

=Second Partial Project=
Calculus I

**APPLICATIONS OF MOTION:
POSITION, VELOCITY AND
ACCELERATION**

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Calculus is the wide range area of mathematics dealing with topics at instantaneous rates of change, areas under curves, and sequences and series. We will analyze the situation when a function represents the position of an object, in two dimension motions, vertically, horizontally or a combination. When we have a position function, the first two derivatives have specific meanings. The first derivative is the velocity and the second derivative is the acceleration of the object. We take the derivative with respect to the independent variable, t.

About Position

The total displacement is defined as the final position minus the initial position. When we have a position function, the first two derivatives have specific meanings. The first derivative is the velocity and the second derivative is the acceleration of the object. We take the derivative with respect to the independent variable, t.

About Velocity

There is a difference between velocity and speed. Velocity can be negative and includes direction information. Speed is the magnitude of velocity and is always positive. Speed does not include direction. If you are familiar with vectors, velocity is a vector, speed is a scalar whose value is the magnitude of a velocity vector.

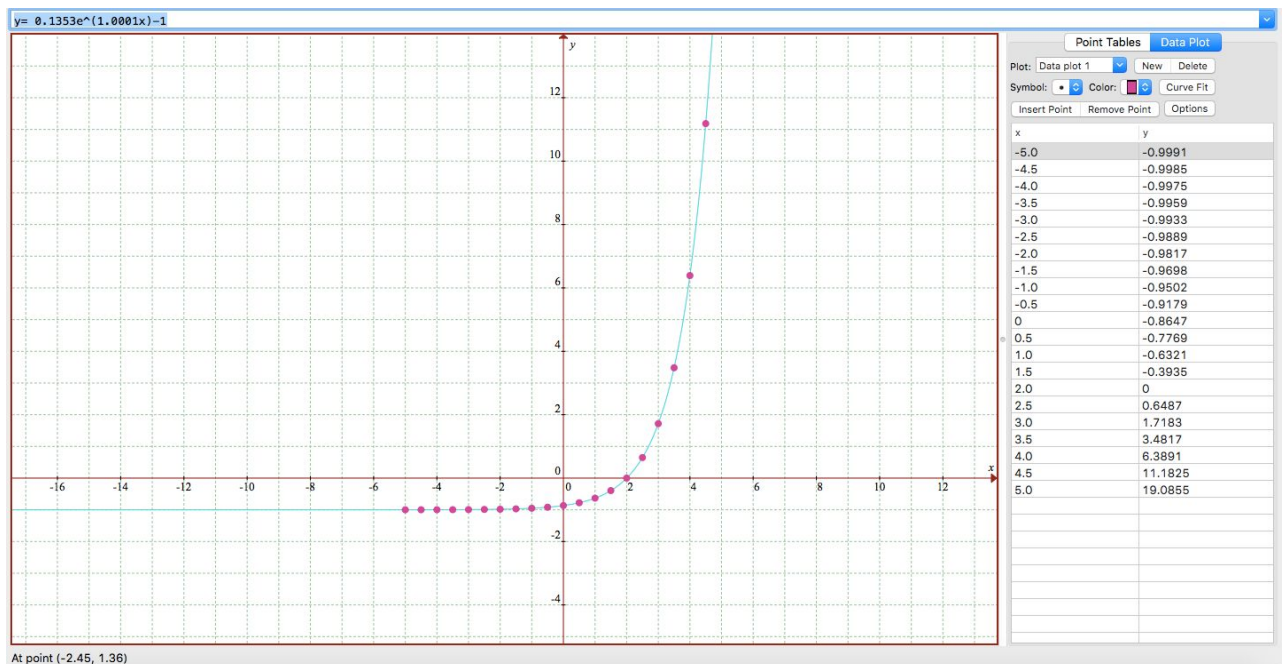
About Acceleration

We assumed acceleration is constant. This is true in many problems, especially ones where we talk about the acceleration due to gravity near the surface of the earth, for example. However, this is not true in ALL problems. So you need to pay attention to what is going on in the problem statement if you are required to set up these equations.

