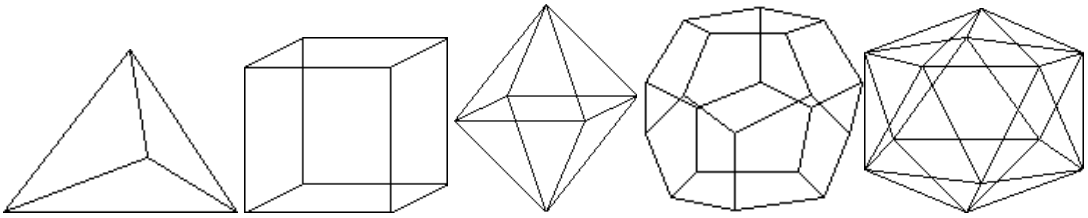
Grade / Age: 10 - 14
Topic: mathematics STEAM
Subject area: geometry, spatial geometry
Keywords: regular (Platonic) solids
Single/ teamwork: both
Language: (English or Local) English
Duration: 3 x 60 min

Description of the Task:

1. Definition of Platonic solids

The following solids are also called regular or Platonic solids. There are 5 such bodies. In order tetrahedron, hexahedron (cube), octahedron, dodecahedron, icosahedron. They are named after the Greek names for the number of their faces.



All their edges have the same edge angle and face angle. In other words, all their faces are congruent regular polygons.

2. Platonic solids in GeoGebra

You can draw Platonic solids in GeoGebra with 2 instructions. In the view menu, switch on the 3D Graphics view. Draw a regular triangle, then in the command line of the algebra window start typing tetrahedron, type the name of your regular triangle in brackets after the tetrahedron, this is usually polygon1, and you have your tetrahedron. We can also look at the net of the tetrahedron using the net command by clicking on the tetrahedron. In a similar way we get the other regular solids, only we start editing with a quadrilateral instead of a regular triangle for the cube and a pentagon for the dodecahedron.

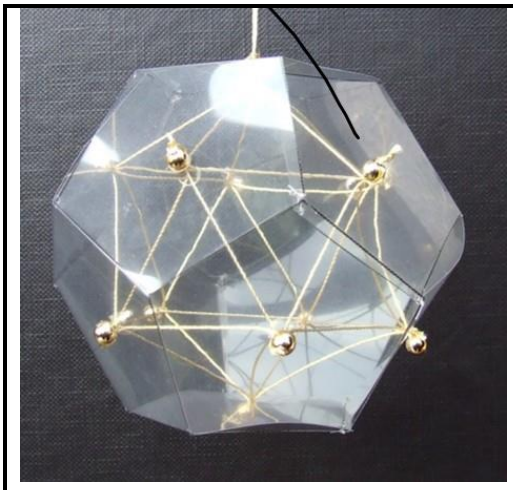
3. Making paper models of Platonic solids

Based on the nets, we can also make paper models of the bodies. Be careful not to forget to put "tabs" in the right place for gluing.

4. Duality in GeoGebra and on paper models

There is a very interesting relationship between Platonic solids. If you connect the centres of their faces you get another (or the same) Platonic solid. This is called duality. Let's draw dual solids in GeoGebra.

You can also use dual solids to make beautiful Christmas tree ornaments.




5. Browse the Internet, where can you find Platonic solids?

Solutions of the Task:

