 - 5x$

Solving Linear Equations Symbolically

Exercises 13–38: Solve the equation and check your answer.

13. $2x - 8 = 0$
14. $4x - 8 = 0$
15. $-5x + 3 = 23$
16. $-9x - 3 = 24$
17. $4(z - 8) = z$
18. $-3(2z - 1) = 2z$
19. $-5(3 - 4t) = 65$
20. $6(5 - 3t) = 66$
21. $k + 8 = 5k - 4$
22. $2k - 3 = k + 3$
23. $2(1 - 3x) + 1 = 3x$
24. $5(x - 2) = -2(1 - x)$
25. $-5(3 - 2x) - (1 - x) = 4(x - 3)$
26. $-3(5 - x) - (x - 2) = 7x - 2$
27. $-4(5x - 1) = 8 - (x + 2)$
28. $6(3 - 2x) = 1 - (2x - 1)$
29. $\frac{2}{7}n + \frac{1}{5} = \frac{4}{7}$
30. $\frac{6}{11} - \frac{2}{33}n = \frac{5}{11}n$
31. $\frac{1}{2}(d - 3) - \frac{2}{3}(2d - 5) = \frac{5}{12}$

32. $\frac{7}{3}(2d - 1) - \frac{2}{5}(4 - 3d) = \frac{1}{5}d$

33. $\dfrac{x - 5}{3} + \dfrac{3 - 2x}{2} = \dfrac{5}{4}$

34. $\dfrac{3x - 1}{5} - 2 = \dfrac{2 - x}{3}$

35. $0.1z - 0.05 = -0.07z$ **36.** $1.1z - 2.5 = 0.3(z - 2)$

37. $0.15t + 0.85(100 - t) = 0.45(100)$

38. $0.35t + 0.65(10 - t) = 0.55(10)$

Exercises 39–48: Complete the following.

(a) Solve the equation symbolically.

(b) Classify the equation as a contradiction, an identity, or a conditional equation.

39. $5x - 1 = 5x + 4$

40. $7 - 9z = 2(3 - 4z) - z$

41. $3(x - 1) = 5$ **42.** $22 = -2(2x + 1.4)$

43. $0.5(x - 2) + 5 = 0.5x + 4$

44. $\frac{1}{2}x - 2(x - 1) = -\frac{3}{2}x + 2$

45. $\dfrac{t + 1}{2} = \dfrac{3t - 2}{6}$ **46.** $\dfrac{2x + 1}{3} = \dfrac{2x - 1}{3}$

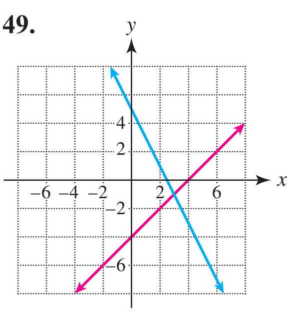
47. $\dfrac{1 - 2x}{4} = \dfrac{3x - 1.5}{-6}$

48. $0.5(3x - 1) + 0.5x = 2x - 0.5$

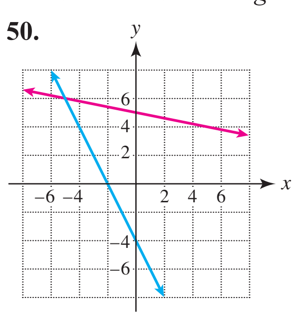
Solving Linear Equations Graphically

Exercises 49 and 50: A linear equation is solved by using the intersection-of-graphs method. Find the solution by interpreting the graph. Assume that the solution is an integer.

49.



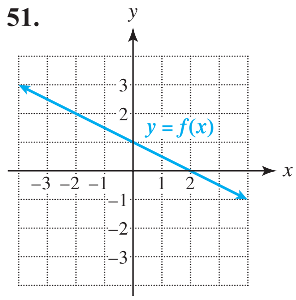
50.



Exercises 51 and 52: Use the graph of $y = f(x)$ to solve each equation.

(a) $f(x) = -1$ *(b)* $f(x) = 0$ *(c)* $f(x) = 2$

51.



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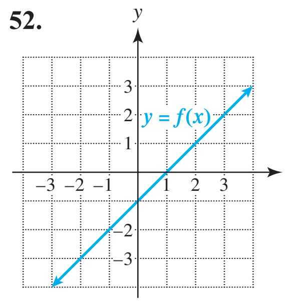


Exercises 53–58: Use the intersection-of-graphs method to solve the equation. Then solve symbolically.

53. $x + 4 = 1 - 2x$ **54.** $2x = 3x - 1$

55. $-x + 4 = 3x$ **56.** $1 - 2x = x + 4$

57. $2(x - 1) - 2 = x$ **58.** $-(x + 1) - 2 = 2x$

Exercises 59–66: Solve the linear equation with the intersection-of-graphs method. Approximate the solution to the nearest thousandth whenever appropriate.

59. $5x - 1.5 = 5$ **60.** $8 - 2x = 1.6$

61. $3x - 1.7 = 1 - x$ **62.** $\sqrt{2}x = 4x - 6$

63. $3.1(x - 5) = \frac{1}{5}x - 5$ **64.** $65 = 8(x - 6) - 5.5$

65. $\dfrac{6 - x}{7} = \dfrac{2x - 3}{3}$

66. $\pi(x - \sqrt{2}) = 1.07x - 6.1$

Solving Linear Equations Numerically

Exercises 67–74: Use tables to solve the equation numerically to the nearest tenth.

67. $2x - 7 = -1$ **68.** $1 - 6x = 7$

69. $2x - 7.2 = 10$ **70.** $5.8x - 8.7 = 0$

71. $\sqrt{2}(4x - 1) + \pi x = 0$

72. $\pi(0.3x - 2) + \sqrt{2}x = 0$

73. $0.5 - 0.1(\sqrt{2} - 3x) = 0$

74. $\sqrt{5} - \pi(\pi + 0.3x) = 0$

Solving Linear Equations by More Than One Method

Exercises 75–82: Solve the equation (to the nearest tenth)

(a) symbolically,
(b) graphically, and
(c) numerically.

75. $5 - (x + 1) = 3$

76. $7 - (3 - 2x) = 1$

77. $\sqrt{3}(2 - \pi x) + x = 0$

78. $3(\pi - x) + \sqrt{2} = 0$

79. $x - 3 = 2x + 1$

80. $3(x - 1) = 2x - 1$

81. $6x - 8 = -7x + 18$

82. $5 - 8x = 3(x - 7) + 37$

Solving for a Variable

Exercises 83–90: Solve the equation for the specified variable.

83. $A = LW$ for W

84. $E = IR + 2$ for R

85. $P = 2L + 2W$ for L

86. $V = 2\pi rh + \pi r^2$ for h

87. $3x + 2y = 8$ for y

88. $5x - 4y = 20$ for y

89. $y = 3(x - 2) + x$ for x

90. $y = 4 - (8 - 2x)$ for x

Applications

91. ***Income*** The per capita (per person) income from 1980 to 2006 can be modeled by

$$f(x) = 1000(x - 1980) + 10{,}000,$$

where x is the year. Determine the year when the per capita income was \$19,000. (**Source:** Bureau of the Census.)

92. ***Tuition and Fees*** In Example 10 of Section 2.2, we modeled tuition and fees in dollars during year x by

$$f(x) = 630.8(x - 1980) + 3617.$$

Use $f(x)$ to determine when tuition and fees reached \$13,700.

93. ***Vinyl and CD Sales*** During the 1980s, sales of compact discs surpassed vinyl record sales. From 1985 to 1990, sales of compact discs in millions can be modeled by the formula $f(x) = 51.6(x - 1985) + 9.1$, whereas sales of vinyl LP records in millions can be modeled by $g(x) = -31.9(x - 1985) + 167.7$. Approximate the year x when sales of LP records and compact discs were equal by using the intersection-of-graphs method. (**Source:** Recording Industry Association of America.)

94. ***Median Age*** The median age A in the United States during year x, where $2000 \le x \le 2050$, is projected to be

$$A(x) = 0.07(x - 2000) + 35.3.$$

Use $A(x)$ to estimate when the median age may reach 37 years. (**Source:** Bureau of the Census.)

95. ***Population Density*** In 1980 the population density of the United States was 64 people per square mile, and in 2000 it was 80 people per square mile. Use a linear function to estimate when the U.S. population density reached 87 people per square mile.

96. ***Value of a Home*** In 1999 the value of a house was \$180,000, and in 2009 it was \$245,000.

(a) Find a linear function V that approximates the value of the house during year x.

(b) What does the slope of the graph of V represent?

(c) Use V to estimate the year when the house was worth \$219,000.

97. ***Sale Price*** A store is discounting all regularly priced merchandise by 25%. Find a function f that computes the sale price of an item having a regular price of x. If an item normally costs \$56.24, what is its sale price?

98. ***Sale Price*** Continuing Exercise 97, use f to find the regular price of an item that costs \$19.62 on sale.

99. ***Skin Cancer*** Approximately 4.5% of all cancer cases diagnosed in 2007 were skin cancer. (**Source:** American Cancer Society.)

(a) If x cases of cancer were diagnosed, how many of these were skin cancer?

(b) There were 65,000 cases of skin cancer diagnosed in 2007. Find the total number of cancer cases in 2007.

100. ***Grades*** In order to receive an A in a college course it is necessary to obtain an average of 90% correct on three 1-hour exams of 100 points each and on one final exam of 200 points. If a student scores 82, 88, and 91 on the 1-hour exams, what is the minimum score that the person can receive on the final exam and still earn an A?

101. ***Working Together*** Suppose that a lawn can be raked by one gardener in 3 hours and by a second gardener in 5 hours.

(a) Mentally estimate how long it will take the two gardeners to rake the lawn working together.

(b) Solve part (a) symbolically.

102. ***Pumping Water*** Suppose that a large pump can empty a swimming pool in 50 hours and a small pump can empty the pool in 80 hours. How long will it take to empty the pool if both pumps are used?

103. ***Motion*** A car went 372 miles in 6 hours, traveling part of the time at 55 miles per hour and part of the time at 70 miles per hour. How long did the car travel at each speed?

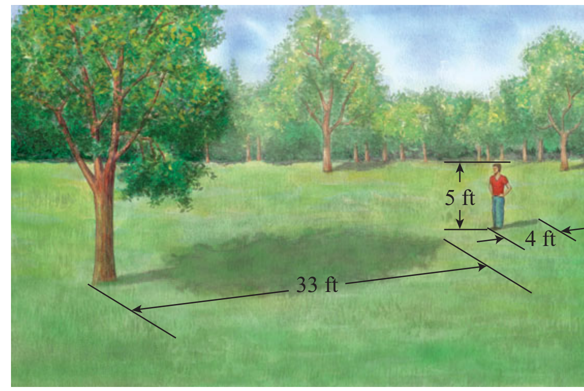
104. ***Mixing Candy*** Two types of candy sell for \$2.50 per pound and \$4.00 per pound. A store clerk is trying to make a 5-pound mixture worth \$17.60. How much of each type of candy should be included in the mixture?

105. ***Running*** At 2:00 P.M. a runner heads north on a highway, jogging at 10 miles per hour. At 2:30 P.M. a driver heads north on the same highway to pick up the runner. If the car travels at 55 miles per hour, how long will it take the driver to catch the runner?

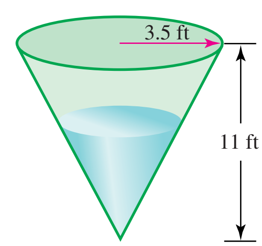
106. ***Investments*** A total of \$5000 was invested in two accounts. One pays 5% annual interest, and the second pays 7% annual interest. If the first-year interest is \$325, how much was invested in each account?

107. ***Shadow Length*** A person 66 inches tall is standing 15 feet from a streetlight. If the person casts a shadow 84 inches long, how tall is the streetlight?

108. ***Height of a Tree*** In the accompanying figure, a person 5 feet tall casts a shadow 4 feet long. A nearby tree casts a shadow 33 feet long. Find the height of the tree by solving a linear equation.



109. ***Conical Water Tank*** A water tank in the shape of an inverted cone has a height of 11 feet and a radius of 3.5 feet, as illustrated in the figure. If the volume of the cone is $V = \frac{1}{3}\pi r^2 h$, find the volume of the water in the tank when the water is 7 feet deep. (*Hint:* Consider using similar triangles.)



110. ***Dimension of a Cone*** (Refer to Exercise 109.) A conical water tank holds 100 cubic feet of water and has a diameter of 6 feet. Estimate its height to the nearest tenth of a foot.

111. ***Chemistry*** Determine how much pure water should be mixed with 5 liters of a 40% solution of sulfuric acid to make a 15% solution of sulfuric acid.

112. ***Mixing Antifreeze*** A radiator holds 5 gallons of fluid. If it is full with a 15% solution, how much fluid should be drained and replaced with a 65% antifreeze mixture to result in a 40% antifreeze mixture?

113. ***Window Dimensions*** A rectangular window has a length that is 18 inches more than its width. If its perimeter is 180 inches, find its dimensions.

114. ***Online Holiday Shopping*** In 2003 online holiday sales were \$17 billion, and in 2006 they were \$26 billion. (**Source:** Digital Lifestyles.)

(a) Find a linear function S that models these data.

(b) Interpret the slope of the graph of S.

(c) Predict when online holiday sales might reach \$41 billion.

115. ***Sales of CRT and LCD Screens*** In 2002, 75 million CRT (cathode ray tube) monitors were sold and 29 million flat LCD (liquid crystal display) monitors were sold. In 2006 the numbers were 45 million for CRT monitors and 88 million for LCD monitors. (**Source:** International Data Corporation.)

(a) Find a linear function C that models these data for CRT monitors and another linear function L that models these data for LCD monitors. Let x be the year.

(b) Interpret the slopes of the graphs of C and of L.

(c) Determine graphically the year when sales of these two types of monitors were equal.

(d) Solve part (c) symbolically.

(e) Solve part (c) numerically.

116. ***Geometry*** A 174-foot-long fence is being placed around the perimeter of a rectangular swimming pool that has a 3-foot-wide sidewalk around it. The actual swimming pool without the sidewalk is twice as long as it is wide. Find the dimensions of the pool without the sidewalk.

117. ***Temperature Scales*** The Celsius and Fahrenheit scales are related by the equation $C = \frac{5}{9}(F - 32)$. These scales have the same temperature reading at a unique value where $F = C$. Find this temperature.

118. ***Business*** A company manufactures compact discs with recorded music. The master disc costs \$2000 to produce and copies cost \$0.45 each. If a company spent \$2990 producing compact discs, how many copies did the company manufacture?

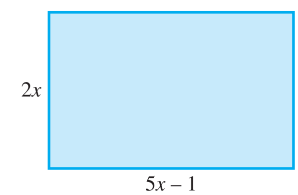
119. ***Two-Cycle Engines*** Two-cycle engines, used in snowmobiles, chain saws, and outboard motors, require a mixture of gasoline and oil. For certain engines the amount of oil in pints that should be added to x gallons of gasoline is computed by $f(x) = 0.16x$. (**Source:** Johnson Outboard Motor Company.)

(a) Why is it reasonable to expect f to be linear?

(b) Evaluate $f(3)$ and interpret the answer.

(c) How much gasoline should be mixed with 2 pints of oil?

120. ***Perimeter*** Find the length of the longest side of the rectangle if its perimeter is 25 feet.



Modeling Data with Linear Functions

Exercises 121 and 122: The following data can be modeled by a linear function. Estimate the value of x when $y = 2.99$.

121.

x	2	4	6	8
y	0.51	1.23	1.95	2.67

122.

x	1	2	3	4
y	−1.66	2.06	5.78	9.50

Linear Regression

123. ***Ring Size*** The table lists ring size S for a finger with circumference x in centimeters.

x (cm)	4.65	5.40	5.66	6.41
S (size)	4	7	8	11

Source: Overstock.

(a) Find a linear function S that models the data.

(b) Find the circumference of a finger with a ring size of 6.

124. ***Hat Size*** The table lists hat size H for a head with circumference x in inches.

x (in.)	$21\frac{1}{8}$	$21\frac{7}{8}$	$22\frac{5}{8}$	25
S (size)	$6\frac{3}{4}$	7	$7\frac{1}{4}$	8

Source: Brentblack.

(a) Find a linear function S that models the data.

(b) Find the circumference of a head with a hat size of $7\frac{1}{2}$.

125. ***Super Bowl Ads*** The table lists the cost in millions of dollars for a 30-second Super Bowl commercial for selected years.

Year	1990	1994	1998	2004	2008
Cost	0.8	1.2	1.6	2.3	2.7

Source: MSNBC.

(a) Find a linear function f that models the data.

(b) Estimate the cost in 1987 and compare the estimate to the actual value of \$0.6 million. Did your estimate involve interpolation or extrapolation?

(c) Use f to predict the year when the cost could reach \$3.2 million.

126. ***Women in Politics*** The table lists the percentage P of women in state legislatures during year x.

x	1993	1997	2001	2005	2007
P	20.5	21.6	22.4	22.7	23.5

Source: National Women's Political Caucus.

(a) Find a linear function P that models the data.

(b) Estimate this percentage in 2003 and compare the estimate to the actual value of 22.4%. Did your estimate involve interpolation or extrapolation?

(c) Use P to predict the year when this percentage could reach 25%.

Writing about Mathematics

127. Describe a basic graphical method used to solve a linear equation. Give examples.

128. Describe verbally how to solve $ax + b = 0$. What assumptions have you made about the value of a?

EXTENDED AND DISCOVERY EXERCISES

1. ***Geometry*** Suppose that two rectangles are similar and the sides of the first rectangle are twice as long as the corresponding sides of the second rectangle.

(a) Is the perimeter of the first rectangle twice the perimeter of the second rectangle? Explain.