TABLE E

$$\rightarrow f(t) = 0.1353e^{(1.0001x)}$$

Graph



Procedure

$f(t)$

$$y = ae^{bx} - 1$$
$$\begin{aligned} -0.8647 &= ae^{0b} - 1 & 0 &= 0.1353e^{2b} - 1 \\ -0.8647 &= a - 1 & 1 &= 0.1353e^{2b} \\ a &= 0.1353 & (1/0.1353) &= e^{2b} \\ & & \ln(1/0.1353) &= 2b \\ & & [\ln(1/0.1353)]/2 &= b \\ & & b &= 1.0001 \end{aligned}$$
$$\underline{y = 0.1353e^{1.0001x} - 1}$$

Analysis

It is an **exponential function**, it has an asymptote at (-1) , since it is positive its values go to positive infinity.

Equation of Position

$$f(t) = 0.1353e^{(1.0001x)}$$

Equation of Velocity

$$v(t) = 0.1353e^{(1.0001x)}$$

$$\begin{aligned} f(t) &= 0.1353e^{1.0001x} \\ v(t) &= (1.0001)(0.1353)e^{1.0001x} \\ v(t) &= 0.1353e^{1.0001x} \end{aligned}$$

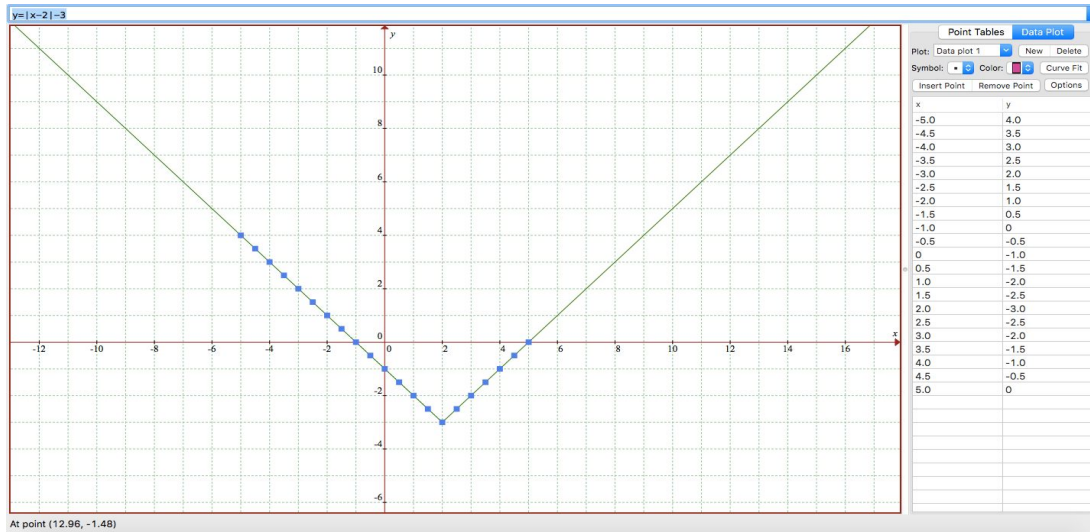
Equation of Acceleration

$$a(t) = 0.1353e^{(1.0001x)}$$

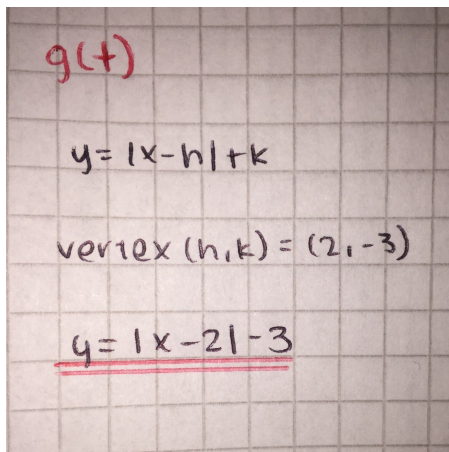
$$\begin{aligned} v(t) &= 0.1353e^{1.0001x} \\ a(t) &= (1.0001)(0.1353)e^{1.0001x} \\ a(t) &= 0.1353e^{1.0001x} \end{aligned}$$

$$\rightarrow g(t) = |x-2|-3$$

Graph



Procedure



Analysis

It is an **absolute value function**, its vertex is at (2,-3). Since it is an absolute value function, its minimum value is the vertex and all of the other values are greater than that.