5 Platonic solids, net

https://www.geogebra.org/classic/n5a2quxn

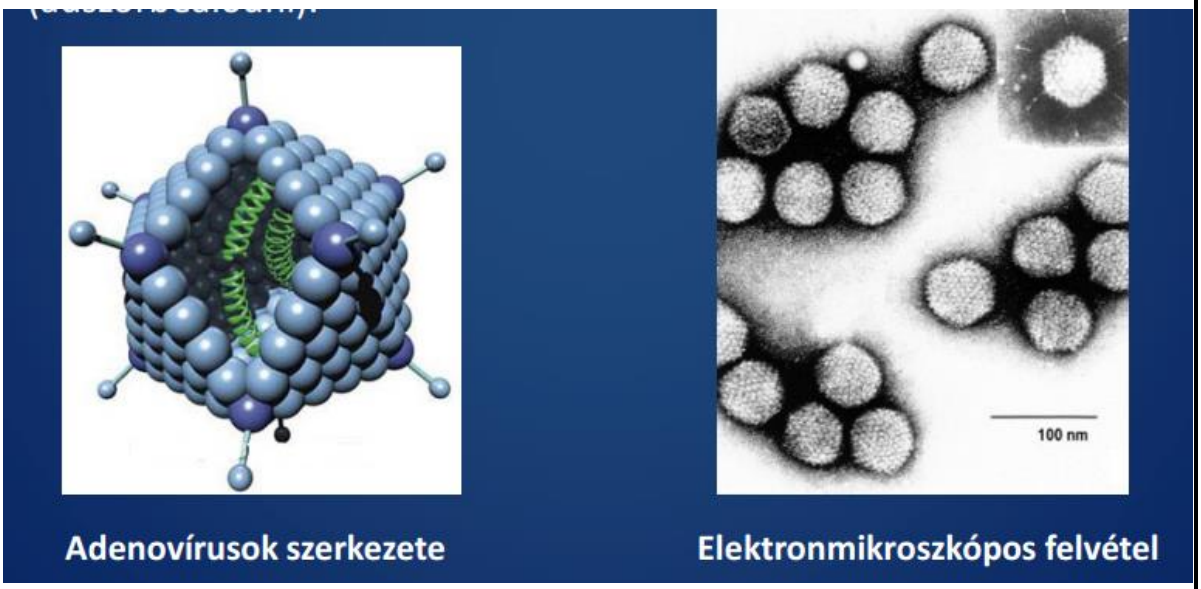
Duality:

https://www.geogebra.org/classic/x6aaqvtm
https://www.geogebra.org/classic/mjg3bpwq
https://www.geogebra.org/classic/qrzhhea7
https://www.geogebra.org/classic/uwpb98tu
https://www.geogebra.org/classic/jbktjwxh

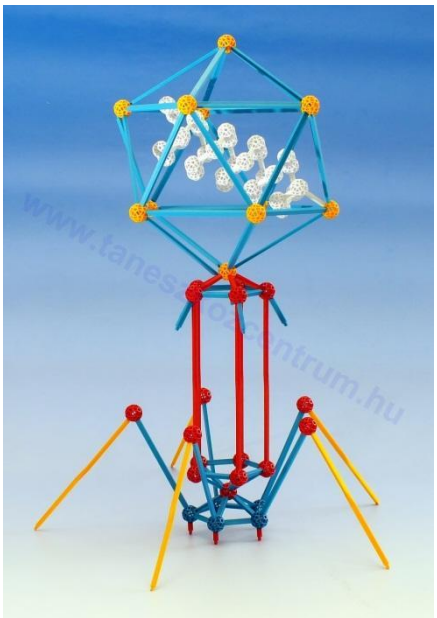
Platonic solids made of gingerbread:

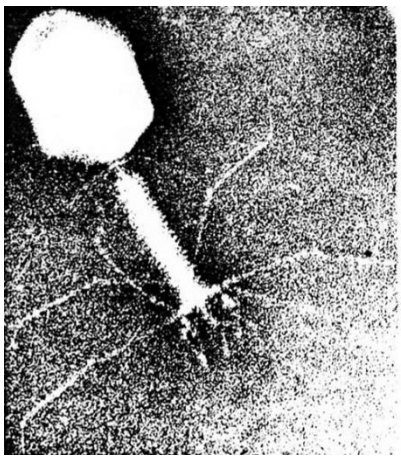
Ideas for point 5:

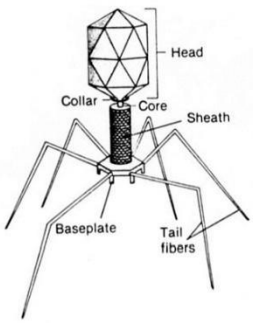
a) Viruses: there are viruses with icosahedron and dodecahedron structures
Adenoviruses are double-stranded DNA viruses of the Adenoviridae family, 60-90 nm in size, icosahedral in shape. They most commonly cause respiratory disease, such as upper respiratory tract infections, bronchitis or pneumonia, but can also cause conjunctivitis and gastroenteritis.