(b) Is the area of the first rectangle twice the area of the second rectangle? Explain.

2. ***Geometry*** Repeat the previous exercise for an equilateral triangle. Try to make a generalization. (*Hint:* The area of an equilateral triangle is $A = \frac{\sqrt{3}}{4}x^2$, where x is the length of a side.) What will happen to the circumference of a circle if the radius is doubled? What will happen to its area?

3. ***Indoor Air Pollution*** Formaldehyde is an indoor air pollutant formerly found in plywood, foam insulation, and carpeting. When concentrations in the air reach 33 micrograms per cubic foot (μg/ft^3), eye irritation can occur. One square foot of new plywood could emit 140 μg per hour. (**Source:** A. Hines, *Indoor Air Quality & Control*.)

(a) A room has 100 square feet of new plywood flooring. Find a linear function f that computes the amount of formaldehyde in micrograms that could be emitted in x hours.

(b) The room contains 800 cubic feet of air and has no ventilation. Determine how long it would take for concentrations to reach 33 μg/ft^3.

4. ***Temperature and Volume*** The table shows the relationship between the temperature of a sample of helium and its volume.

Temperature (°C)	0	25	50	75	100
Volume (in^3)	30	32.7	35.4	38.1	40.8

(a) Make a scatterplot of the data.

(b) Write a formula for a function f that receives the temperature x as input and outputs the volume y of the helium.

(c) Find the volume when the temperature is 65°C.

(d) Find the temperature if the volume is 25 cubic inches. Did your answer involve interpolation or extrapolation? Do you believe that your answer is accurate?

2.4 Linear Inequalities

- Understand basic terminology related to inequalities
- Solve linear inequalities symbolically
- Solve linear inequalities graphically and numerically
- Solve compound inequalities

Introduction

If a person who weighs 143 pounds needs to purchase a life preserver for whitewater rafting, it is doubtful that there is one designed exactly for this weight. Life preservers are manufactured to support a range of body weights. A vest that is approved for weights between 120 and 160 pounds would be appropriate. Every airplane has a maximum weight allowance. It is important that this weight limit be accurately determined. However, most people feel more comfortable at takeoff if that maximum has not been reached, because any weight that is less than the maximum is also safe and allows a greater margin of error. Both of these situations involve the concept of inequality.

In mathematics much effort is expended in solving equations and determining equality. One reason is that equality is frequently a boundary between *greater than* and *less than*. The solution to an inequality often can be found by first locating where two expressions are equal. Since equality and inequality are closely related, many of the techniques used to solve equations also can be applied to inequalities.

Inequalities

Inequalities result whenever the equals sign in an equation is replaced with any one of the symbols $<$, $\leq$, $>$, or $\geq$. Some examples of inequalities include

$$x + 15 < 9x - 1, \qquad x^2 - 2x + 1 \geq 2x, \qquad z + 5 > 0,$$
$$xy + x^2 \leq y^3 + x, \qquad \text{and} \qquad 2 + 3 > 1.$$

The first three inequalities have one variable, the fourth inequality contains two variables, and the fifth inequality has only constants. As with linear equations, our discussion focuses on inequalities with one variable.

To **solve** an inequality means to find all values for the variable that make the inequality a true statement. Such values are **solutions**, and the set of all solutions is the **solution set** to the inequality. Two inequalities are **equivalent** if they have the same solution set. It is common for an inequality to have infinitely many solutions. For instance, the inequality $x - 1 > 0$ has infinitely many solutions because any real number x satisfying $x > 1$ is a solution. The solution set is $\{x \mid x > 1\}$.

Like functions and equations, inequalities in one variable can be classified as *linear* or *nonlinear*.

Linear Inequality in One Variable

A **linear inequality** in one variable is an inequality that can be written in the form

$$ax + b > 0,$$

where $a \neq 0$. (The symbol $>$ may be replaced by $\geq$, $<$, or $\leq$.)

Examples of linear inequalities include

$$3x - 4 < 0, \qquad 7x + 5 \geq x, \qquad x + 6 > 23, \qquad \text{and} \qquad 7x + 2 \leq -3x + 6.$$

Using techniques from algebra, we can transform these inequalities into one of the forms $ax + b > 0$, $ax + b \geq 0$, $ax + b < 0$, or $ax + b \leq 0$. For example, by subtracting x

from each side of $7x + 5 \geq x$, we obtain the equivalent inequality $6x + 5 \geq 0$. If an inequality is not a linear inequality, it is called a **nonlinear inequality**.

MAKING CONNECTIONS

Linear Functions, Equations, and Inequalities These concepts are closely related.

$f(x) = ax + b$	Linear function
$ax + b = 0, a \neq 0$	Linear equation
$ax + b > 0, a \neq 0$	Linear inequality

NOTE This relationship is used again in the next chapter to define quadratic functions, equations, and inequalities.

Properties of Inequalities

Let a, b, and c be real numbers.

1. $a < b$ and $a + c < b + c$ are equivalent.
   (The same number may be added to or subtracted from each side of an inequality.)
2. If $c > 0$, then $a < b$ and $ac < bc$ are equivalent.
   (Each side of an inequality may be multiplied or divided by the same positive number.)
3. If $c < 0$, then $a < b$ and $ac > bc$ are equivalent.
   (Each side of an inequality may be multiplied or divided by the same negative number provided the inequality symbol is reversed.)

Replacing $<$ with $\leq$ and $>$ with $\geq$ results in similar properties.

The following examples illustrate each property.

Property 1: To solve $x - 5 < 6$, add 5 to each side to obtain $x < 11$.

Property 2: To solve $5x < 10$, divide each side by 5 to obtain $x < 2$.

Property 3: To solve $-5x < 10$, divide each side by -5 to obtain $x > -2$. (Whenever you multiply or divide an inequality by a negative number, reverse the inequality symbol.)

Review of Interval Notation In Section 1.5 interval notation was introduced as an efficient way to express intervals on the real number line. For example, the interval $3 \leq x \leq 5$ is written as $[3, 5]$, whereas the interval $3 < x < 5$ is written as $(3, 5)$. A bracket, [or], is used when an endpoint is included, and a parenthesis, (or), is used when an endpoint is not included. The interval $x \geq 2$ is written as $[2, \infty)$, where ∞ denotes infinity, and the interval $x < 2$ is written as $(-\infty, 2)$

In the next example we solve linear equalities and express the solution set in both set-builder and interval notation.

EXAMPLE 1 Solving linear inequalities symbolically

Solve each inequality. Write the solution set in set-builder and interval notation.

(a) $2x - 3 < \dfrac{x + 2}{-3}$ **(b)** $-3(4z - 4) \geq 4 - (z - 1)$

SOLUTION

(a) Use Property 3 by multiplying each side by -3 to clear fractions. Remember to reverse the inequality symbol when multiplying by a negative number.

$2x - 3 < \dfrac{x + 2}{-3}$	Given inequality
$-6x + 9 > x + 2$	Property 3: Multiply by -3 and reverse the inequality symbol.
$9 > 7x + 2$	Property 1: Add $6x$.
$7 > 7x$	Property 1: Add -2 (or subtract 2).
$1 > x$	Property 2: Divide by 7.

In set-builder notation the solution set is $\{x \mid x < 1\}$, and in interval notation it is written as $(-\infty, 1)$.

(b) Begin by applying the distributive property.

$-3(4z - 4) \geq 4 - (z - 1)$	Given inequality
$-12z + 12 \geq 4 - z + 1$	Distributive property
$-12z + 12 \geq -z + 5$	Simplify.
$-12z + z \geq 5 - 12$	Property 1: Add z and -12.
$-11z \geq -7$	Simplify.
$z \leq \dfrac{7}{11}$	Property 3: Divide by -11 and reverse inequality symbol.

In set-builder notation the solution set is $\left\{z \mid z \leq \frac{7}{11}\right\}$, and in interval notation it is written as $\left(-\infty, \frac{7}{11}\right]$.

Now Try Exercises 15 and 17 ◀

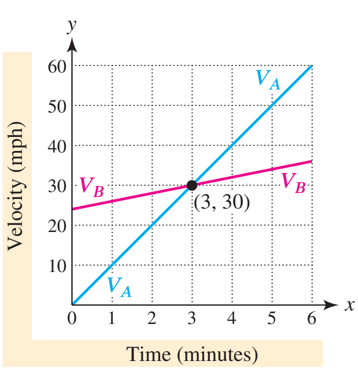


Figure 2.44 Velocities of Two Cars

Graphical Solutions The intersection-of-graphs method can be extended to solve inequalities. Figure 2.44 shows the velocity of two cars in miles per hour after x minutes. V_A denotes the velocity of car A, and V_B denotes the velocity of car B. The domains of V_A and V_B are both assumed to be $0 \leq x \leq 6$.

At 3 minutes $V_A = V_B$, and both cars are traveling at 30 miles per hour. To the left of $x = 3$ the graph of V_A is below the graph of V_B, so car A is traveling slower than car B. Thus

$$V_A < V_B \quad \text{when} \quad 0 \leq x < 3.$$

To the right of $x = 3$ the graph of V_A is above the graph of V_B, so car A is traveling faster than car B. Thus

$$V_A > V_B \quad \text{when} \quad 3 < x \leq 6.$$

This technique is used in the next example.

EXAMPLE 2 Solving a linear inequality graphically

Graph $y_1 = \frac{1}{2}x + 2$ and $y_2 = 2x - 1$ by hand. Use the graph to solve the linear inequality $\frac{1}{2}x + 2 > 2x - 1$.

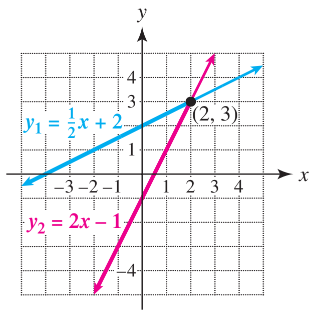


Figure 2.45

SOLUTION The graphs of $y_1 = \frac{1}{2}x + 2$ and $y_2 = 2x - 1$ are shown in Figure 2.45. The graphs intersect at the point (2, 3). The graph of $y_1 = \frac{1}{2}x + 2$ is above the graph of $y_2 = 2x - 1$ to the left of the point of intersection, or when $x < 2$. Thus the solution set to the inequality $\frac{1}{2}x + 2 > 2x - 1$ is $\{x \mid x < 2\}$, or $(-\infty, 2)$.

Now Try Exercise 39

An Application In the next example we use graphical techniques to solve an application from meteorology.

EXAMPLE 3 Using the intersection-of-graphs method

When the air temperature reaches the dew point, fog may form. This phenomenon also causes clouds to form. See Figure 2.46. Both the air temperature and the dew point often decrease at a constant rate as the altitude above ground level increases. If the ground-level Fahrenheit temperature and dew point are T_0 and D_0, the air temperature can sometimes be approximated by $T(x) = T_0 - 19x$ and the dew point by $D(x) = D_0 - 5.8x$ at an altitude of x miles.

(a) If $T_0 = 75°F$ and $D_0 = 55°F$, determine the altitudes where clouds will not form.

(b) The slopes of the graphs for the functions T and D are called *lapse rates*. Interpret their meanings. Explain how these two slopes ensure a strong likelihood of clouds forming. (**Source:** A. Miller and R. Anthes, *Meteorology*.)

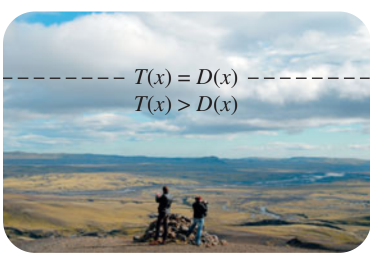


Figure 2.46

SOLUTION

(a) Since $T_0 = 75$ and $D_0 = 55$, let $T(x) = 75 - 19x$ and $D(x) = 55 - 5.8x$. Clouds will not form when the air temperature is greater than the dew point. Therefore we must solve the inequality $T(x) > D(x)$. Graph $T = 75 - 19x$ and $D = 55 - 5.8x$, as shown in Figure 2.47. The graphs intersect near (1.52, 46.2). This means that the air temperature and dew point are both 46.2°F at about 1.52 miles above ground level. Clouds will not form below this altitude, or when the graph of T is above the graph of D. The solution set is $\{x \mid 0 \le x < 1.52\}$, where the endpoint 1.52 has been approximated.

[0, 5, 1] by [0, 80, 10]

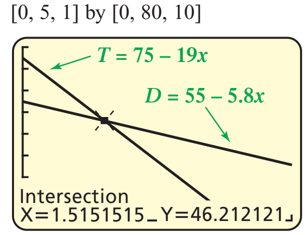


Figure 2.47

(b) The slope of the graph of T is -19. This means that for each 1-mile increase in altitude, the air temperature *decreases* by 19°F. Similarly, the slope of the graph of D is -5.8. The dew point *decreases* by 5.8°F for every 1-mile increase in altitude. As the altitude increases, the air temperature decreases at a faster rate than the dew point. As a result, the air temperature typically cools to the dew point at higher altitudes and clouds may form.

Now Try Exercise 85

CLASS DISCUSSION

How does the difference between the air temperature and the dew point at ground level affect the altitude at which clouds may form? Explain.

***x*-Intercept Method** If a linear inequality can be written as $y_1 > 0$, where $>$ may be replaced by $\ge$, $\le$, or $<$, then we can solve this inequality by using the ***x*-intercept method**. To apply this method for $y_1 > 0$, graph y_1 and find the x-intercept. The solution set includes x-values where the graph of y_1 is **above** the x-axis.

EXAMPLE 4 Applying the x-intercept method

Solve the inequality $1 - x > \frac{1}{2}x - 2$ by using the x-intercept method. Write the solution set in set-builder and interval notation. Then solve the inequality symbolically.

SOLUTION

Graphical Solution Start by subtracting $\frac{1}{2}x - 2$ from each side to obtain the inequality $1 - x - \left(\frac{1}{2}x - 2\right) > 0$. Then graph $Y_1 = 1 - X - (X/2 - 2)$, as shown in Figure 2.48, where the x-intercept is 2. The graph of y_1 is above the x-axis when $x < 2$. Therefore the solution set to $y_1 > 0$ is $\{x \mid x < 2\}$, or $(-\infty, 2)$.

$[-6, 6, 1]$ by $[-4, 4, 1]$

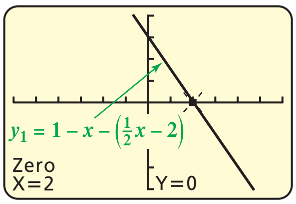


Figure 2.48

Calculator Help

To locate a zero or x-intercept on the graph of a function, see Appendix A (page AP-9).

Symbolic Solution

$$1 - x > \frac{1}{2}x - 2 \quad \text{Given inequality}$$

$$-x > \frac{1}{2}x - 3 \quad \text{Subtract 1 from each side.}$$

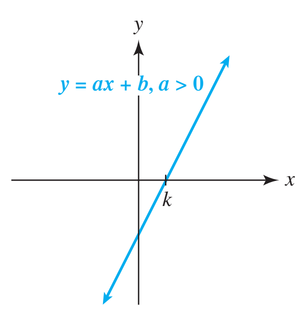
$$-\frac{3}{2}x > -3 \quad \text{Subtract } \tfrac{1}{2}x \text{ from each side.}$$

$$x < 2 \quad \text{Multiply by } -\tfrac{2}{3}\text{; reverse inequality.}$$

The solution set is $\{x \mid x < 2\}$, or $(-\infty, 2)$.

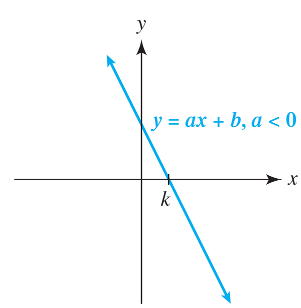
Now Try Exercise 51

Visualizing Solutions Example 4 suggests a general result about linear inequalities. The graph of the equation $y = ax + b$, $a \neq 0$, is a line that intersects the x-axis once and slopes either upward (if $a > 0$) or downward (if $a < 0$). If the x-intercept is k, then the solution set to $ax + b > 0$ satisfies either $x > k$ or $x < k$, as illustrated in Figures 2.49 and 2.50. (Similar remarks hold for linear inequalities that use the symbol $<$, $\leq$, or $\geq$.) Note that the solution to $ax + b = 0$ is k and the value of k is that *boundary* between *greater than* and *less than* in either situation.



Solutions to $ax + b > 0$ satisfy $x > k$ when $a > 0$.

Figure 2.49



Solutions to $ax + b > 0$ satisfy $x < k$ when $a < 0$.

Figure 2.50

Numerical Solutions Inequalities can sometimes be solved by using a table of values. The following example helps to explain the mathematical concept behind this method.

Suppose that it costs a company $5x + 200$ dollars to produce x pairs of headphones and the company receives $15x$ dollars for selling x pairs of headphones. Then the profit P from selling x pairs of headphones is $P = 15x - (5x + 200) = 10x - 200$. A value of $x = 20$ results in $P = 0$, and so $x = 20$ is called the **boundary number** because it represents

the boundary between making money and losing money (the break-even point). To make money, the profit P must be positive, and the inequality

$$10x - 200 > 0$$

must be satisfied. The table of values for $y_1 = 10x - 200$ in Table 2.8 shows the boundary number $x = 20$ along with several **test values**. The test values of $x = 17$, 18, and 19 result in a loss. The test values of $x = 21$, 22, and 23 result in a profit. Therefore the solution set to $10x - 200 > 0$ is $\{x \mid x > 20\}$.

Table 2.8

x	17	18	19	20	21	22	23
$10x - 200$	−30	−20	−10	0	10	20	30

Less than 0 (17–19); Boundary number (20); Greater than 0 (21–23)

EXAMPLE 5 Solving a linear inequality with test values

Solve $3(6 - x) + 5 - 2x < 0$ numerically.