Equation of Position

$$g(t) = |x-2|-3$$

Equation of Velocity

$$v(t) = (-x+2)/(|-x+2|)$$

Handwritten derivation of the velocity equation:

$$g(t) = |-x+2| - 3$$

$$\frac{d}{dx} [|-x+2|] + \frac{d}{dx} [-3]$$

$$[|-x+2|] = f'g(x)g'(x)$$

$$\frac{-x+2}{|-x+2|} + \frac{d}{dx} [-3]$$

$$\frac{-x+2}{|-x+2|} + 0 \quad g'(t) = \frac{-x+2}{|-x+2|}$$

Additional notes: $f = |x|$, $g(x) = -x+2$, $f' = \frac{x}{|x|}$

Equation of Acceleration

$$a(t) = [-(2-x)/(|x-2|)(x-2)] - [1/(|x-2|)]$$

Handwritten derivation of the acceleration equation:

$$\frac{d}{dx} \frac{[2-x] \cdot [x-2] - (2-x)[x-2]}{(x-2)^2}$$

$$\frac{-1[x-2] - (1)(2-x)(x-2)}{|x-2|}$$

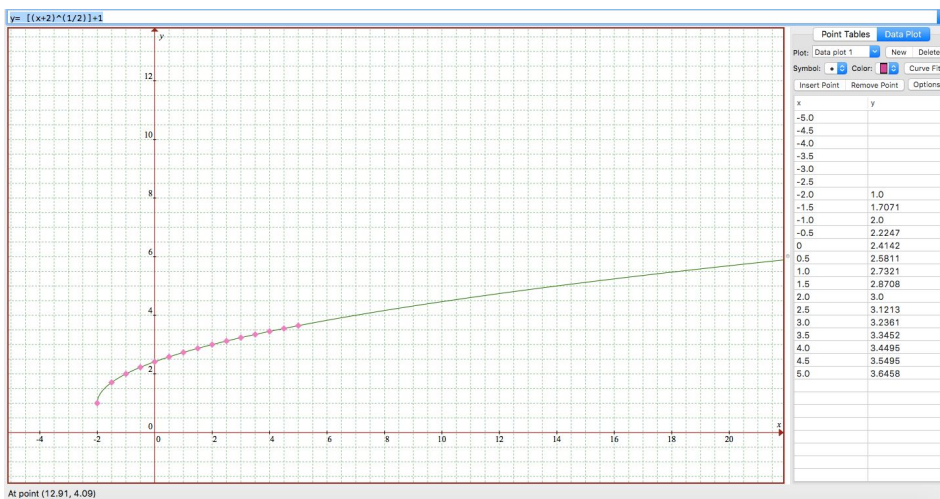
$$\frac{-(1)(2-x)(x-2)}{|x-2|} - |x-2|$$

$$\frac{-(2-x)(x-2)}{(x-2)} - |x-2|$$

$$g'(t) = -\frac{2-x}{|x-2|(x-2)} = \frac{1}{|x-2|}$$

$$\rightarrow h(t) = [(x+2)^{(1/2)}] + 1$$

Graph



Procedure

$h(t)$
 $y = \sqrt{x}$
minimum value = 1
left shift = -2
 $y = \sqrt{(x+2)} + 1$

Analysis

It is a **square root function**. Its minimum value is at 1 and it has a shift of two units to the left which is represented in the square root as $(x+2)$.