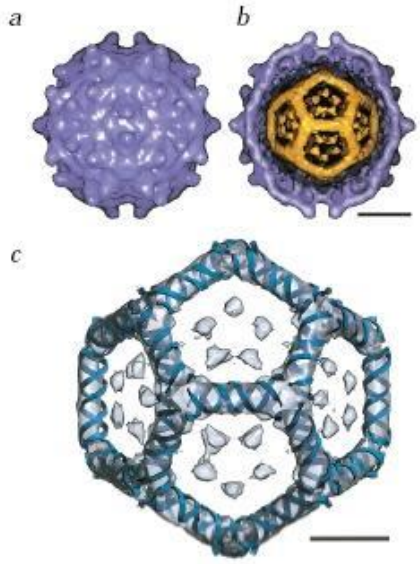


Bacteriophage - viruses that attack bacteria
Liang Tang et al, Pariacoto virus shows dodecahedral structure with duplex RNA, Nature Structural Biology 8 (2001), 77-83.
The first black line is 100 angstroms long (10 -8 meters), while the second is 50 angstroms long. Viruses of arthropods
Virus model from the zometool kit



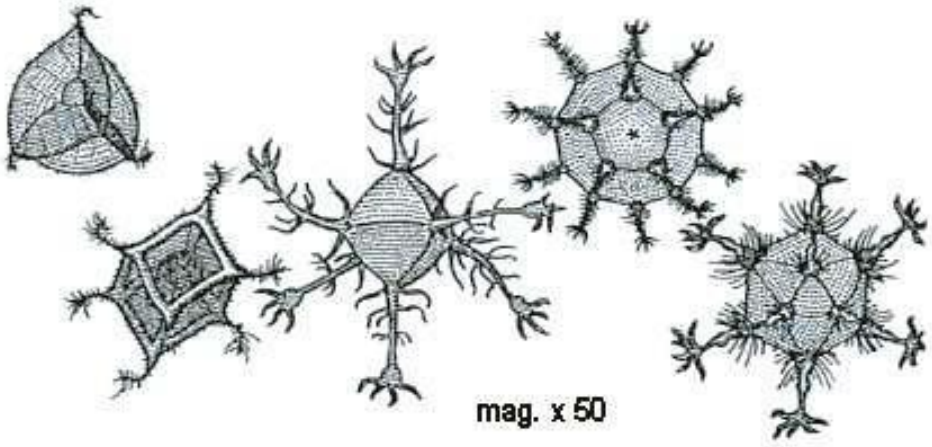






b)

Platonic bodies have been found in marine organisms. The tetrahedron, slightly rounded as a result of internal pressure, takes shape in the protozoan Callimitra agnesae, the cube in Lithocubus geometricus, the octahedron in Circoporus octahedrus, the dodecahedron in Circorrhegma dodecahedrus and the icosahedron in Circognia icosahedrus.



c)

Man first encountered regular bodies in nature. The tetrahedron, the cube and the octahedron are found in the crystals of minerals, while the dodecahedron and the icosahedron are found in the animal kingdom.

One of the minerals of copper, tetrahedrite, crystallizes in tetrahedra, and zinc sulphide crystallizes in truncated tetrahedra.

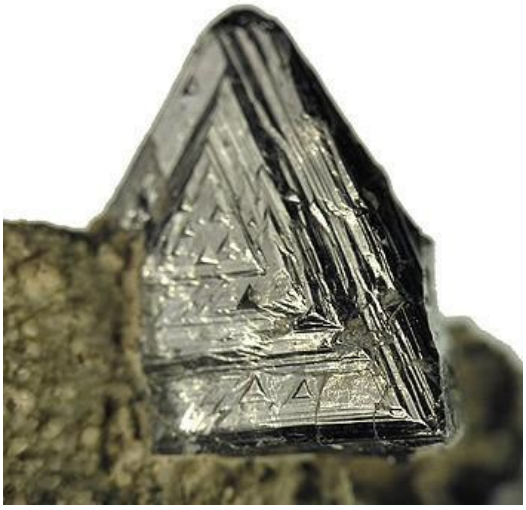



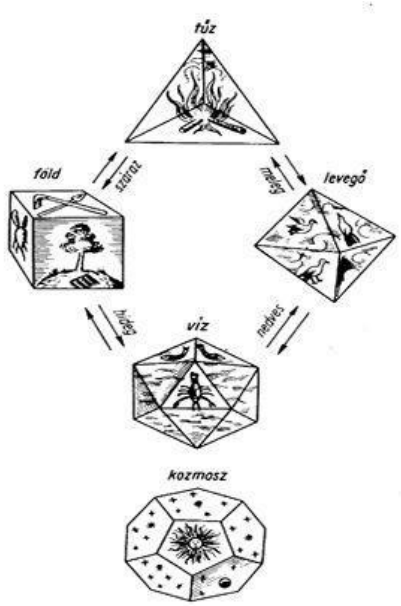
Figure left: tetrahedral tetrahedrite crystal cluster on quartz
Figure right: Growth steps on tetrahedrite crystal
Source:
http://www.tankonyvtar.hu/en/tartalom/tamop425/0033_SCORM_MFFAT6101/sco_21_06.htm
Gold, silver, galenite, for example, crystallize in cubic and octahedral forms.
Crystals of rock salt and galenite can be cubic or truncated cubic.
Kryolite also crystallizes in a truncated cubic form.
Octahedral crystals are common, e.g. in cuprite. twin crystals are common, e.g. in tetrahedrite and pyrite.
Icosahedron crystals can only be produced artificially, e.g. artificial boron crystals.