SOLUTION We begin by making a table of $Y_1 = 3(6 - X) + 5 - 2X$, as shown in Figure 2.51. We can see that the boundary number for this inequality lies between $x = 4$ and $x = 5$. Changing the increment from 1 to 0.1 in Figure 2.52 shows that the boundary number for the inequality is $x = 4.6$. The test values of $x = 4.7$, 4.8, and 4.9 indicate that when $x > 4.6$, the inequality $y_1 < 0$ is true. The solution set is $\{x \mid x > 4.6\}$.

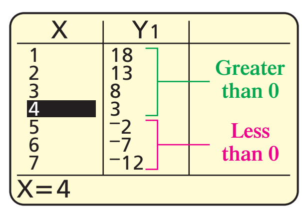


Figure 2.51

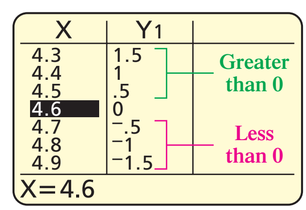


Figure 2.52

Now Try Exercise 71

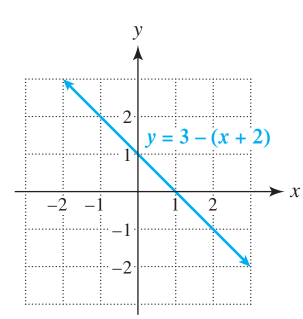
MAKING CONNECTIONS

Symbolic, Graphical, and Numerical Solutions Linear inequalities can be solved symbolically, graphically, or numerically. Each method is used in the following example to solve the inequality $3 - (x + 2) > 0$.

Symbolic Solution

$$3 - (x + 2) > 0$$
$$-(x + 2) > -3$$
$$x + 2 < 3$$
$$x < 1$$

Graphical Solution



The graph of $y = 3 - (x + 2)$ is above the x-axis when $x < 1$.

Numerical Solution

x	$3 - (x + 2)$
−2	3
−1	2
0	1
1	0
2	−1
3	−2

Greater than 0 (x = −2, −1, 0); Equal 0 (x = 1); Less than 0 (x = 2, 3)

The values of $3 - (x + 2)$ are greater than 0 when $x < 1$.

Compound Inequalities

Sometimes a variable must satisfy two inequalities. For example, on a freeway there may be a minimum speed limit of 40 miles per hour and a maximum speed limit of 70 miles per hour. If x represents the speed of a vehicle, then x must satisfy the compound inequality

$$x \geq 40 \quad \text{and} \quad x \leq 70.$$

A **compound inequality** occurs when two inequalities are connected by the word *and* or *or*. When the word *and* connects two inequalities, the two inequalities can sometimes be written as a **three-part inequality**. For example, the previous compound inequality may be written as the three-part inequality

$$40 \leq x \leq 70.$$

Compound inequalities involving the word *or* are discussed in the next section.

EXAMPLE 6 Solving a three-part inequality symbolically

Solve the inequality. Write the solution set in set-builder and interval notation.

(a) $-4 \leq 5x + 1 < 21$ **(b)** $\frac{1}{2} < \frac{1 - 2t}{4} < 2$

SOLUTION

(a) Use properties of inequalities to simplify the three-part inequality.

$$\begin{aligned} -4 \leq 5x + 1 < 21 &\quad \text{Given inequality} \\ -5 \leq 5x < 20 &\quad \text{Add } -1 \text{ to each part.} \\ -1 \leq x < 4 &\quad \text{Divide each part by 5.} \end{aligned}$$

The solution set is $\{x \mid -1 \leq x < 4\}$, or $[-1, 4)$.

(b) Begin by multiplying each part by 4 to clear fractions.

$$\begin{aligned} \frac{1}{2} < \frac{1 - 2t}{4} < 2 &\quad \text{Given inequality} \\ 2 < 1 - 2t < 8 &\quad \text{Multiply each part by 4.} \\ 1 < -2t < 7 &\quad \text{Add } -1 \text{ to each part.} \\ -\frac{1}{2} > t > -\frac{7}{2} &\quad \text{Divide by } -2\text{; reverse inequalities.} \\ -\frac{7}{2} < t < -\frac{1}{2} &\quad \text{Rewrite the inequality.} \end{aligned}$$

The solution set is $\left\{t \mid -\frac{7}{2} < t < -\frac{1}{2}\right\}$, or $\left(-\frac{7}{2}, -\frac{1}{2}\right)$.

Now Try Exercises 23 and 35

NOTE In Example 6, it is correct to write a three-part inequality as either $-\frac{1}{2} > t > -\frac{7}{2}$ or $-\frac{7}{2} < t < -\frac{1}{2}$. However, we usually write the smaller number on the left side and the larger number on the right side.

Three-part inequalities occur in many applications and can often be solved symbolically and graphically. This is demonstrated in the next example.

EXAMPLE 7 Modeling sunset times

In Boston, on the 82nd day (March 22) of 2008 the sun set at 7:00 P.M., and on the 136th day (May 15) the sun set at 8:00 P.M. Use a linear function S to estimate the days when the sun set between 7:15 P.M. and 7:45 P.M., inclusive. Do not consider any days of the year after May 15. (**Source:** R. Thomas, *The Old Farmer's 2008 Almanac*.)

SOLUTION

Getting Started First, find a linear function S whose graph passes through the points (82, 7) and (136, 8). Then solve the compound inequality $7.25 \le S(x) \le 7.75$. Note that 7.25 hours past noon corresponds to 7:15 P.M. and 7.75 hours past noon corresponds to 7:45 P.M. ▶

Symbolic Solution The slope of the line passing through (82, 7) and (136, 8) is given by $\frac{8-7}{136-82} = \frac{1}{54}$. The point-slope form of the line passing through (82, 7) with slope $\frac{1}{54}$ is

$$S(x) = \frac{1}{54}(x - 82) + 7. \qquad \text{Point-slope form}$$

Now solve the following compound inequality.

$$7.25 \le \frac{1}{54}(x - 82) + 7 \le 7.75 \qquad \text{Given inequality}$$

$$0.25 \le \frac{1}{54}(x - 82) \le 0.75 \qquad \text{Subtract 7 from each part.}$$

$$13.5 \le x - 82 \le 40.5 \qquad \text{Multiply each part by 54.}$$

$$95.5 \le x \le 122.5 \qquad \text{Add 82 to each part.}$$

If we round 95.5 and 122.5 up to 96 and 123, then this model predicts that the sun set between 7:15 P.M. and 7:45 P.M. from the 96th day (April 5) to the 123rd day (May 2). (Note that the actual days were April 5 and May 1.)

Graphical Solution Graph $y_1 = 7.25$, $y_2 = \frac{1}{54}(x - 82) + 7$, and $y_3 = 7.75$ and determine their points of intersection, (**95.5**, 7.25) and (**122.5**, 7.75), as shown in Figures 2.53 and 2.54. The graph of y_2 is between the graphs of y_1 and y_3 for $\mathbf{95.5} \le x \le \mathbf{122.5}$. This agrees with the symbolic solution. A different graph showing this solution appears in Figure 2.55.

[80, 150, 10] by [6.5, 8.5, 1]

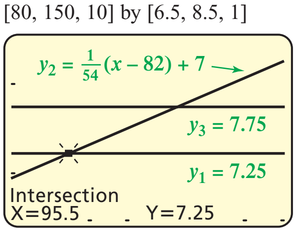


Figure 2.53

[80, 150, 10] by [6.5, 8.5, 1]

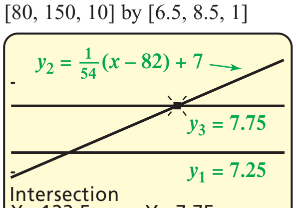


Figure 2.54

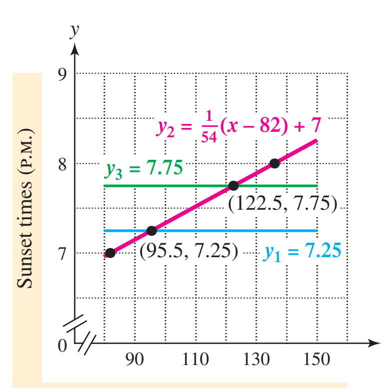


Figure 2.55

Now Try Exercise 93 ◀

EXAMPLE 8 Solving inequalities symbolically

Solve the linear inequalities symbolically. Express the solution set using interval notation.

(a) $-\frac{x}{2} + 1 \leq 3$ **(b)** $-8 < \frac{3x - 1}{2} \leq 5$ **(c)** $5(x - 6) < 2x - 2(1 - x)$

SOLUTION

(a) Simplify the inequality as follows.

$$-\frac{x}{2} + 1 \leq 3 \quad \text{Given inequality}$$

$$-\frac{x}{2} \leq 2 \quad \text{Add } -1\text{, or subtract 1.}$$

$$x \geq -4 \quad \text{Multiply by } -2\text{. Reverse the inequality.}$$

In interval notation the solution set is $[-4, \infty)$.

(b) The parts of this compound inequality can be solved simultaneously.

$$-8 < \frac{3x - 1}{2} \leq 5 \quad \text{Given inequality}$$

$$-16 < 3x - 1 \leq 10 \quad \text{Multiply by 2.}$$

$$-15 < 3x \leq 11 \quad \text{Add 1.}$$

$$-5 < x \leq \frac{11}{3} \quad \text{Divide by 3.}$$

The solution set is $\left(-5, \frac{11}{3}\right]$.

(c) Start by applying the distributive property to each side of the inequality.

$$5(x - 6) < 2x - 2(1 - x) \quad \text{Given inequality}$$

$$5x - 30 < 2x - 2 + 2x \quad \text{Distributive property}$$

$$5x - 30 < 4x - 2 \quad \text{Simplify.}$$

$$x - 30 < -2 \quad \text{Subtract } 4x\text{.}$$

$$x < 28 \quad \text{Add 30.}$$

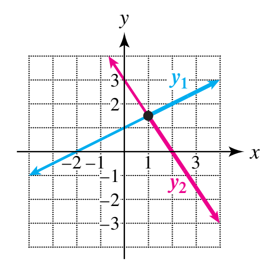
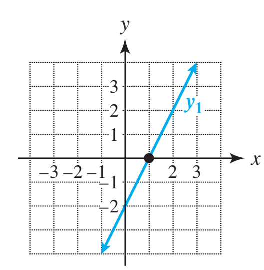
The solution set is $(-\infty, 28)$.

Now Try Exercises 13, 27, and 33

2.4 Putting It All Together

Any linear inequality can be written as $ax + b > 0$ with $a \neq 0$, where $>$ can be replaced by $\geq$, $<$, or $\leq$. The following table includes methods for solving linear inequalities in the form $h(x) > 0$ or $f(x) > g(x)$. Inequalities involving $<$, $\leq$, and $\geq$ are solved in a similar manner.

Concept	Explanation	Examples
Interval notation	An efficient notation for writing the solution set to inequalities To review interval notation, see Section 1.5.	$x < 2$ is equivalent to $(-\infty, 2)$. $x \geq 1$ is equivalent to $[1, \infty)$. $-1 \leq x < 4$ is equivalent to $[-1, 4)$.

<table>
<tr><th>Concept</th><th>Explanation</th><th>Examples</th></tr>
<tr><td>Compound inequality</td><td>Two inequalities connected by the word and or or</td><td>$x \le 4$ or $x \ge 10$
$x \ge -3$ and $x < 4$
$x > 5$ and $x \le 20$ can be written as the three-part inequality $5 < x \le 20$.</td></tr>
<tr><td>Symbolic method</td><td>Use properties of inequalities to simplify $f(x) > g(x)$ to either $x > k$ or $x < k$ for some real number k.</td><td>$\frac{1}{2}x + 1 > 3 - \frac{3}{2}x$ Given inequality
$2x + 1 > 3$ Add $\frac{3}{2}x$.
$2x > 2$ Subtract 1.
$x > 1$ Divide by 2.</td></tr>
<tr><td>Intersection-of-graphs method</td><td>To solve $f(x) > g(x)$, graph $y_1 = f(x)$ and $y_2 = g(x)$. Find the point of intersection. The solution set includes x-values where the graph of y_1 is above the graph of y_2.</td><td>$$\frac{1}{2}x + 1 > 3 - \frac{3}{2}x$$
Graph $y_1 = \frac{1}{2}x + 1$ and $y_2 = 3 - \frac{3}{2}x$. The solution set for $y_1 > y_2$ is $\{x \mid x > 1\}$.

</td></tr>
<tr><td>The x-intercept method</td><td>Write the inequality as $h(x) > 0$. Graph $y_1 = h(x)$. Solutions occur where the graph is above the x-axis.</td><td>$$\frac{1}{2}x + 1 > 3 - \frac{3}{2}x$$
Graph $y_1 = \frac{1}{2}x + 1 - \left(3 - \frac{3}{2}x\right)$. The solution set for $y_1 > 0$ is $\{x \mid x > 1\}$.

</td></tr>
<tr><td>Numerical method</td><td>Write the inequality as $h(x) > 0$. Create a table for $y_1 = h(x)$ and find the boundary number $x = k$ such that $h(k) = 0$. Use the test values in the table to determine if the solution set is $x > k$ or $x < k$.</td><td>$$\frac{1}{2}x + 1 > 3 - \frac{3}{2}x$$
Table $y_1 = \frac{1}{2}x + 1 - \left(3 - \frac{3}{2}x\right)$. The solution set for $y_1 > 0$ is $\{x \mid x > 1\}$.
<table><tr><th>x</th><td>−1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><th>y_1</th><td>−4</td><td>−2</td><td>0</td><td>2</td><td>4</td></tr><tr><td></td><td colspan="2">Less than 0</td><td></td><td colspan="2">Greater than 0</td></tr></table></td></tr>
</table>

2.4 Exercises

Interval Notation

Exercises 1–8: Express the following in interval notation.

1. $x < 2$
2. $x > -3$
3. $x \geq -1$
4. $x \leq 7$
5. $\{x \mid 1 \leq x < 8\}$
6. $\{x \mid -2 < x \leq 4\}$
7. $\{x \mid x \leq 1\}$
8. $\{x \mid x > 5\}$

Solving Linear Inequalities Symbolically

Exercises 9–38: Solve the inequality symbolically. Express the solution set in set-builder or interval notation.

9. $2x + 6 \geq 10$
10. $-4x - 3 < 5$
11. $-2(x - 10) + 1 > 0$
12. $3(x + 5) \leq 0$
13. $\dfrac{t + 2}{3} \geq 5$
14. $\dfrac{2 - t}{6} < 0$
15. $4x - 1 < \dfrac{3 - x}{-3}$
16. $\dfrac{x + 5}{-10} > 2x + 3$
17. $-3(z - 4) \geq 2(1 - 2z)$
18. $-\frac{1}{4}(2z - 6) + z \geq 5$
19. $\dfrac{1 - x}{4} < \dfrac{2x - 2}{3}$
20. $\dfrac{3x}{4} < x - \dfrac{x + 2}{2}$
21. $2x - 3 > \frac{1}{2}(x + 1)$
22. $5 - (2 - 3x) \leq -5x$
23. $5 < 4t - 1 \leq 11$
24. $-1 \leq 2t \leq 4$
25. $3 \leq 4 - x \leq 20$
26. $-5 < 1 - 2x < 40$
27. $-7 \leq \dfrac{1 - 4x}{7} < 12$
28. $0 < \dfrac{7x - 5}{3} \leq 4$
29. $5 > 2(x + 4) - 5 > -5$
30. $\frac{8}{3} \geq \frac{4}{3} - (x + 3) \geq \frac{2}{3}$
31. $3 \leq \frac{1}{2}x + \frac{3}{4} \leq 6$
32. $-4 \leq 5 - \frac{4}{5}x < 6$
33. $5x - 2(x + 3) \geq 4 - 3x$
34. $3x - 1 < 2(x - 3) + 1$
35. $\dfrac{1}{2} \leq \dfrac{1 - 2t}{3} < \dfrac{2}{3}$
36. $-\dfrac{3}{4} < \dfrac{2 - t}{5} < \dfrac{3}{4}$
37. $\frac{1}{2}z + \frac{2}{3}(3 - z) - \frac{5}{4}z \geq \frac{3}{4}(z - 2) + z$
38. $\dfrac{2}{3}(1 - 2z) - \dfrac{3}{2}z + \dfrac{5}{6}z \geq \dfrac{2z - 1}{3} + 1$

Solving Linear Inequalities Graphically

Exercises 39–46: (Refer to Example 2.) Solve the inequality graphically. Use set-builder notation.

39. $x + 2 \geq 2x$
40. $2x - 1 \leq x$
41. $\frac{2}{3}x - 2 > -\frac{4}{3}x + 4$
42. $-2x \geq -\frac{5}{3}x + 1$
43. $-1 \leq 2x - 1 \leq 3$
44. $-2 < 1 - x < 2$
45. $-3 < x - 2 \leq 2$
46. $-1 \leq 1 - 2x < 5$

Exercises 47–50: Use the given graph of $y = ax + b$ to solve each equation and inequality. Write the solution set to each inequality in set-builder or interval notation.