Equation of Position

$$[(x+2)^{(1/2)}]+1$$

Equation of Velocity

$$v(t) = [1/(2(x+2)^{1/2})]$$

② $h(t) = [(x+2)^{1/2}] + 1$
 $\frac{d}{dx} (\sqrt{x+2}) \circ \frac{d}{dx} (1)$
 $\frac{1}{2} (x+2)^{-1/2} \circ (x+2) + 0$
 $h'(t) = \frac{1}{2\sqrt{x+2}}$

Equation of Acceleration

$$a(t) = [1/(4(x+2)^{3/2})]$$

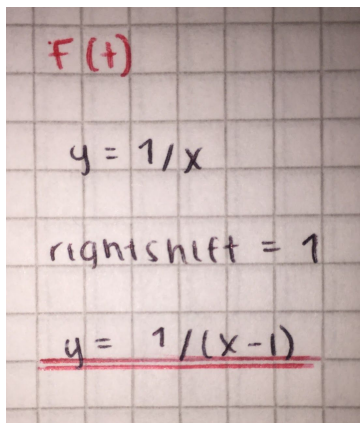
2.2
 $\frac{1}{2} \circ \frac{1}{\sqrt{x+2}}$
 $(-\frac{1}{2})(x+2)^{-1/2-1} \circ \frac{dx}{dx} [x+2]$
 $\frac{\frac{d}{dx} x + \frac{d}{dx} 2}{4(x+2)^{3/2}} = \frac{1}{4(x+2)^{3/2}}$

$$\rightarrow F(t) = 1/(x-1)$$

Graph



Procedure



Analysis

It is a **rational/inverse function**. It has a shift of one unit to the right that is represented in the denominator as $(x-1)$.

Equation of Position

$$1/(x-1)$$

Equation of Velocity

$$v(t) = -1/(x-1)^2$$

$$\textcircled{3} F(t) = \frac{1}{(x-1)}$$

$$\frac{\frac{d}{dx} [x-1]}{(x-1)^2} = -\frac{1}{(x-1)^2}$$

Equation of Acceleration

$$a(t) = 2/(x-1)^3$$

3.2

$$-\frac{d}{dx} \left[\frac{1}{(x-1)^2} \right]$$

$$= -(-2)(x-1)^3 \cdot \frac{d}{dx} (x-1)$$

$$= \frac{2(1)}{(x-1)^3}$$

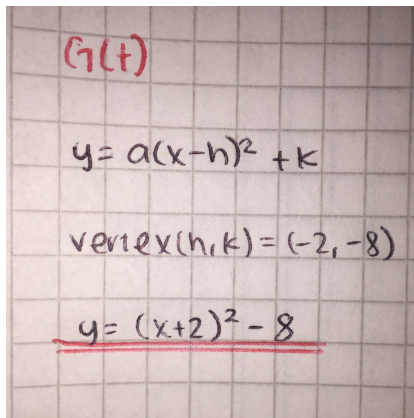
$$= \frac{2}{(x-1)^3}$$

$$\rightarrow G(t) = [(x+2)^2]-8$$

Graph



Procedure



$g(t)$

$$y = a(x-h)^2 + k$$
$$\text{vertex } (h, k) = (-2, -8)$$
$$\underline{y = (x+2)^2 - 8}$$

Analysis

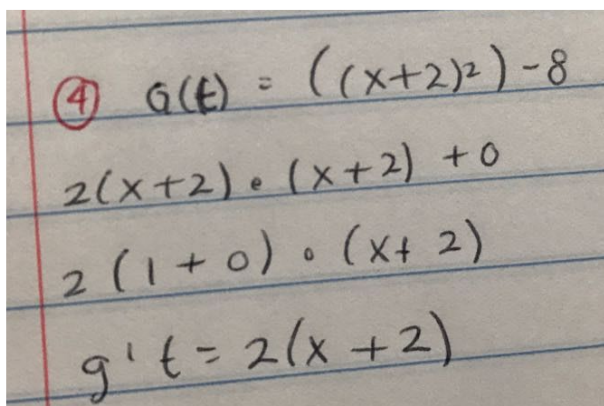
It is a **quadratic function**. Its vertex is at $(-2, -8)$.