Figure left: gold
Figure right: rock salt
d)
The history of polyhedra dates back to the earliest times, polyhedra was much used in antiquity too.
The British Museum's Egyptian collection includes two icosahedron-shaped playing cubes from the Ptolemy dynasty. In Italy, an Etruscan children's toy in the shape of a dodecahedron, made of soapstone from before 500 BC, was found in the Monta Loffa excavations near Padua.


Figure left: Roman rock crystal D20 from the 1st or 2nd century (British Museum)
Figure right: Egyptian D20 from the Metropolitan Museum of Art, New York.

e)

Ancient philosophers Plato and Aristotle equated the four primordial elements with four Platonic bodies, the tetrahedron with fire, the hexahedron with earth, the octahedron with air, the icosahedron with water, and the dodecahedron with the entire cosmos because of its perfection. According to Aristotle, the four primordial elements (fire, water, earth, air) are the most primitive forms of substances, and objects are created by some differentiation or assemblage of their mixture.



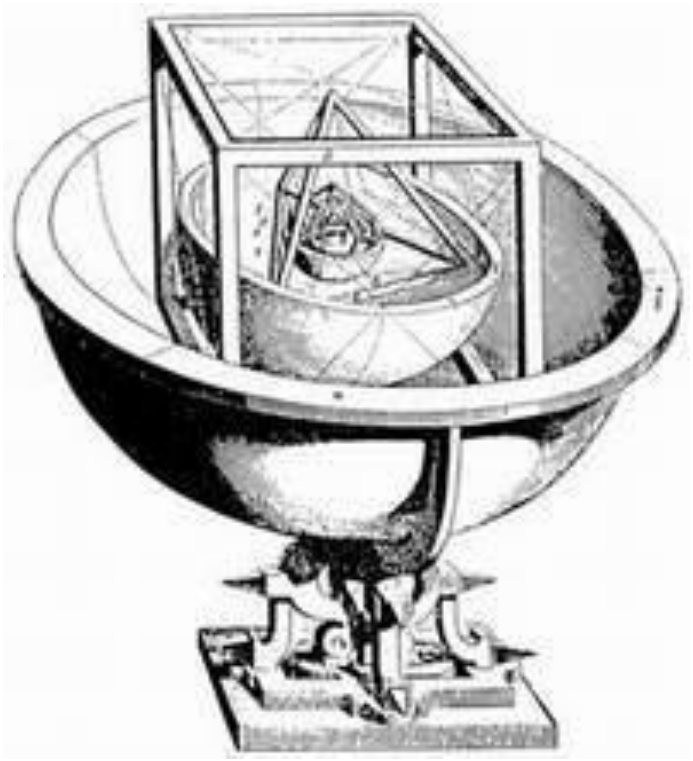
f)

Kepler's model of the universe was based on regular solids written inside each other. The interesting thing is that, although this has no scientific basis, it was accurate enough to be measured at that time.

The model was as follows:

The measure of all things is the sphere of the Earth.

Draw a dodecahedron around it: the sphere around it will be the sphere of Mars.
Now draw a tetrahedron around the sphere of Mars: the sphere around this tetrahedron will be Jupiter's one.
Draw a cube around the sphere of Jupiter: the sphere around this cube belongs to Saturn.
Now draw an icosahedron inside the Earth's sphere: the sphere inside this one belongs to Venus.
Draw an octahedron in the sphere of Venus: the sphere drawn in this octahedron will be the sphere of Mercury.
Here is the explanation of the number of planets.
(Kepler: Mysterium Cosmographicum, 1596)



If somebody has the opportunity, this model can be built from a Zometool kit. Look for it under the name Kepler's Cosmos.

Prior knowledge:
Polygons, regular polygons

Comments:

Connection to other subjects/topics/areas:
mathematics, art, biology, chemistry (minerals), astronomy (Kepler), archaeology, informatics (GeoGebra)