(a) $ax + b = 0$ *(b)* $ax + b < 0$ *(c)* $ax + b \geq 0$

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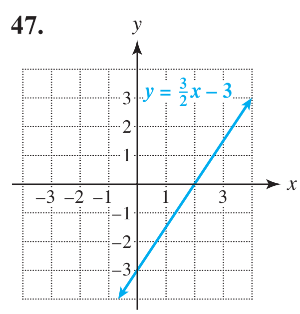

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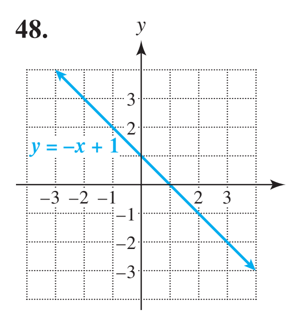

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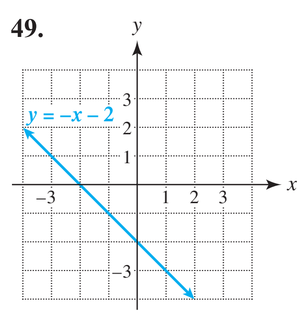

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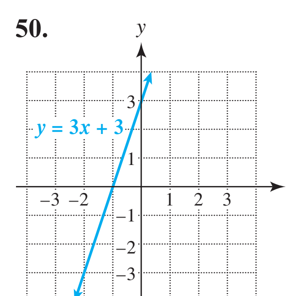


Exercises 51–54: ***x-Intercept Method*** *(Refer to Example 4.) Use the x-intercept method to solve the inequality. Write the solution set in set-builder or interval notation. Then solve the inequality symbolically.*

51. $x - 3 \le \frac{1}{2}x - 2$ **52.** $x - 2 \le \frac{1}{3}x$

53. $2 - x < 3x - 2$ **54.** $\frac{1}{2}x + 1 > \frac{3}{2}x - 1$

Exercises 55–60: Solve the linear inequality graphically. Write the solution set in set-builder notation. Approximate endpoints to the nearest hundredth whenever appropriate.

55. $5x - 4 > 10$

56. $-3x + 6 \le 9$

57. $-2(x - 1990) + 55 \ge 60$

58. $\sqrt{2}x > 10.5 - 13.7x$

59. $\sqrt{5}(x - 1.2) - \sqrt{3}x < 5(x + 1.1)$

60. $1.238x + 0.998 \le 1.23(3.987 - 2.1x)$

Exercises 61–66; Solve the compound linear inequality graphically. Write the solution set in set-builder or interval notation, and approximate endpoints to the nearest tenth whenever appropriate.

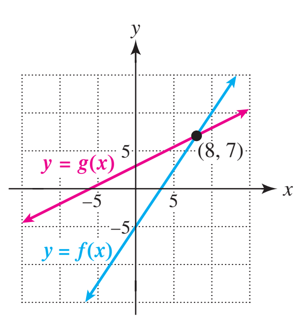
61. $3 \le 5x - 17 < 15$ **62.** $-4 < \dfrac{55 - 3.1x}{4} < 17$

63. $1.5 \le 9.1 - 0.5x \le 6.8$ **64.** $0.2x < \dfrac{2x - 5}{3} < 8$

65. $x - 4 < 2x - 5 < 6$ **66.** $-3 \le 1 - x \le 2x$

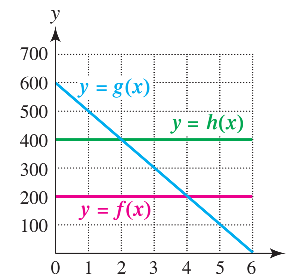
67. The graphs of two linear functions f and g are shown.
(a) Solve the equation $g(x) = f(x)$.

(b) Solve the inequality $g(x) > f(x)$.



68. Use the figure to solve each equation or inequality.
(a) $f(x) = g(x)$ (b) $g(x) = h(x)$

(c) $f(x) < g(x) < h(x)$ (d) $g(x) > h(x)$



Solving Linear Inequalities Numerically

Exercises 69 and 70: Assume y_1 represents a linear function with the set of real numbers for its domain. Use the table to solve the inequalities. Use set-builder notation.

69. $y_1 > 0, y_1 \le 0$ **70.** $y_1 < 0, y_1 \ge 0$

X	Y1
0	220
1	165
2	110
3	55
4	0
5	−55
6	−110
X=0	

X	Y1
−5	−32
−4	−16
−3	0
−2	16
−1	32
0	48
1	64
X=−5	

Exercises 71–78: Solve each inequality numerically. Write the solution set in set-builder or interval notation, and approximate endpoints to the nearest tenth when appropriate.

71. $-4x - 6 > 0$ **72.** $1 - 2x \ge 9$

73. $1 \le 3x - 2 \le 10$ **74.** $-5 < 2x - 1 < 15$

75. $-\dfrac{3}{4} < \dfrac{2 - 5x}{3} \le \dfrac{3}{4}$ **76.** $\dfrac{3x - 1}{5} < 15$

77. $(\sqrt{11} - \pi)x - 5.5 \le 0$

78. $1.5(x - 0.7) + 1.5x < 1$

You Decide the Method

Exercises 79–82: Solve the inequality. Approximate the endpoints to the nearest thousandth when appropriate.

79. $2x - 8 > 5$ **80.** $5 < 4x - 2.5$

81. $\pi x - 5.12 \le \sqrt{2}x - 5.7(x - 1.1)$

82. $5.1x - \pi \ge \sqrt{3} - 1.7x$

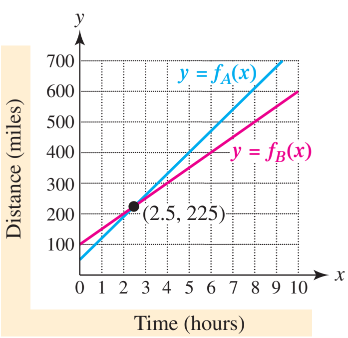
Applications

83. ***Distance between Cars*** Cars A and B are both traveling in the same direction. Their distances in miles north of St. Louis after x hours are computed by the functions f_A and f_B, respectively. The graphs of f_A and f_B are shown in the figure for $0 \le x \le 10$.

(a) Which car is traveling faster? Explain.

(b) How many hours elapse before the two cars are the same distance from St. Louis? How far are they from St. Louis when this occurs?

(c) During what time interval is car B farther from St. Louis than car A?



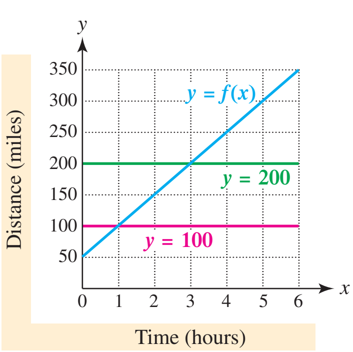
84. ***Distance*** Function f computes the distance y in miles between a car and the city of Omaha after x hours, where $0 \le x \le 6$. The graphs of f and the horizontal lines $y = 100$ and $y = 200$ are shown in the figure.

(a) Is the car moving toward or away from Omaha? Explain.

(b) Determine the times when the car is 100 miles and 200 miles from Omaha.

(c) Determine when the car is from 100 to 200 miles from Omaha.

(d) When is the car's distance from Omaha greater than 100 miles?



85. ***Clouds and Temperature*** (Refer to Example 3.) Suppose the ground-level temperature is 65°F and the dew point is 50°F.

(a) Use the intersection-of-graphs method to estimate the altitudes where clouds will not form.

(b) Solve part (a) symbolically.

86. ***Temperature and Altitude*** Suppose the Fahrenheit temperature x miles above ground level is given by the formula $T(x) = 85 - 19x$.

(a) Use the intersection-of-graphs method to estimate the altitudes where the temperature is below freezing. Assume that the domain of T is $0 \le x \le 6$.

(b) What does the x-intercept on the graph of $y = T(x)$ represent?

(c) Solve part (a) symbolically.

87. ***Prices of Homes*** The median prices of a single-family home in the United States from 1990 to 2005 can be approximated by the formula $P(x) = 8667x + 90{,}000$, where $x = 0$ corresponds to 1990 and $x = 15$ to 2005. (**Source:** National Association of Realtors.)

(a) Interpret the slope of the graph of P.

(b) Estimate the years when the median price range was from \$142,000 to \$194,000.

88. ***Population Density*** The population density D of the United States in people per square mile during year x from 1900 to 2000 can be approximated by the formula $D(x) = 0.58x - 1080$. (**Source:** Bureau of the Census.)

(a) Interpret the slope of the graph of D.

(b) Estimate when the density varied between 50 and 75 people per square mile.

89. ***Broadband Internet Connections*** The number of households using broadband Internet connections, such as cable and DSL, increased from 6 million in 2000 to 30 million in 2004. (**Source:** eMarketer.)

(a) Find a linear function given by

$$B(x) = m(x - x_1) + y_1$$

that models these data, where x is the year.

(b) Use $B(x)$ to estimate the years when the number of households using broadband Internet connections was 24 million or more. Assume that the domain of B is 2000 to 2006.

90. ***Online Betting*** Consumer gambling losses from online betting were \$4 billion in 2002 and \$10 billion in 2005. (**Source:** Christiansen Capital Advisors.)

(a) Find a linear function given by

$$B(x) = m(x - x_1) + y_1$$

that models these data, where x is the year.

(b) Use $B(x)$ to estimate the years when consumer losses from online betting were more than \$6 billion. Assume that the domain of B is 2002 to 2007.

91. ***Consumer Spending*** In 2005 consumers used credit and debit cards to pay for 40% of all purchases. This percentage is projected to be 55% in 2011. (**Source:** Bloomburg.)

(a) Find a linear function P that models the data.

(b) Estimate when this percentage was between 45% and 50%.

92. ***VISA Cards*** Annual transactions on VISA cards increased from \$400 billion in 2002 to \$635 billion in 2007. (**Source:** CardWeb.)

(a) Find a linear function V that models the data.

(b) Estimate when this number was between \$450 billion and \$540 billion.

93. ***Modeling Sunrise Times*** In Boston, on the 90th day (March 30) of 2008 the sun rose at 6:30 A.M., and on the 129th day (May 8) the sun rose at 5:30 A.M. Use a linear function to estimate the days when the sun rose between 5:45 A.M. and 6:00 A.M. Do not consider days after May 8. (**Source:** R. Thomas.)

94. ***Modeling Sunrise Times*** In Denver, on the 77th day (March 17) of 2008 the sun rose at 7:00 A.M., and on the 112th day (April 21) the sun rose at 6:00 A.M. Use a linear function to estimate the days when the sun rose between 6:10 A.M. and 6:40 A.M. Do not consider days after April 21. (**Source:** R. Thomas.)

95. ***Error Tolerances*** Suppose that an aluminum can is manufactured so that its radius r can vary from 1.99 inches to 2.01 inches. What range of values is possible for the circumference C of the can? Express your answer by using a three-part inequality.

96. ***Error Tolerances*** Suppose that a square picture frame has sides that vary between 9.9 inches and 10.1 inches. What range of values is possible for the perimeter P of the picture frame? Express your answer by using a three-part inequality.

97. ***Modeling Data*** The following data are exactly linear.

x	0	2	4	6
y	−1.5	4.5	10.5	16.5

(a) Find a linear function f that models the data.

(b) Solve the inequality $f(x) > 2.25$.

98. ***Modeling Data*** The following data are exactly linear.

x	1	2	3	4	5
y	0.4	3.5	6.6	9.7	12.8

(a) Find a linear function f that models the data.

(b) Solve the inequality $2 \leq f(x) \leq 8$.

Linear Regression

99. ***Cell Phone Subscribers*** The table lists the number N of cell phone subscribers worldwide in millions for selected years x.

x	2001	2002	2003	2004	2005
N	128	141	159	182	208

Source: CTIA—The Wireless Association.

(a) Find a linear function N that models the data.

(b) Estimate the years when this number was from 43 million to 83 million.

(c) Did your estimate involve interpolation or extrapolation?

100. ***Home Ownership Rates*** The table lists the percentage P of U.S. homes that are owned by their occupant rather than rented for selected years x.

x	1900	1950	1980	2006
P	47%	55%	64%	69%

Source: Bureau of the Census.

(a) Find a linear function P that models the data.

(b) Estimate the years when this percentage was from 58% to 60%.

(c) Did your estimate involve interpolation or extrapolation?

Writing about Mathematics

101. Suppose the solution to the equation $ax + b = 0$ with $a > 0$ is $x = k$. Discuss how the value of k can be used to help solve the linear inequalities $ax + b > 0$ and $ax + b < 0$. Illustrate this process graphically. How would the solution sets change if $a < 0$?

102. Describe how to numerically solve the linear inequality $ax + b \leq 0$. Give an example.

103. If you multiply each part of a three-part inequality by the same negative number, what must you make sure to do? Explain by using an example.

104. Explain how a linear function, a linear equation, and a linear inequality are related. Give an example.

EXTENDED AND DISCOVERY EXERCISES

1. ***Arithmetic Mean*** The arithmetic mean of two numbers a and b is given by $\frac{a + b}{2}$. Use properties of inequalities to show that if $a < b$, then $a < \frac{a + b}{2} < b$.

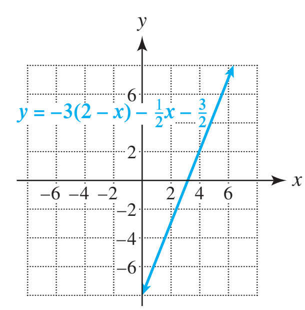
2. ***Geometric Mean*** The **geometric mean** of two numbers a and b is given by $\sqrt{ab}$. Use properties of inequalities to show that if $0 < a < b$, then $a < \sqrt{ab} < b$.

1. Solve the linear equation $4(x - 2) = 2(5 - x) - 3$ by using each method. Compare your results.
(a) Graphical **(b)** Numerical **(c)** Symbolic

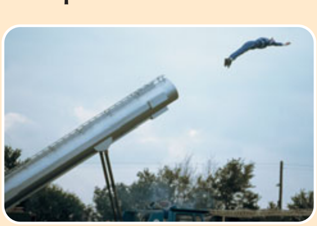
2. Solve the inequality $2(x - 4) > 1 - x$. Express the solution set in set-builder notation.

3. Solve the compound inequality $-2 \leq 1 - 2x \leq 3$. Use set-builder or interval notation.

4. Use the graph to the right to solve each equation and inequality. Then solve each part symbolically. Use set-builder or interval notation when possible.
(a) $-3(2 - x) - \frac{1}{2}x - \frac{3}{2} = 0$
(b) $-3(2 - x) - \frac{1}{2}x - \frac{3}{2} > 0$
(c) $-3(2 - x) - \frac{1}{2}x - \frac{3}{2} \leq 0$



- **Evaluate and graph the absolute value function**
- **Solve absolute value equations**
- **Solve absolute value inequalities**



Introduction

A margin of error can be very important in many aspects of life, including being fired out of a cannon. The most dangerous part of the feat, first done by a human in 1875, is to land squarely on a net. For a human cannonball who wants to fly 180 feet in the air and then land in the center of a net with a 60-foot-long safe zone, there is a margin of error of ± 30 feet. That is, the horizontal distance D traveled by the human cannonball can vary between $180 - 30 = 150$ feet and $180 + 30 = 210$ feet. (**Source:** Ontario Science Center.)

This margin of error can be expressed mathematically by using the *absolute value inequality*

$$|D - 180| \leq 30.$$

The absolute value is necessary because D can be either less than or greater than 180, but by not more than 30 feet.

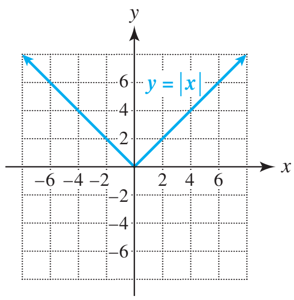


Figure 2.56 The Absolute Value Function

The Absolute Value Function

The graph of $y = |x|$ is shown in Figure 2.56. It is V-shaped and cannot be represented by a single linear function. However, it can be represented by the lines $y = x$ (when $x \geq 0$) and $y = -x$ (when $x < 0$). This suggests that the absolute value function can be defined symbolically using a piecewise-linear function. The absolute value function is decreasing for $x \leq 0$ and increasing for $x \geq 0$.

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

There is another formula for $|x|$. Consider the following examples.

$$\sqrt{3^2} = \sqrt{9} = 3 \quad \text{and} \quad \sqrt{(-3)^2} = \sqrt{9} = 3.$$
$$\sqrt{7^2} = \sqrt{49} = 7 \quad \text{and} \quad \sqrt{(-7)^2} = \sqrt{49} = 7.$$

That is, regardless of whether a real number x is positive or negative, the expression $\sqrt{x^2}$ equals the *absolute value* of x. This statement is summarized by

$$\sqrt{x^2} = |x| \qquad \textbf{for all real numbers } x.$$

For example, $\sqrt{y^2} = |y|$, $\sqrt{(x-1)^2} = |x-1|$, and $\sqrt{(2x)^2} = |2x|$.

EXAMPLE 1 Analyzing the graph of $y = |ax + b|$

For each linear function f, graph $y = f(x)$ and $y = |f(x)|$ separately. Discuss how the absolute value affects the graph of f.

(a) $f(x) = x + 2$ **(b)** $f(x) = -2x + 4$

SOLUTION

(a) The graphs of $y_1 = x + 2$ and $y_2 = |x + 2|$ are shown in Figures 2.57 and 2.58, respectively. The graph of y_1 is a line with x-intercept -2. The graph of y_2 is V-shaped. The graphs are identical for $x > -2$. For $x < -2$, the graph of $y_1 = f(x)$ passes below the x-axis, and the graph of $y_2 = |f(x)|$ is the *reflection* of $y_1 = f(x)$ across the x-axis. The graph of $y_2 = |f(x)|$ does not dip below the x-axis because an absolute value is *never* negative.