Equation of Position

$$[(x+2)^2] - 8$$

Equation of Velocity

$$v(t) = 2(x+2)$$



④ $g(t) = ((x+2)^2) - 8$

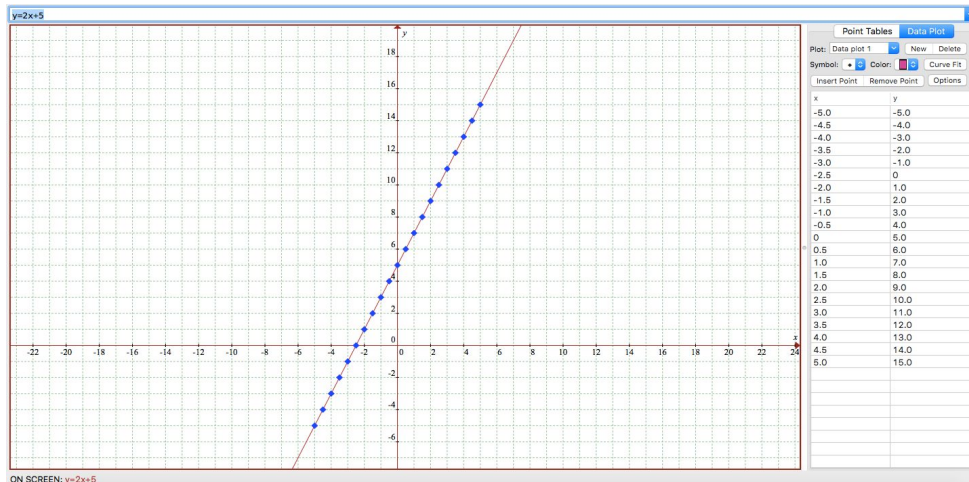
$$2(x+2) \cdot (x+2) + 0$$
$$2(1+0) \cdot (x+2)$$
$$g'(t) = 2(x+2)$$

Equation of Acceleration

$$a(t) = 2$$

$$\rightarrow H(t) = 2x+5$$

Graph



Procedure

$H(t)$

$$y = mx + b$$
$$m = \frac{11-9}{3-2} = \frac{2}{1} = 2$$
$$b = y - mx$$
$$b = 7 - 2(1)$$
$$b = 5$$
$$\underline{y = 2x + 5}$$

Analysis

It is a **linear function**. It has a slope of 2 and a y-intercept at (5).

Equation of Position

$$2x+5$$

Equation of Velocity

$$v(t) = 2$$

⑤ $H(t) = 2x + 5$

$$H'(t) = 2$$

Equation of Acceleration

$$a(t) = 0$$

Team Conclusions

Alejandra: With this project, we could apply previous knowledge on functions combined with what we are currently learning which are derivatives and with all of these, solve situations. We could use graphs, equations and formulas in order to solve all of what was asked for in the document. We could apply derivatives to find, from a position equation, the velocity and acceleration equation. which is really helpful to us not only in this course but also in others.

Carolina: With this project we discovered and learned about the different applications of derivatives and calculus, as well as how to apply derivatives in different scenario. Velocity and acceleration are derivatives of the position, which shows us how calculus can be applied in our daily life, calculus helps us simplify the process of knowing certain quantities based on others we have.

Nina: At the end we can see how this project helped us understand more about the importance of position, velocity, and acceleration in calculus. And with some research made we looked how it is also very related with physics. The motion is represented in the graph of position and the derivatives in the velocity and acceleration.

Daniela: By doing this project, we realize the importance of the three applications of motion: position, velocity and acceleration. The first tells you where something or someone is, the second tells you how fast it is going and the third one tells you whether it is going faster or slower. The understanding of this three can help you with any kind of metrics and you can use it whenever you need to know more than just what your position is.

Mariana: We conclude that the graphs told us about the importance of calculus, by finding the derivatives we can see how they relate to the graph of position, which is the graph for the first formula. The formula for velocity is the derivative of the position formula, and the formula for acceleration is the derivative of the velocity. We learned the velocity can be negative and that the speed only zero or positive. The graphs represent the position, and the acceleration most of the times is constant.

References:

- Linear Motion (Position, Velocity and Acceleration). (2015, July 27). Retrieved October 10, 2017, from <https://17calculus.com/integrals/linear-motion/>
- How to find position - Calculus 1. (n.d.). Retrieved October 10, 2017, from https://www.varsitytutors.com/calculus_1-help/how-to-find-position
- Taking derivatives. (n.d.). Retrieved October 11, 2017, from <https://www.khanacademy.org/math/calculus-home/taking-derivatives-calc>