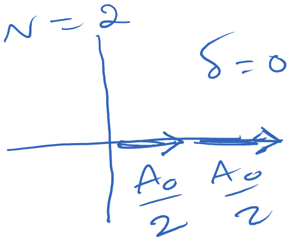
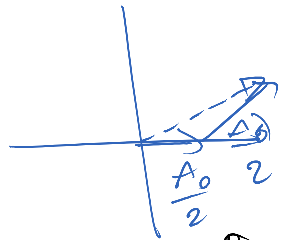
N - Slit Interference

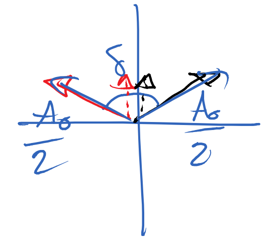
Thursday, October 12, 2017 8:11 AM

$N = 2$





$\Rightarrow$ exploit symmetry



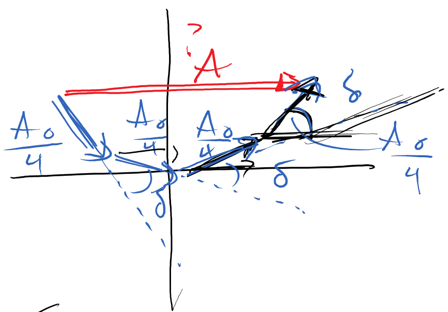
Law of cosines vs.

$$A = \left(\frac{A_0}{\cancel{2}} \cos\left(\frac{\delta}{2}\right)\right)\cancel{2}$$

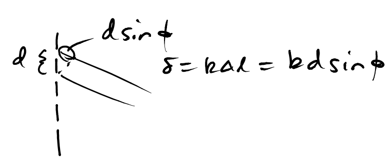
$$A = A_0 \cos\left(\frac{\delta}{2}\right)$$

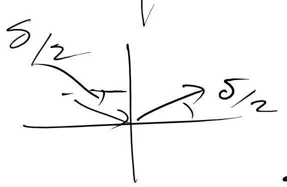
$N = 4$

$\frac{A_0}{4},\ \frac{A_0}{4},\ \frac{A_0}{4},\ \frac{A_0}{4}$



$A =$

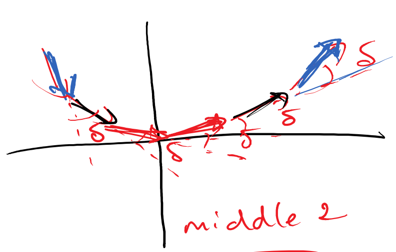




$$2\,\frac{A_0}{4}\cos\left(\frac{\delta}{2}\right) + 2\,\frac{A_0}{4}\cos\left(\frac{\delta}{2} + \delta\right)$$

$$A = 2\,\frac{A_0}{4}\cos\left(\frac{\delta}{2}\right) + 2\,\frac{A_0}{4}\cos\left(\frac{3\delta}{2}\right) \quad ; \; N = 4$$

$N = 6$



$$A = \underbrace{2\,\frac{A_0}{6}\cos\left(\frac{\delta}{2}\right)} + 2\,\frac{A_0}{6}\cos\left(\frac{3\delta}{2}\right) + 2\,\frac{A_0}{6}\cos\left(\frac{5\delta}{2}\right) \; ; \; N = 6$$

$$\boxed{A = \sum_{i=1}^{N/2} 2\,\frac{A_0}{N}\cos\left(\left(i - \frac{1}{2}\right)\delta\right) \quad ; \; \text{even } N}$$

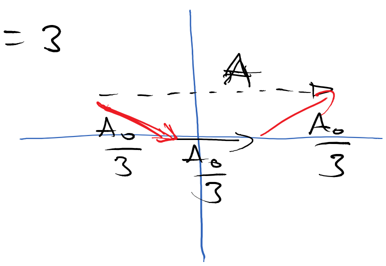
$$A^2 = \left[\sum_{i=1}^{N/2} \frac{2}{N}\cos\left(\left(i - \frac{1}{2}\right)\delta\right)\right]^2 \quad ; \; \text{even } N$$

$$\frac{I}{I_0} = \frac{A^2}{A_0^2} = \left[\sum_{i=1}^{N/2} \frac{2}{N} \cos\left(\left(i-\tfrac{1}{2}\right)\delta\right)\right]^2 \; ; \text{ even } N$$

$$\boxed{I = I_0 \left[\sum_{i=1}^{N/2} \frac{2}{N} \cos\left(\left(i-\tfrac{1}{2}\right)\delta\right)\right]^2 \; ; \text{ even } N}$$

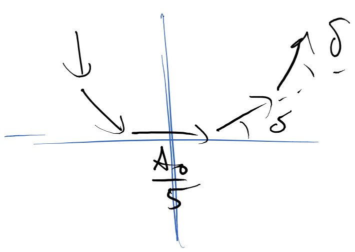
For odd N we need a different solution.

$N = 3$



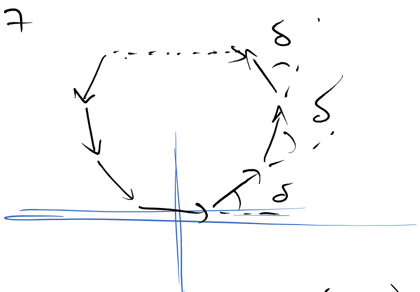
$$A = \frac{A_0}{3} + 2\frac{A_0}{3}\cos(\delta)$$

$N = 5$



$$A = \frac{A_0}{5} + 2\frac{A_0}{5}\cos(\delta) + 2\frac{A_0}{5}\cos(2\delta)$$

$N = 7$



$$A = \frac{A_0}{7} + 2\frac{A_0}{7}\cos(\delta) + 2\frac{A_0}{7}\cos(2\delta) + 2\frac{A_0}{7}\cos(3\delta)$$

N odd $\quad A = \dfrac{A_0}{N} + \displaystyle\sum_{i=1}^{\frac{(N-1)}{2}} \frac{2A_0}{N}\cos(i\delta)$

$$\frac{I}{I_0} = \frac{A^2}{A_0^2} = \left[\frac{A_0}{N} + \sum_{i=1}^{\frac{(N-1)}{2}} \frac{2\cos(i\delta)}{N}\right]^2$$

$$\boxed{I = I_0\left[\frac{A_0}{N} + \sum_{i=1}^{\frac{(N-1)}{2}} \frac{2\cos(i\delta)}{N}\right]^2 \; ; \; N \text{ odd}}$$

$\delta = k\Delta l = kd\sin\phi$ for both even and odd.