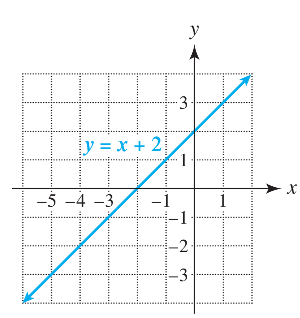


Figure 2.57

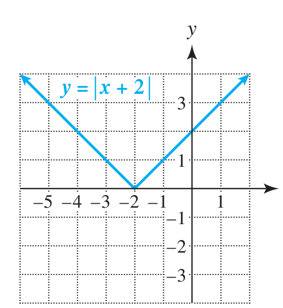


Figure 2.58

NOTE In general, the graph of $y = |f(x)|$ is a reflection of the graph of $y = f(x)$ across the x-axis whenever $f(x) < 0$. Otherwise (whenever $f(x) \geq 0$), their graphs are identical.

Calculator Help
To access the absolute value function, see Appendix A (page AP-10).

(b) The graphs of $y_1 = -2x + 4$ and $y_2 = |-2x + 4|$ are shown in Figures 2.59 and 2.60. Again, the graph of y_2 is V-shaped. The graph of $y_2 = |f(x)|$ is the reflection of f across the x-axis whenever the graph of $y_1 = f(x)$ is below the x-axis.

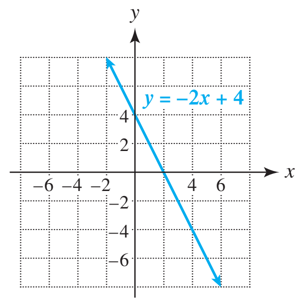


Figure 2.59

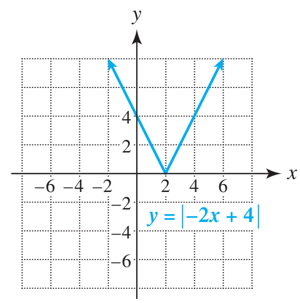


Figure 2.60

Now Try Exercises 13 and 17

Example 1 illustrates the fact that the graph of $y = |ax + b|$ with $a \neq 0$ is V-shaped and is never located below the x-axis. The vertex (or point) of the V-shaped graph corresponds to the x-intercept, which can be found by solving the linear equation $ax + b = 0$.

Absolute Value Equations

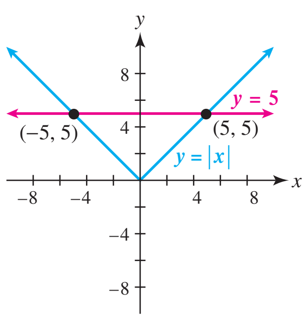


Figure 2.61

The equation $|x| = 5$ has two solutions: ± 5. This fact is shown visually in Figure 2.61, where the graph of $y = |x|$ intersects the horizontal line $y = 5$ at the points $(\pm 5, 5)$. In general, the solutions to $|x| = k$ with $k > 0$ are given by $x = \pm k$. Thus if $y = ax + b$, then $|ax + b| = k$ has two solutions given by $ax + b = \pm k$.

These concepts can be illustrated visually. The graph of $y = |ax + b|$ with $a \neq 0$ is V-shaped. It intersects the horizontal line $y = k$ twice whenever $k > 0$, as illustrated in Figure 2.62. Thus there are two solutions to the equation $|ax + b| = k$. This V-shaped graph intersects the line $y = 0$ once, as shown in Figure 2.63. As a result, equation $|ax + b| = 0$ has one solution, which corresponds to the x-intercept. When $k < 0$, the line $y = k$ lies *below* the x-axis and there are no points of intersection, as shown in Figure 2.64. Thus the equation $|ax + b| = k$ with $k < 0$ has no solutions.

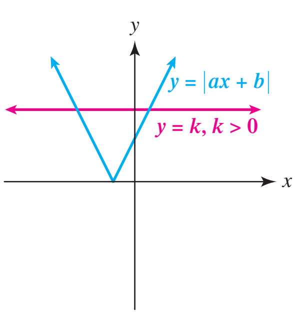


Figure 2.62 Two Solutions

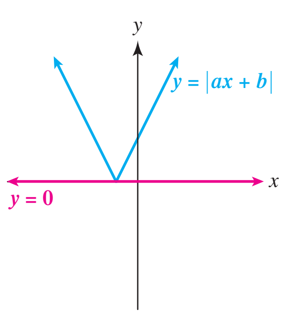


Figure 2.63 One Solution

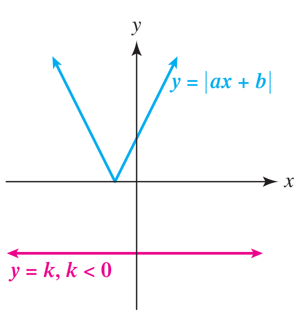


Figure 2.64 No Solutions

Absolute Value Equations

Let k be a positive number. Then

$$|ax + b| = k \quad \text{is equivalent to} \quad ax + b = \pm k.$$

EXAMPLE 2 Solving absolute value equations

Solve each equation.

(a) $\left|\frac{3}{4}x - 6\right| = 15$ **(b)** $|1 - 2x| = -3$ **(c)** $|3x - 2| - 5 = -2$

SOLUTION

(a) The equation $\left|\frac{3}{4}x - 6\right| = 15$ is satisfied when $\frac{3}{4}x - 6 = \pm 15$.

$$\begin{array}{rcrl} \frac{3}{4}x - 6 = 15 & \text{or} & \frac{3}{4}x - 6 = -15 & \text{Equations to solve} \\ \frac{3}{4}x = 21 & \text{or} & \frac{3}{4}x = -9 & \text{Add 6 to each side.} \\ x = 28 & \text{or} & x = -12 & \text{Multiply by } \frac{4}{3}. \end{array}$$

The solutions are -12 and 28.

(b) Because an absolute value is never negative, $|1 - 2x| \geq 0$ for all x and can never equal -3. There are no solutions. This is illustrated graphically in Figure 2.65.

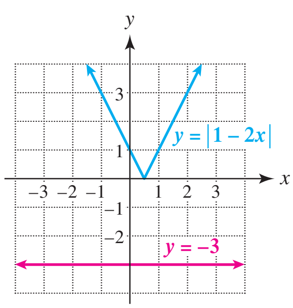


Figure 2.65 No Solutions

(c) Because the right side of the equation is a negative number, it might appear at first glance that there were no solutions. However, if we add 5 to each side of the equation,

$$|3x - 2| - 5 = -2 \quad \text{becomes} \quad |3x - 2| = 3.$$

This equation is equivalent to $3x - 2 = \pm 3$ and has two solutions.

$$\begin{array}{rcrl} 3x - 2 = 3 & \text{or} & 3x - 2 = -3 & \text{Equations to solve} \\ 3x = 5 & \text{or} & 3x = -1 & \text{Add 2 to each side.} \\ x = \frac{5}{3} & \text{or} & x = -\frac{1}{3} & \text{Divide by 3.} \end{array}$$

The solutions are $-\frac{1}{3}$ and $\frac{5}{3}$.

Now Try Exercises 21, 29, and 31

EXAMPLE 3 Solving an equation with technology

Solve the equation $|2x + 5| = 2$ graphically, numerically, and symbolically.

SOLUTION

Graphical Solution Graph $Y_1 = \text{abs}(2X + 5)$ and $Y_2 = 2$. The V-shaped graph of y_1 intersects the horizontal line at the points $(-3.5, 2)$ and $(-1.5, 2)$, as shown in Figures 2.66 and 2.67. The solutions are -3.5 and -1.5.

Numerical Solution Table $Y_1 = \text{abs}(2X + 5)$ and $Y_2 = 2$, as shown in Figure 2.68. The solutions to $y_1 = y_2$ are -3.5 and -1.5.

Calculator Help

To find a point of intersection, see Appendix A (page AP-8).

[−9, 9, 1] by [−6, 6, 1]

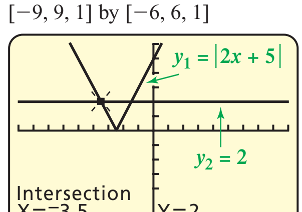


Figure 2.66

[−9, 9, 1] by [−6, 6, 1]

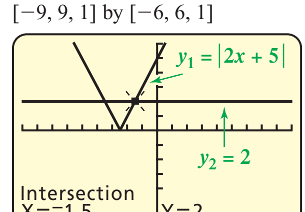


Figure 2.67

X	Y1	Y2	
−4	3	2	
−3.5	2	2	← $y_1 = y_2$
−3	1	2	
−2.5	0	2	
−2	1	2	
−1.5	2	2	← $y_1 = y_2$
−1	3	2	

Y1 = abs(2X+5)

Figure 2.68

Symbolic Solution The equation $|2x + 5| = 2$ is satisfied when $2x + 5 = \pm 2$.

$$\begin{aligned} 2x + 5 &= 2 &\quad\text{or}\quad&& 2x + 5 &= -2 &\quad& \text{Equations to solve} \\ 2x &= -3 &\quad\text{or}\quad&& 2x &= -7 && \text{Subtract 5 from each side.} \\ x &= -\frac{3}{2} &\quad\text{or}\quad&& x &= -\frac{7}{2} && \text{Divide by 2.} \end{aligned}$$

Now Try Exercise 45 ◀

EXAMPLE 4 Describing speed limits with absolute values

The lawful speeds S on an interstate highway satisfy $|S - 55| \leq 15$. Find the maximum and minimum speed limits by solving the equation $|S - 55| = 15$.

SOLUTION The equation $|S - 55| = 15$ is equivalent to $S - 55 = \pm 15$.

$$\begin{aligned} S - 55 &= 15 &\quad\text{or}\quad&& S - 55 &= -15 &\quad& \text{Equations to solve} \\ S &= 70 &\quad\text{or}\quad&& S &= 40 && \text{Add 55 to each side.} \end{aligned}$$

The maximum speed limit is 70 miles per hour and the minimum is 40 miles per hour.

Now Try Exercise 73 ◀

An Equation with Two Absolute Values Sometimes more than one absolute value sign occurs in an equation. For example, an equation might be in the form

$$|ax + b| = |cx + d|.$$

In this case there are two possibilities:

$$\text{either} \quad ax + b = cx + d \quad \text{or} \quad ax + b = -(cx + d).$$

This symbolic technique is demonstrated in the next example.

EXAMPLE 5 Solving an equation involving two absolute values

Solve the equation $|x - 2| = |1 - 2x|$.

SOLUTION We must solve both of the following equations.

$$\begin{aligned} x - 2 &= 1 - 2x &\quad\text{or}\quad&& x - 2 &= -(1 - 2x) \\ 3x &= 3 &\quad\text{or}\quad&& x - 2 &= -1 + 2x \\ x &= 1 &\quad\text{or}\quad&& -1 &= x \end{aligned}$$

There are two solutions: -1 and 1.

Now Try Exercise 35 ◀

Absolute Value Inequalities

In Figure 2.69 the solutions to $|ax + b| = k$ are labeled s_1 and s_2. The V-shaped graph of $y = |ax + b|$ is below the horizontal line $y = k$ between s_1 and s_2, or when $s_1 < x < s_2$. The solution set for the inequality $|ax + b| < k$ is green on the x-axis. In Figure 2.70 the V-shaped graph is above the horizontal line $y = k$ left of s_1 or right of s_2, that is, when $x < s_1$ or $x > s_2$. The solution set for the inequality $|ax + b| > k$ is green on the x-axis.

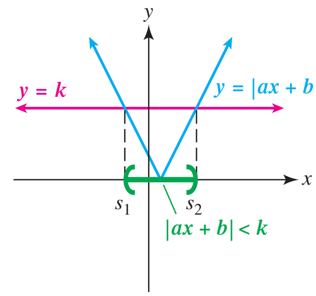


Figure 2.69

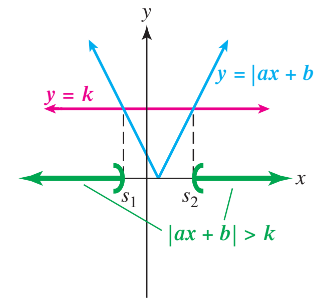


Figure 2.70

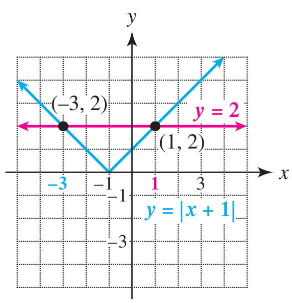


Figure 2.71

Note that in both figures equality (determined by s_1 and s_2) is the boundary between *greater than* and *less than*. For this reason, s_1 and s_2 are called *boundary numbers*.

For example, the graphs of $y = |x + 1|$ and $y = 2$ are shown in Figure 2.71. These graphs intersect at the points $(-3, 2)$ and $(1, 2)$. It follows that the two solutions to

$$|x + 1| = 2$$

are $s_1 = -3$ and $s_2 = 1$. The solutions to $|x + 1| < 2$ lie between $s_1 = -3$ and $s_2 = 1$, which can be written as $-3 < x < 1$. Furthermore, the solutions to $|x + 1| > 2$ lie "outside" $s_1 = -3$ and $s_2 = 1$. This can be written as $x < -3$ or $x > 1$.

These results are generalized as follows.

Absolute Value Inequalities

Let the solutions to $|ax + b| = k$ be s_1 and s_2, where $s_1 < s_2$ and $k > 0$.

1. $|ax + b| < k$ is equivalent to $s_1 < x < s_2$.

2. $|ax + b| > k$ is equivalent to $x < s_1$ or $x > s_2$.

Similar statements can be made for inequalities involving $\le$ or $\ge$.

NOTE The union symbol $\cup$ may be used to write $x < s_1$ or $x > s_2$ in interval notation. For example, $x < -3$ or $x > 1$ is written as $(-\infty, -3) \cup (1, \infty)$ in interval notation. This indicates that the solution set includes all real numbers in either $(-\infty, -3)$ or $(1, \infty)$.

EXAMPLE 6 Solving inequalities involving absolute values symbolically

Solve each inequality symbolically. Write the solution set in interval notation.

(a) $|2x - 5| \le 6$ **(b)** $|5 - x| > 3$

SOLUTION

(a) Begin by solving $|2x - 5| = 6$, or equivalently, $2x - 5 = \pm 6$.

$$\begin{aligned} 2x - 5 &= 6 &\quad \text{or} \quad&& 2x - 5 &= -6 \\ 2x &= 11 &\quad \text{or} \quad&& 2x &= -1 \\ x &= \frac{11}{2} &\quad \text{or} \quad&& x &= -\frac{1}{2} \end{aligned}$$

The solutions to $|2x - 5| = 6$ are $-\frac{1}{2}$ and $\frac{11}{2}$. The solution set for the inequality $|2x - 5| \le 6$ includes all real numbers x satisfying $-\frac{1}{2} \le x \le \frac{11}{2}$. In interval notation this is written as $\left[-\frac{1}{2}, \frac{11}{2}\right]$.

(b) To solve $|5 - x| > 3$, begin by solving $|5 - x| = 3$, or equivalently, $5 - x = \pm 3$.

$$\begin{aligned} 5 - x &= 3 \quad &\text{or}\quad 5 - x &= -3 \\ -x &= -2 \quad &\text{or}\quad -x &= -8 \\ x &= 2 \quad &\text{or}\quad x &= 8 \end{aligned}$$

The solutions to $|5 - x| = 3$ are 2 and 8. The solution set for $|5 - x| > 3$ includes all real numbers x left of 2 or right of 8. Thus $|5 - x| > 3$ is equivalent to $x < 2$ or $x > 8$. In interval notation this is written as $(-\infty, 2) \cup (8, \infty)$.

Now Try Exercises 55 and 63

EXAMPLE 7 Analyzing the temperature range in Santa Fe

The inequality $|T - 49| \le 20$ describes the range of monthly average temperatures T in degrees Fahrenheit for Santa Fe, New Mexico. (**Source:** A. Miller and J. Thompson, *Elements of Meteorology*.)

(a) Solve this inequality graphically and symbolically.

(b) The high and low monthly average temperatures satisfy the absolute value equation $|T - 49| = 20$. Use this fact to interpret the results from part (a).

SOLUTION

(a) ***Graphical Solution*** Graph $Y_1 = \text{abs}(X - 49)$ and $Y_2 = 20$, as in Figure 2.72. The V-shaped graph of y_1 intersects the horizontal line at the points (29, 20) and (69, 20). See Figures 2.73 and 2.74. The graph of y_1 is below the graph of y_2 between these two points. Thus the solution set consists of all temperatures T satisfying $29 \le T \le 69$.

[20, 80, 5] by [0, 35, 5]

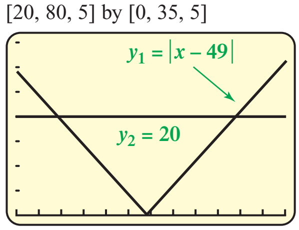


Figure 2.72

[20, 80, 5] by [0, 35, 5]

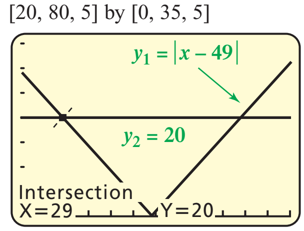


Figure 2.73

[20, 80, 5] by [0, 35, 5]

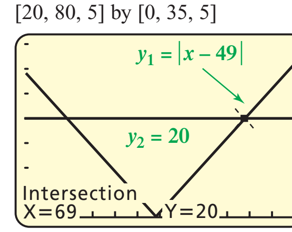


Figure 2.74

Symbolic Solution First solve the related equation $|T - 49| = 20$.

$$\begin{aligned} T - 49 &= -20 \quad &\text{or}\quad T - 49 &= 20 \\ T &= 29 \quad &\text{or}\quad T &= 69 \end{aligned}$$

Thus by our previous discussion $|T - 49| \le 20$ is equivalent to $29 \le T \le 69$.

(b) The solutions to $|T - 49| = 20$ are 29 and 69. Therefore the monthly average temperatures in Sante Fe vary between a low of 29°F (January) and a high of 69°F (July). The monthly averages are always within 20 degrees of 49°F. ***Now Try Exercise 77***

An Alternative Method There is a second symbolic method that can be used to solve absolute value inequalities. This method is often used in advanced mathematics courses, such as calculus. It is based on the following two properties.

Absolute Value Inequalities (Alternative Method)

Let k be a positive number.

1. $|ax + b| < k$ is equivalent to $-k < ax + b < k$.

2. $|ax + b| > k$ is equivalent to $ax + b < -k$ or $ax + b > k$.

Similar statements can be made for inequalities involving $\le$ or $\ge$.

EXAMPLE 8 Using an alternative method

Solve each absolute value inequality. Write your answer in interval notation.

(a) $|4 - 5x| \le 3$ **(b)** $|-4x - 6| > 2$

SOLUTION

(a) $|4 - 5x| \le 3$ is equivalent to the following three-part inequality.

$-3 \le 4 - 5x \le 3$	Equivalent inequality
$-7 \le -5x \le -1$	Subtract 4 from each part.
$\frac{7}{5} \ge x \ge \frac{1}{5}$	Divide each part by -5; reverse the inequality.

In interval notation the solution is $\left[\frac{1}{5}, \frac{7}{5}\right]$.

(b) $|-4x - 6| > 2$ is equivalent to the following compound inequality.

$-4x - 6 < -2$	or	$-4x - 6 > 2$	Equivalent compound inequality
$-4x < 4$	or	$-4x > 8$	Add 6 to each side.
$x > -1$	or	$x < -2$	Divide each by -4; reverse the inequality.

In interval notation the solution set is $(-\infty, -2) \cup (-1, \infty)$.

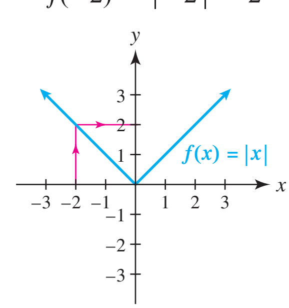
Now Try Exercises 57 and 61

CLASS DISCUSSION

Sketch the graphs of $y = ax + b$, $y = |ax + b|$, $y = -k$, and $y = k$ on one xy-plane. Now use these graphs to explain why the alternative method for solving absolute value inequalities is correct.

2.5 Putting It All Together

The following table summarizes some important concepts from this section.

Concept	Explanation	Examples
Absolute value function	$f(x) = \lvert x \rvert$: The output from the absolute value function is never negative. $f(x) = \lvert x \rvert$ is equivalent to $f(x) = \sqrt{x^2}$.	$f(-2) = \lvert -2 \rvert = 2$ 

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Concept	Explanation	Examples
Absolute value equations	**1.** If $k > 0$, then $\lvert ax + b \rvert = k$ has two solutions, given by $ax + b = \pm k$. **2.** If $k = 0$, then $\lvert ax + b \rvert = k$ has one solution, given by $ax + b = 0$. **3.** If $k < 0$, then $\lvert ax + b \rvert = k$ has no solutions.	**1.** Solve $\lvert 3x - 5 \rvert = 4$. $3x - 5 = -4$ or $3x - 5 = 4$ $3x = 1$ or $3x = 9$ $x = \frac{1}{3}$ or $x = 3$ **2.** Solve $\lvert x - 1 \rvert = 0$. $x - 1 = 0$ implies $x = 1$. **3.** $\lvert 4x - 9 \rvert = -2$ has no solutions.
Absolute value inequalities	To solve $\lvert ax + b \rvert < k$ or $\lvert ax + b \rvert > k$ with $k > 0$, first solve $\lvert ax + b \rvert = k$. Let these solutions be s_1 and s_2, where $s_1 < s_2$. **1.** $\lvert ax + b \rvert < k$ is equivalent to $s_1 < x < s_2$. **2.** $\lvert ax + b \rvert > k$ is equivalent to $x < s_1$ or $x > s_2$.	To solve $\lvert x - 5 \rvert < 4$ or $\lvert x - 5 \rvert > 4$, first solve $\lvert x - 5 \rvert = 4$ to obtain the solutions $s_1 = 1$ and $s_2 = 9$. **1.** $\lvert x - 5 \rvert < 4$ is equivalent to $1 < x < 9$. **2.** $\lvert x - 5 \rvert > 4$ is equivalent to $x < 1$ or $x > 9$.
Alternative method for solving absolute value inequalities	**1.** $\lvert ax + b \rvert < k$ with $k > 0$ is equivalent to $-k < ax + b < k$. **2.** $\lvert ax + b \rvert > k$ with $k > 0$ is equivalent to $ax + b < -k$ or $ax + b > k$.	**1.** $\lvert x - 1 \rvert < 5$ is solved as follows. $-5 < x - 1 < 5$ $-4 < x < 6$ **2.** $\lvert x - 1 \rvert > 5$ is solved as follows. $x - 1 < -5$ or $x - 1 > 5$ $x < -4$ or $x > 6$

2.5

Basic Concepts

Exercises 1–8: Let $a \neq 0$.

1. Solve $\lvert x \rvert = 3$.

2. Solve $\lvert x \rvert \leq 3$.

3. Solve $\lvert x \rvert > 3$.

4. Solve $\lvert ax + b \rvert \leq -2$.

5. Describe the graph of $y = \lvert ax + b \rvert$.

6. Solve $\lvert ax + b \rvert = 0$.

7. Rewrite $\sqrt{36a^2}$ by using an absolute value.

8. Rewrite $\sqrt{(ax + b)^2}$ by using an absolute value.

Absolute Value Graphs

Exercises 9–12: Graph by hand.

(a) Find the x-intercept.
(b) Determine where the graph is increasing and where it is decreasing.

9. $y = \lvert x + 1 \rvert$

10. $y = \lvert 1 - x \rvert$

11. $y = \lvert 2x - 3 \rvert$

12. $y = \lvert \frac{1}{2}x + 1 \rvert$

Exercises 13–18: (Refer to Example 1.) Do the following.

(a) Graph $y = f(x)$.
(b) Use the graph of $y = f(x)$ to sketch a graph of the equation $y = \lvert f(x) \rvert$.
(c) Determine the x-intercept for the graph of the equation $y = \lvert f(x) \rvert$.

13. $y = 2x$

14. $y = \frac{1}{2}x$

15. $y = 3x - 3$

16. $y = 2x - 4$

17. $y = 6 - 2x$

18. $y = 2 - 4x$

Absolute Value Equations and Inequalities

Exercises 19–40: Solve the absolute value equation.

19. $|-2x| = 4$ **20.** $|3x| = -6$

21. $|5x - 7| = 2$ **22.** $|-3x - 2| = 5$

23. $|3 - 4x| = 5$ **24.** $|2 - 3x| = 1$

25. $|-6x - 2| = 0$ **26.** $|6x - 9| = 0$

27. $|7 - 16x| = 0$ **28.** $|-x - 4| = 0$

29. $|17x - 6| = -3$ **30.** $|-8x - 11| = -7$

31. $|1.2x - 1.7| - 1 = 3$ **32.** $|3 - 3x| - 2 = 2$

33. $|4x - 5| + 3 = 2$ **34.** $|4.5 - 2x| + 1.1 = 9.7$

35. $|2x - 9| = |8 - 3x|$ **36.** $|x - 3| = |8 - x|$

37. $\left|\frac{3}{4}x - \frac{1}{4}\right| = \left|\frac{3}{4} - \frac{1}{4}x\right|$ **38.** $\left|\frac{1}{2}x + \frac{3}{2}\right| = \left|\frac{3}{2}x - \frac{7}{2}\right|$

39. $|15x - 5| = |35 - 5x|$

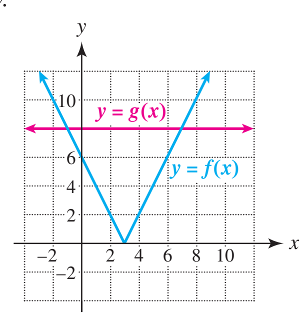
40. $|20x - 40| = |80x - 20|$

Exercises 41 and 42. The graphs of f and g are shown. Solve each equation and inequality.

41. **(a)** $f(x) = g(x)$

(b) $f(x) < g(x)$

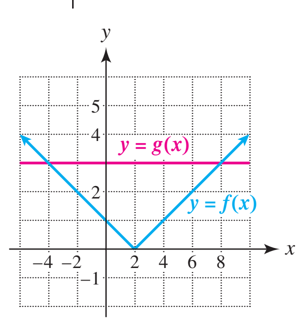
(c) $f(x) > g(x)$



42. **(a)** $f(x) = g(x)$

(b) $f(x) \le g(x)$

(c) $f(x) \ge g(x)$



Exercises 43 and 44: Solve each equation or inequality.

43. **(a)** $|2x - 3| = 1$ **44.** **(a)** $|5 - x| = 2$

(b) $|2x - 3| < 1$ **(b)** $|5 - x| \le 2$

(c) $|2x - 3| > 1$ **(c)** $|5 - x| \ge 2$

Exercises 45–48: Solve the equation

(a) graphically,
(b) numerically, and
(c) symbolically.

Then solve the related inequality.

45. $|2x - 5| = 10$, $|2x - 5| < 10$

46. $|3x - 4| = 8$, $|3x - 4| \le 8$

47. $|5 - 3x| = 2$, $|5 - 3x| > 2$

48. $|4x - 7| = 5$, $|4x - 7| \ge 5$

Exercises 49–54: Solve the equation symbolically. Then solve the related inequality.

49. $|2.1x - 0.7| = 2.4$, $|2.1x - 0.7| \ge 2.4$

50. $\left|\frac{1}{2}x - \frac{3}{4}\right| = \frac{7}{4}$, $\left|\frac{1}{2}x - \frac{3}{4}\right| \le \frac{7}{4}$

51. $|3x| + 5 = 6$, $|3x| + 5 > 6$

52. $|x| - 10 = 25$, $|x| - 10 < 25$

53. $\left|\frac{2}{3}x - \frac{1}{2}\right| = -\frac{1}{4}$, $\left|\frac{2}{3}x - \frac{1}{2}\right| \le -\frac{1}{4}$

54. $|5x - 0.3| = -4$, $|5x - 0.3| > -4$

You Decide the Method

Exercises 55–66: Solve the inequality. Write the solution in interval notation.

55. $|3x - 1| < 8$ **56.** $|15 - x| < 7$

57. $|7 - 4x| \le 11$ **58.** $|-3x + 1| \le 5$

59. $|0.5x - 0.75| < 2$ **60.** $|2.1x - 5| \le 8$

61. $|2x - 3| > 1$ **62.** $|5x - 7| > 2$

63. $|-3x + 8| \ge 3$ **64.** $|-7x - 3| \ge 5$

65. $|0.25x - 1| > 3$ **66.** $|-0.5x + 5| \ge 4$

Domain and Range

67. If $f(k) = -6$, what is the value of $|f(k)|$?

68. If $f(k) = 17$, what is the value of $|f(k)|$?

69. If the domain of $f(x)$ is given by $[-2, 4]$, what is the domain of $|f(x)|$?

70. If the domain of $f(x)$ is given by $(-\infty, 0]$, what is the domain of $|f(x)|$?

71. If the range of $f(x)$ is given by $(-\infty, 0]$, what is the range of $|f(x)|$?

72. If the range of $f(x)$ is given by $(-4, 5)$, what is the range of $|f(x)|$?

Applications

73. ***Speed Limits*** The lawful speeds S on an interstate highway satisfy $|S - 57.5| \leq 17.5$. Find the maximum and minimum speed limits by solving the absolute value equation $|S - 57.5| = 17.5$.

74. ***Human Cannonball*** A human cannonball plans to travel 180 feet and land squarely on a net with a 70-foot-long safe zone.

(a) What distances D can this performer travel and still land safely on the net?

(b) Use an absolute value inequality to describe the restrictions on D.

75. ***Temperature and Altitude*** Air temperature decreases as altitude increases. If the ground temperature is 80°F, then the air temperature x miles high is $T = 80 - 19x$.

(a) Determine the altitudes x where the air temperature T is between 0°F and 32°F, inclusive.

(b) Use an absolute value inequality to describe these altitudes.

76. ***Dew Point and Altitude*** The dew point decreases as altitude increases. If the ground temperature is 80°F, then the dew point x miles high is $D = 80 - \frac{29}{5}x$.

(a) Determine the altitudes x where the dew point D is between 50°F and 60°F, inclusive.

(b) Use an absolute value inequality to describe these altitudes.

Exercises 77–82: ***Average Temperatures*** *(Refer to Example 7.) The inequality describes the range of monthly average temperatures T in degrees Fahrenheit at a certain location.*

(a) Solve the inequality.

(b) If the high and low monthly average temperatures satisfy equality, interpret the inequality.

77. $|T - 43| \leq 24$, Marquette, Michigan

78. $|T - 62| \leq 19$, Memphis, Tennessee

79. $|T - 50| \leq 22$, Boston, Massachusetts

80. $|T - 10| \leq 36$, Chesterfield, Canada

81. $|T - 61.5| \leq 12.5$, Buenos Aires, Argentina

82. $|T - 43.5| \leq 8.5$, Punta Arenas, Chile

83. ***Error in Measurements*** Products are often manufactured to be within a specified tolerance of a given size. For instance, if an aluminum can is supposed to have a diameter of 3 inches, either 2.99 inches or 3.01 inches might be acceptable. If the maximum error in the diameter d of a can is limited to 0.004 inch, then d must satisfy the absolute value inequality

$$|d - 3| \leq 0.004.$$

Solve this inequality and interpret the results.

84. ***Error in Measurements*** (Refer to Exercise 83.) Suppose that a 12-inch ruler must have the correct length L to within 0.0002 inch.

(a) Write an absolute value inequality for L that describes this requirement.

(b) Solve this inequality and interpret the results.

85. ***Relative Error*** If a quantity is measured to be Q and its exact value is A, then the relative error in Q is

$$\left|\frac{Q - A}{A}\right|.$$

If the exact value is $A = 35$ and you want the relative error in Q to be less than or equal to 0.02 (or 2%), what values for Q are possible?

86. ***Relative Error*** (Refer to Exercise 85.) The exact perimeter P of a square is 50 feet. What measured lengths are possible for the side S of the square to have relative error in the perimeter that is less than or equal to 0.04 (or 4%)?

Writing about Mathematics

87. Explain how to solve $|ax + b| = k$ with $k > 0$ symbolically. Give an example.

88. Explain how you can use the solutions to $|ax + b| = k$ with $k > 0$ to solve the inequalities $|ax + b| < k$ and $|ax + b| > k$. Give an example.

EXTENDED AND DISCOVERY EXERCISES

1. Let δ be a positive number and let x and c be real numbers. Write an absolute value inequality that expresses that the distance between x and c on the number line is less than δ.

2. Let ϵ be a positive number, L be a real number, and f be a function. Write an absolute value inequality that expresses that the distance between $f(x)$ and L on the number line is less than ϵ.

CHECKING BASIC CONCEPTS FOR SECTION 2.5

1. Rewrite $\sqrt{4x^2}$ by using an absolute value.

2. Graph $y = |3x - 2|$ by hand.

3. **(a)** Solve the equation $|2x - 1| = 5$.

(b) Use part (a) to solve the absolute value inequalities $|2x - 1| \leq 5$ and $|2x - 1| > 5$.

4. Solve each equation or inequality. For each inequality, write the solution set in interval notation.

(a) $|2 - 5x| - 4 = -1$

(b) $|3x - 5| \leq 4$

(c) $|\frac{1}{2}x - 3| > 5$

5. Solve $|x + 1| = |2x|$.

2 Summary

CONCEPT	EXPLANATION AND EXAMPLES
SECTION 2.1 LINEAR FUNCTIONS AND MODELS	
Linear Function	A linear function can be written as $f(x) = ax + b$. Its graph is a line. **Example:** $f(x) = -2x + 5$; slope $= -2$, y-intercept $= 5$
Linear Model	If a quantity increases or decreases by a constant amount for each unit increase in x, then it can be modeled by a linear function given by $f(x) = (\text{constant rate of change})x + (\text{initial amount})$. **Example:** If water is pumped from a full 100-gallon tank at 7 gallons per minute, then $A(t) = 100 - 7t$ gives the gallons of water in the tank after t minutes.
Piecewise-Defined Function	A function defined by more than one formula on its domain **Examples:** Step function, greatest integer function, absolute value function, and $f(x) = \begin{cases} 4 - x & \text{if } -4 \leq x < 1 \\ 3x & \text{if } 1 \leq x \leq 5 \end{cases}$ It follows that $f(2) = 6$ because if $1 \leq x \leq 5$ then $f(x) = 3x$. Note that f is continuous on its domain of $[-4, 5]$.
Linear Regression	One way to determine a linear function or a line that models data is to use the method of least squares. This method determines a unique line that can be found with a calculator. The correlation coefficient r $(-1 \leq r \leq 1)$ measures how well a line fits the data. **Example:** The line of least squares modeling the data (1, 1), (3, 4), and (4, 6) is given by $y \approx 1.643x - 0.714$, with $r \approx 0.997$.

CONCEPT	EXPLANATION AND EXAMPLES
SECTION 2.2 EQUATIONS OF LINES	
Point-Slope Form	If a line with slope m passes through (x_1, y_1), then $$y = m(x - x_1) + y_1 \quad \text{or} \quad y - y_1 = m(x - x_1).$$ **Example:** $y = -\frac{3}{4}(x + 4) + 5$ has slope $-\frac{3}{4}$ and passes through $(-4, 5)$.
Slope-Intercept Form	If a line has slope m and y-intercept b, then $y = mx + b$. **Example:** $y = 3x - 4$ has slope 3 and y-intercept -4.
Determining Intercepts	To find the x-intercept(s), let $y = 0$ in the equation and solve for x. To find the y-intercept(s), let $x = 0$ in the equation and solve for y. **Examples:** The x-intercept on the graph of $3x - 4y = 12$ is 4 because $3x - 4(0) = 12$ implies that $x = 4$. The y-intercept on the graph of $3x - 4y = 12$ is -3 because $3(0) - 4y = 12$ implies that $y = -3$.
Horizontal and Vertical Lines	A horizontal line passing through the point (a, b) is given by $y = b$, and a vertical line passing through (a, b) is given by $x = a$. **Examples:** The horizontal line $y = -3$ passes through $(6, -3)$. The vertical line $x = 4$ passes through $(4, -2)$.
Parallel and Perpendicular Lines	Parallel lines have equal slopes satisfying $m_1 = m_2$, and perpendicular lines have slopes satisfying $m_1m_2 = -1$, provided neither line is vertical. **Examples:** The lines $y_1 = 3x - 1$ and $y_2 = 3x + 4$ are parallel. The lines $y_1 = 3x - 1$ and $y_2 = -\frac{1}{3}x + 4$ are perpendicular.
Direct Variation	A quantity y is directly proportional to a quantity x, or y varies directly with x, if $y = kx$, where $k \neq 0$. If data vary directly, the ratios $\frac{y}{x}$ are equal to the constant of variation k. **Example:** If a person works for \$8 per hour, then that person's pay P is directly proportional to, or varies directly with, the number of hours H that the person works by the equation $P = 8H$, where the constant of variation is $k = 8$.
SECTION 2.3 LINEAR EQUATIONS	
Linear Equation	Can be written as $ax + b = 0$ with $a \neq 0$ and has one solution **Example:** The solution to $2x - 4 = 0$ is 2 because $2(2) - 4 = 0$.

CONCEPT	EXPLANATION AND EXAMPLES

SECTION 2.3 LINEAR EQUATIONS (CONTINUED)

Properties of Equality

Addition: $a = b$ is equivalent to $a + c = b + c$.

Multiplication: $a = b$ is equivalent to $ac = bc$, provided $c \neq 0$.

Example:

$\frac{1}{2}x - 4 = 3$ Given equation

$\frac{1}{2}x = 7$ Addition property; add 4.

$x = 14$ Multiplication property; multiply by 2.

Contradiction, Identity, and Conditional Equation

A contradiction has no solutions, an identity is true for all (meaningful) values of the variable, and a conditional equation is true for some, but not all, values of the variable.

Examples:

$3(1 - 2x) = 3 - 6x$ Identity

$x + 5 = x$ Contradiction

$x - 1 = 4$ Conditional equation

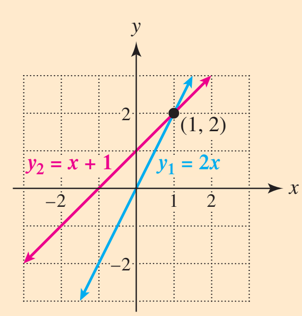
Intersection-of-Graphs and *x*-Intercept Methods

Intersection-of-graphs method: Set y_1 equal to the left side of the equation and set y_2 equal to the right side. The x-coordinate of a point of intersection is a solution.

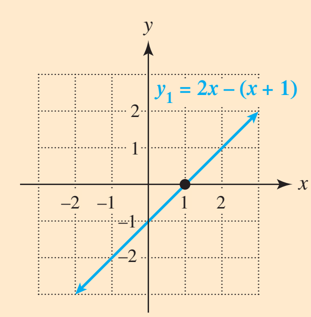
Example: The graphs of $y_1 = 2x$ and $y_2 = x + 1$ intersect at (1, 2), so the solution to the linear equation $2x = x + 1$ is 1. See the figure below on the left.

x-intercept method: Move all terms to the left side of the equation. Set y_1 equal to the left side of the equation. The solutions are the x-intercepts.

Example: Write $2x = x + 1$ as $2x - (x + 1) = 0$. Graph $y_1 = 2x - (x + 1)$. The only x-intercept is 1, as shown in the figure on the right.



Point of Intersection (1, 2)



***x*-intercept: 1**

Problem-Solving Strategies

STEP 1: Read the problem and make sure you understand it. Assign a variable to what you are being asked to find. If necessary, write other quantities in terms of this variable.

STEP 2: Write an equation that relates the quantities described in the problem. You may need to sketch a diagram and refer to known formulas.

STEP 3: Solve the equation and determine the solution.

STEP 4: Look back and check your solution. Does it seem reasonable?

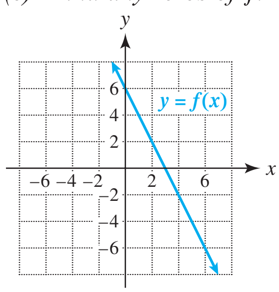
CONCEPT	EXPLANATION AND EXAMPLES
SECTION 2.4 LINEAR INEQUALITIES	
Linear Inequality	Can be written as $ax + b > 0$ with $a \neq 0$, where $>$ can be replaced by $<$, $\leq$, or $\geq$. If the solution to $ax + b = 0$ is k, then the solution to the linear inequality $ax + b > 0$ is either the interval $(-\infty, k)$ or the interval (k, ∞). **Example:** $3x - 1 < 2$ is linear since it can be written as $3x - 3 < 0$. The solution set is $\{x \mid x < 1\}$, or $(-\infty, 1)$.
Properties of Inequality	*Addition:* $a < b$ is equivalent to $a + c < b + c$. *Multiplication:* $a < b$ is equivalent to $ac < bc$ when $c > 0$. $a < b$ is equivalent to $ac > bc$ when $c < 0$. **Example:** $-3x - 4 < 14$ Given equation $-3x < 18$ Addition property; add 4. $x > -6$ Multiplication property; divide by -3. Reverse the inequality symbol.
Compound Inequality	**Example:** $x \geq -2$ and $x \leq 4$, or equivalently, $-2 \leq x \leq 4$. This is called a three-part inequality.
SECTION 2.5 ABSOLUTE VALUE EQUATIONS AND INEQUALITIES	
Absolute Value Function	An absolute value function is defined by $f(x) = \lvert x \rvert$. Its graph is V-shaped. An equivalent formula is $f(x) = \sqrt{x^2}$. **Examples:** $f(-9) = \lvert -9 \rvert = 9$; $\sqrt{(2x + 1)^2} = \lvert 2x + 1 \rvert$
Absolute Value Equations	$\lvert ax + b \rvert = k$ with $k > 0$ is equivalent to $ax + b = \pm k$. **Example:** $\lvert 2x - 3 \rvert = 4$ is equivalent to $2x - 3 = 4$ or $2x - 3 = -4$. The solutions are $\frac{7}{2}$ and $-\frac{1}{2}$.
Absolute Value Inequalities	Let the solutions to $\lvert ax + b \rvert = k$ be s_1 and s_2, where $s_1 < s_2$ and $k > 0$. **1.** $\lvert ax + b \rvert < k$ is equivalent to $s_1 < x < s_2$. **2.** $\lvert ax + b \rvert > k$ is equivalent to $x < s_1$ or $x > s_2$. Similar statements can be made for inequalities involving $\leq$ or $\geq$. **Example:** The solutions to $\lvert 2x + 1 \rvert = 5$ are given by $x = -3$ and $x = 2$. The solutions to $\lvert 2x + 1 \rvert < 5$ are given by $-3 < x < 2$. The solutions to $\lvert 2x + 1 \rvert > 5$ are given by $x < -3$ or $x > 2$.

2 Review Exercises

Exercises 1 and 2: The graph of a linear function f is shown.

(a) Identify the slope, y-intercept, and x-intercept.
(b) Write a formula for f.
(c) Find any zeros of f.

1.



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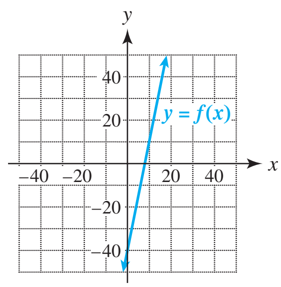


Exercises 3 and 4: Find $f(x) = ax + b$ so that f models the data exactly.

3.

x	1	2	3	4
$f(x)$	2.5	0	−2.5	−5

4.

x	−3	6	15	24
$f(x)$	−1.65	−1.2	−0.75	−0.3

Exercises 5 and 6: Graph the linear function.

5. $f(x) = -\frac{2}{3}x$ **6.** $g(x) = 4 - 2x$

7. Write a formula for a linear function f whose graph has slope -2 and passes through $(-2, 3)$.

8. Find the average rate of change of $f(x) = -3x + 8$ from -2 to 3.

Exercises 9–14: Find the slope-intercept form of the equation of a line satisfying the conditions.

9. Slope 7, passing through $(-3, 9)$

10. Passing through $(2, -4)$ and $(7, -3)$

11. Passing through $(1, -1)$, parallel to $y = -3x + 1$

12. Passing through the point $(-2, 1)$, perpendicular to the line $y = 2(x + 5) - 22$

13. Parallel to the line segment connecting $(0, 3.1)$ and $(5.7, 0)$, passing through $(1, -7)$

14. Perpendicular to $y = -\frac{5}{7}x$, passing through $\left(\frac{6}{7}, 0\right)$

Exercises 15–20: Find an equation of the specified line.

15. Parallel to the y-axis, passing through $(6, -7)$

16. Parallel to the x-axis, passing through $(-3, 4)$

17. Horizontal, passing through $(1, 3)$

18. Vertical, passing through $(1.5, 1.9)$

19. Vertical with x-intercept 2.7

20. Horizontal with y-intercept -8

Exercises 21 and 22: Determine the x- and y-intercepts for the graph of the equation. Graph the equation.

21. $5x - 4y = 20$ **22.** $\dfrac{x}{3} - \dfrac{y}{2} = 1$

Exercises 23–28: Solve the linear equation either symbolically or graphically.

23. $5x - 22 = 10$ **24.** $5(4 - 2x) = 16$

25. $-2(3x - 7) + x = 2x - 1$

26. $5x - \frac{1}{2}(4 - 3x) = \frac{3}{2} - (2x + 3)$

27. $\pi x + 1 = 6$

28. $\dfrac{x - 4}{2} = x + \dfrac{1 - 2x}{3}$

Exercises 29 and 30: Use a table to solve each linear equation numerically to the nearest tenth.

29. $3.1x - 0.2 - 2(x - 1.7) = 0$

30. $\sqrt{7} - 3x - 2.1(1 + x) = 0$

Exercises 31–34: Complete the following.

(a) Solve the equation symbolically.
(b) Classify the equation as a contradiction, an identity, or a conditional equation.

31. $4(6 - x) = -4x + 24$

32. $\frac{1}{2}(4x - 3) + 2 = 3x - (1 + x)$

33. $5 - 2(4 - 3x) + x = 4(x - 3)$

34. $\dfrac{x - 3}{4} + \dfrac{3}{4}x - 5(2 - 7x) = 36x - \dfrac{43}{4}$

Exercises 35–38: Express the inequality in interval notation.

35. $x > -3$

36. $x \le 4$

37. $-2 \le x < \frac{3}{4}$

38. $x \le -2$ or $x > 3$

Exercises 39–44: Solve the linear inequality. Write the solution set in set-builder or interval notation.

39. $3x - 4 \le 2 + x$

40. $-2x + 6 \le -3x$

41. $\dfrac{2x - 5}{2} < \dfrac{5x + 1}{5}$

42. $-5(1 - x) > 3(x - 3) + \frac{1}{2}x$

43. $-2 \le 5 - 2x < 7$

44. $-1 < \dfrac{3x - 5}{-3} < 3$

Exercises 45 and 46: Solve the inequality graphically.

45. $2x > x - 1$

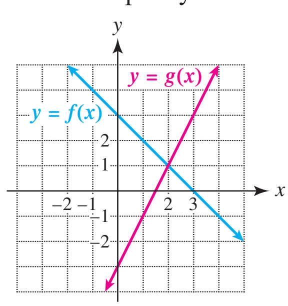
46. $-1 \le 1 + x \le 2$

47. The graphs of two linear functions f and g are shown in the figure. Solve each equation or inequality.

(a) $f(x) = g(x)$

(b) $f(x) < g(x)$

(c) $f(x) > g(x)$



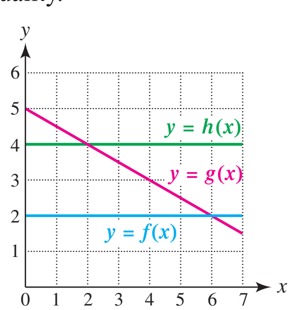
48. The graphs of three linear functions f, g, and h with domains $D = \{x | 0 \le x \le 7\}$ are shown in the figure. Solve each equation or inequality.

(a) $f(x) = g(x)$

(b) $g(x) = h(x)$

(c) $f(x) < g(x) < h(x)$

(d) $g(x) > h(x)$



49. Use $f(x)$ to complete the following.

$$f(x) = \begin{cases} 8 + 2x & \text{if } -3 \le x \le -1 \\ 5 - x & \text{if } -1 < x \le 2 \\ x + 1 & \text{if } \quad 2 < x \le 5 \end{cases}$$

(a) Evaluate f at $x = -2, -1, 2,$ and 3.

(b) Sketch a graph of f. Is f continuous on its domain?

(c) Determine the x-value(s) where $f(x) = 3$.

50. If $f(x) = [\![2x - 1]\!]$, evaluate $f(-3.1)$ and $f(2.5)$.

Exercises 51–54: Solve the equation.

51. $|2x - 5| - 1 = 8$

52. $|3 - 7x| = 10$

53. $|6 - 4x| = -2$

54. $|9 + x| = |3 - 2x|$

Exercises 55–58: Solve the equation. Use the solutions to help solve the related inequality.

55. $|x| = 3, \quad |x| > 3$

56. $|-3x + 1| = 2, \quad |-3x + 1| < 2$

57. $|3x - 7| = 10, \quad |3x - 7| > 10$

58. $|4 - x| = 6, \quad |4 - x| \le 6$

Exercises 59–62: Solve the inequality.

59. $|3 - 2x| < 9$

60. $|-2x - 3| > 3$

61. $|\frac{1}{3}x - \frac{1}{6}| \ge 1$

62. $|\frac{1}{2}x| - 3 \le 5$

Applications

63. ***Median Income*** The median U.S. family income between 1980 and 2005 can be modeled by the formula $f(x) = 1450(x - 1980) + 20{,}000$, where x is the year. (**Source:** Department of Commerce.)

(a) Solve the equation $f(x) = 34{,}500$ graphically and interpret the solution.

(b) Solve part (a) symbolically.

64. ***Course Grades*** In order to receive a B grade in a college course, it is necessary to have an overall average of 80% correct on two 1-hour exams of 75 points each and one final exam of 150 points. If a person scores 55 and 72 on the 1-hour exams, what is the minimum score that the person can receive on the final exam and still earn a B?

65. ***Medicare Costs*** Estimates for Medicare costs in billions of dollars in year x can be modeled by the formula $f(x) = 18x - 35{,}750$, where $1995 \le x \le 2007$. Determine when Medicare costs were from \$268 to \$358 billion. (**Source:** Office of Management and Budget.)

66. ***Temperature Scales*** The table shows equivalent temperatures in degrees Celsius and degrees Fahrenheit.

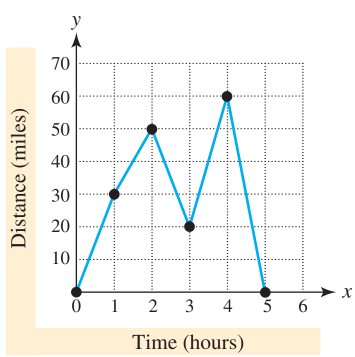
°F	−40	32	59	95	212
°C	−40	0	15	35	100

(a) Plot the data with Fahrenheit temperature on the x-axis and Celsius temperature on the y-axis. What type of relation exists between the data?

(b) Find a function C that receives the Fahrenheit temperature x as input and outputs the corresponding Celsius temperature. Interpret the slope.

(c) If the temperature is 83°F, what is it in degrees Celsius?

67. ***Distance from Home*** The graph depicts the distance y that a person driving a car on a straight road is from home after x hours. Interpret the graph. What speeds did the car travel?



68. ***ACT Scores*** The table lists the average composite ACT scores for selected years. Note that in 2007 the average score was 21.2. (**Source:** The American College Testing Program.)

Year	1989	1990	1991	1992	1993	1994
Score	20.6	20.6	20.6	20.6	20.7	20.8

(a) Make a line graph of the data.

(b) Let this line graph represent a piecewise-linear function f. Find a formula for f.

(c) What is the domain of f?

69. ***Population Estimates*** In 2004 the population of a city was 143,247, and in 2008 it was 167,933. Estimate the population in 2006.

70. ***Distance*** A driver of a car is initially 455 miles from home, traveling toward home on a straight freeway at 70 miles per hour.

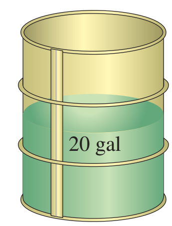
(a) Write a formula for a linear function f that models the distance between the driver and home after x hours.

(b) Graph f. What is an appropriate domain?

(c) Identify the x- and y-intercepts. Interpret each.

71. ***Working Together*** Suppose that one worker can shovel snow from a storefront sidewalk in 50 minutes and another worker can shovel it in 30 minutes. How long will it take if they work together?

72. ***Antifreeze*** Initially, a tank contains 20 gallons of a 30% antifreeze solution. How many gallons of an 80% antifreeze solution should be added to the tank in order to increase the concentration of the antifreeze in the tank to 50%?



73. ***Running*** An athlete traveled 13.5 miles in 1 hour and 48 minutes, jogging at 7 miles per hour and then at 8 miles per hour. How long did the runner jog at each speed?

74. ***Least-Squares Fit*** The table lists the actual annual cost y to drive a midsize car 15,000 miles per year for selected years x.

x	1960	1970	1980	1990	2000
y	\$1394	\$1763	\$3176	\$5136	\$6880

Source: Runzheimer International.

(a) Predict whether the correlation coefficient is positive, negative, or zero.

(b) Find a least-squares regression line that models these data. What is the correlation coefficient?

(c) Estimate the cost of driving a midsize car in 1995.

(d) Estimate the year when the cost could reach \$8000.

75. The table lists data that are exactly linear.

x	−3	−2	−1	1	2
y	6.6	5.4	4.2	1.8	0.6

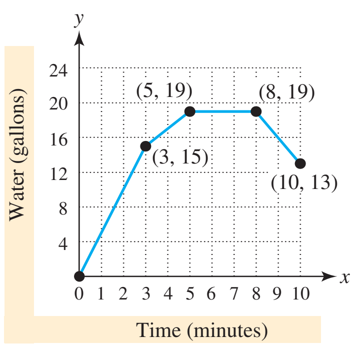
(a) Determine the slope-intercept form of the line that passes through these data points.

(b) Predict y when $x = -1.5$ and 3.5. State whether these calculations involve interpolation or extrapolation.

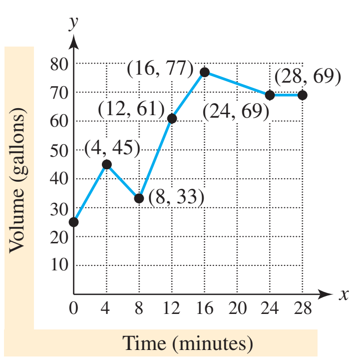
(c) Predict x when $y = 1.3$.

76. ***Geometry*** A rectangle is twice as long as it is wide and has a perimeter of 78 inches. Find the width and length of this rectangle.

77. ***Flow Rates*** A water tank has an inlet pipe with a flow rate of 5 gallons per minute and an outlet pipe with a flow rate of 3 gallons per minute. A pipe can be either closed or completely open. The graph shows the number of gallons of water in the tank after x minutes have elapsed. Use the concept of slope to interpret each piece of this graph.



78. ***Flow Rates*** (Refer to Exercise 77.) Suppose the tank is modified so that it has a second inlet pipe, which flows at a rate of 2 gallons per minute. Interpret the graph by determining when each inlet and outlet pipe is open or closed.



79. ***Air Temperature*** For altitudes up to 4 kilometers, moist air will cool at a rate of about 6°C per kilometer. If the ground temperature is 25°C, at what altitudes would the air temperature be from 5°C to 15°C? (**Source:** A Miller and R. Anthes, *Meteorology.*)

80. ***Water Pollution*** At one time the Thames River in England supported an abundant community of fish. Pollution then destroyed all the fish in a 40-mile stretch near its mouth for a 45-year period beginning in 1915. Since then, improvement of sewage treatment facilities and other ecological steps have resulted in a dramatic increase in the number of different fish present. The number of species present from 1967 to 1978 can be modeled by $f(x) = 6.15x - 12{,}059$, where x is the year.

(a) Estimate the year when the number of species first exceeded 70.

(b) Estimate the years when the number of species was between 50 and 100.

81. ***Relative Error*** The actual length of a side of a building is 52.3 feet. How accurately must an apprentice carpenter measure this side to have the relative error in the measurement be less than 0.003 (0.3%)? (*Hint:* Use $\left|\frac{C - A}{A}\right|$, where C is the carpenter's measurement and A is the actual length.)

82. ***Brown Trout*** Due to acid rain, the percentage of lakes in Scandinavia that lost their population of brown trout increased dramatically between 1940 and 1975. Based on a sample of 2850 lakes, this percentage can be approximated by the following piecewise-linear function. (**Source:** C. Mason, *Biology of Freshwater Pollution.*)

$$f(x) = \begin{cases} \frac{11}{20}(x - 1940) + 7 & \text{if } 1940 \le x < 1960 \\ \frac{32}{15}(x - 1960) + 18 & \text{if } 1960 \le x \le 1975 \end{cases}$$

(a) Determine the percentage of lakes that lost brown trout by 1947 and by 1972.

(b) Sketch a graph of f.

(c) Is f a continuous function on its domain?

EXTENDED AND DISCOVERY EXERCISES

1. ***Archeology*** It is possible for archeologists to estimate the height of an adult based only on the length of the humerus, a bone located between the elbow and the shoulder. The approximate relationship between the height y of an individual and the length x of the humerus is shown in the table for both males and females. All measurements are in inches. Although individual values may vary, tables like this are the result of measuring bones from many skeletons.

x	8	9	10	11
y (females)	50.4	53.5	56.6	59.7
y (males)	53.0	56.0	59.0	62.0

x	12	13	14
y (females)	62.8	65.9	69.0
y (males)	65.0	68.0	71.0

(a) Find the estimated height of a female with a 12-inch humerus.

(b) Plot the ordered pairs (x, y) for both sexes. What type of relation exists between the data?

(c) For each 1-inch increase in the length of the humerus, what are the corresponding increases in the heights of females and of males?

(d) Determine linear functions f and g that model these data for females and males, respectively.

(e) Suppose a humerus from a person of unknown sex is estimated to be between 9.7 and 10.1 inches long. Use f and g to approximate the range for the height of a female and a male.

2. Continuing with Exercise 1, have members of the class measure their heights and the lengths of their humeri (plural of *humerus*) in inches.

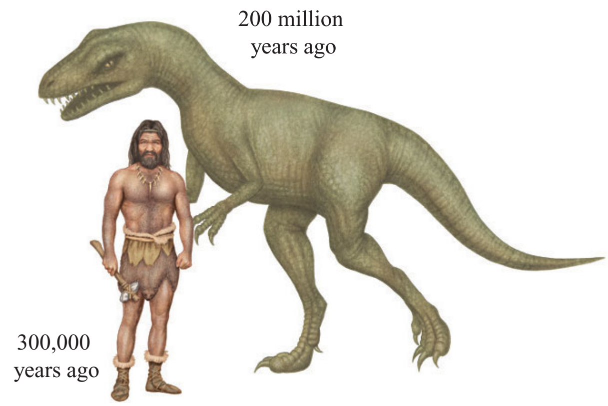
(a) Make a table of the results.

(b) Find regression lines that fit the data points for males and females.

(c) Compare your results with the table in Exercise 1.

3. ***A Puzzle*** Three people leave for a city 15 miles away. The first person walks 4 miles per hour, and the other two people ride in a car that travels 28 miles per hour. After some time, the second person gets out of the car and walks 4 miles per hour to the city while the driver goes back and picks up the first person. The driver takes the first person to the city. If all three people arrive in the city at the same time, how far did each person walk?

4. ***Comparing Ages*** The age of Earth is approximately 4.45 billion years. The earliest evidence of dinosaurs dates back 200 million years, whereas the earliest evidence of *Homo sapiens* dates back 300,000 years. If the age of Earth were condensed into one year, determine the approximate times when dinosaurs and *Homo sapiens* first appeared.



5. ***Limit Notation*** Let ϵ and δ be positive numbers; let x, c, and L be real numbers; and let f be a function. Consider the following: "If the distance between x and c is less than δ, then the distance between $f(x)$ and L is less than ϵ." Rewrite this sentence by using two absolute value inequalities.

1–2 than then the distance between $f(x)$ and L is less than " Rewrite this sentence by using two absolute value inequa

1. Write 123,000 and 0.0051 in scientific notation.

2. Write 6.7×10^6 and 1.45×10^{-4} in standard form.

3. Evaluate $\frac{4 + \sqrt{2}}{4 - \sqrt{2}}$. Round your answer to the nearest hundredth.

4. The table represents a relation S.

x	-1	0	1	2	3
y	6	4	3	0	0

(a) Does S represent a function?

(b) Determine the domain and range of S.

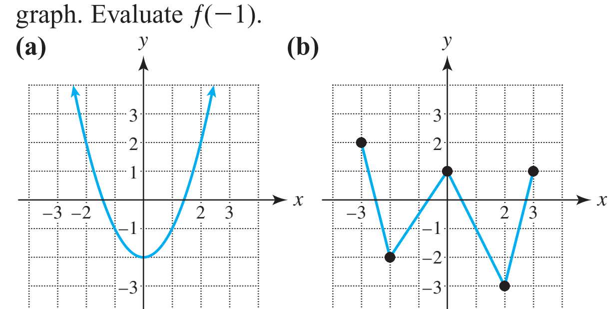
5. Find the standard equation of a circle with center $(-2, 3)$ and radius 7.

6. Evaluate $-5^2 - 2 - \frac{10 - 2}{5 - 1}$ by hand.

7. Find the exact distance between $(-3, 5)$ and $(2, -3)$.

8. Find the midpoint of the line segment with endpoints $(5, -2)$ and $(-3, 1)$.

9. Find the domain and range of the function shown in the graph. Evaluate $f(-1)$.



10. Graph f by hand.

(a) $f(x) = 3 - 2x$ **(b)** $f(x) = |x + 1|$

(c) $f(x) = x^2 - 3$ **(d)** $f(x) = \sqrt{x + 2}$

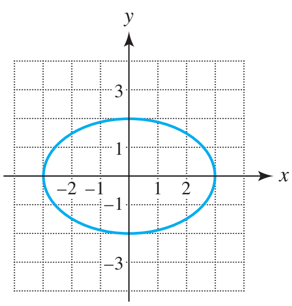
Exercises 11 and 12: Complete the following.

(a) Evaluate $f(2)$ and $f(a - 1)$.
(b) Determine the domain of f.

11. $f(x) = 5x - 3$

12. $f(x) = \sqrt{2x - 1}$

13. Determine if the graph represents a function. Explain your answer.



14. Write a formula for a function f that computes the cost of taking x credits if credits cost \$80 each and fees are fixed at \$89.

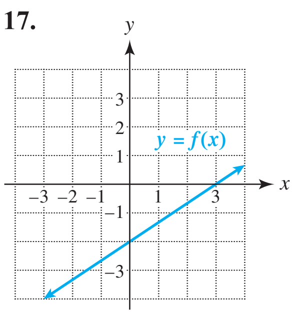
15. Find the average rate of change of $f(x) = x^2 - 2x + 1$ from $x = 1$ to $x = 2$.

16. Find the difference quotient for $f(x) = 2x^2 - x$.

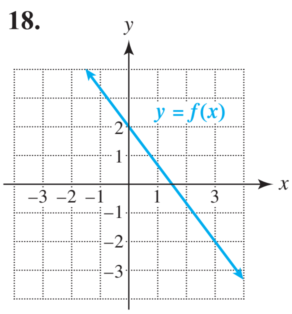
Exercises 17 and 18: The graph of a linear function f is shown.

(a) Identify the slope, y-intercept, and x-intercept.
(b) Write a formula for f.
(c) Find any zeros of f.

17.



18.



19. Write a formula for a linear function whose graph has slope -3 and passes through $\left(\frac{2}{3}, -\frac{2}{3}\right)$.

20. If $G(t) = 200 - 10t$ models the gallons of water in a tank after t minutes, interpret the numbers 200 and -10 in the formula for G.

Exercises 21–26: Write an equation of a line satisfying the given conditions. Use slope-intercept form whenever possible.

21. Passing through $(1, -5)$ and $(-3, \frac{1}{2})$

22. Passing through the point $(-3, 2)$ and perpendicular to the line $y = \frac{2}{3}x - 7$

23. Parallel to the y-axis and passing through $(-1, 3)$

24. Slope 30, passing through (2002, 50)

25. Passing through $(-3, 5)$ and parallel to the line segment connecting (2.4, 5.6) and (3.9, 8.6)

26. Perpendicular to the y-axis and passing through the origin

Exercises 27 and 28: Determine the x- and y-intercepts on the graph of the equation. Graph the equation.

27. $-2x + 3y = 6$ **28.** $x = 2y - 3$

Exercises 29–32: Solve the equation.

29. $4x - 5 = 1 - 2x$ **30.** $\dfrac{2x - 4}{2} = \dfrac{3x}{7} - 1$

31. $\frac{2}{3}(x - 2) - \frac{4}{5}x = \frac{4}{15} + x$

32. $-0.3(1 - x) - 0.1(2x - 3) = 0.4$

33. Solve $x + 1 = 2x - 2$ graphically and numerically.

34. Solve $2x - (5 - x) = \frac{1 - 4x}{2} + 5(x - 2)$. Is this equation either an identity or a contradiction?

Exercises 35–38: Express each inequality in interval notation.

35. $x < 5$ **36.** $-2 \leq x \leq 5$

37. $x < -2$ or $x > 2$ **38.** $x \geq -3$

Exercises 39 and 40: Solve the inequality. Write the solution set in set-builder or interval notation.

39. $-3(1 - 2x) + x \leq 4 - (x + 2)$

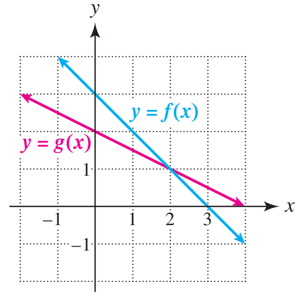
40. $\dfrac{1}{3} \leq \dfrac{2 - 3x}{2} < \dfrac{4}{3}$

41. The graphs of two linear functions f and g are shown. Solve each equation or inequality.

(a) $f(x) = g(x)$

(b) $f(x) > g(x)$

(c) $f(x) \le g(x)$



42. Graph f. Is f continuous on the interval $[-4, 4]$?

$$f(x) = \begin{cases} 2 - x & \text{if } -4 \le x < -2 \\ \frac{1}{2}x + 5 & \text{if } -2 \le x < 2 \\ 2x + 1 & \text{if } \quad 2 \le x \le 4 \end{cases}$$

Exercises 43–46: Solve the equation.

43. $|d + 1| = 5$

44. $|3 - 2x| = 7$

45. $|2t| - 4 = 10$

46. $|11 - 2x| = |3x + 1|$

Exercises 47 and 48: Solve the inequality.

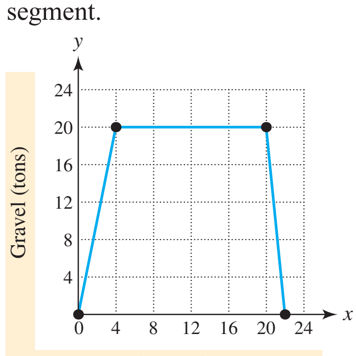
47. $|2t - 5| \le 5$

48. $|5 - 5t| > 7$

Applications

49. ***Volume of a Cylinder*** The volume V of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the cylinder and h is its height. If an aluminum can has a volume of 24 cubic inches and a radius of 1.5 inches, find its height to the nearest hundredth of an inch.

50. ***Interpreting Slope*** The figure shows the weight of a load of gravel in a dump truck. Interpret the slope of each line segment.



51. ***Cost*** A company's cost C in dollars for making x computers is $C(x) = 500x + 20{,}000$.

(a) Evaluate $C(1500)$. Interpret the result.

(b) Find the slope of the graph of C. Interpret the slope.

52. ***Distance*** At midnight car A is traveling north at 60 miles per hour and is located 40 miles south of car B. Car B is traveling west at 70 miles per hour. Approximate the distance between the cars at 1:15 A.M. to the nearest tenth of a mile.

53. ***Average Rate of Change*** On a warm summer day the Fahrenheit temperature x hours past noon is given by the formula $T(x) = 70 + \frac{3}{2}x^2$.

(a) Find the average rate of change of T from 2:00 P.M. to 4:00 P.M.

(b) Interpret this average rate of change.

54. ***Distance from Home*** A driver is initially 270 miles from home, traveling toward home on a straight interstate at 72 miles per hour.

(a) Write a formula for a function D that models the distance between the driver and home after x hours.

(b) What is an appropriate domain for D? Graph D.

(c) Identify the x- and y-intercepts. Interpret each.

55. ***Working Together*** Suppose one person can mow a large lawn in 5 hours with a riding mower and it takes another person 12 hours to mow the lawn with a push mower. How long will it take to mow the lawn if the two people work together?

56. ***Running*** An athlete traveled 15 miles in 1 hour and 45 minutes, running at 8 miles per hour and then 10 miles per hour. How long did the athlete run at each speed?

57. ***Chicken Consumption*** In 2001 Americans ate, on average, 56 pounds of chicken annually. This amount is expected to increase to 61 pounds in 2012. (**Source:** Department of Agriculture.)

(a) Determine a formula

$$f(x) = m(x - x_1) + y_1$$

that models these data. Let x be the year.

(b) Estimate the annual chicken consumption in 2007.

58. ***Relative Error*** If the actual value of a quantity is A and its measured value is M, then the relative error in measurement M is $\left|\frac{M - A}{A}\right|$. If $A = 65$, determine the range of values for M to have a relative error that is less than or equal to 0.03 (3%).

59. The table lists per capita income.

Year	1970	1980	1990	2000
Income	\$4095	\$10,183	\$19,572	\$29,760

Source: Bureau of Economic Analysis.

(a) Find the least-squares regression line for the data.

(b) Estimate the per capita income in 1995. Did this calculation involve interpolation or extrapolation?

60. ***Modeling Data*** According to government guidelines, the recommended minimum weight for a person 58 inches tall is 91 pounds, and for a person 64 inches tall it is 111 pounds.

(a) Find an equation of the line that passes through the points (58, 91) and (64, 111).

(b) Use this line to estimate the minimum weight for someone 61 inches tall. Then use the midpoint formula to find this minimum weight. Are the results the same? Why?