Find the parameters required and then sketch the graph of each rational function below:

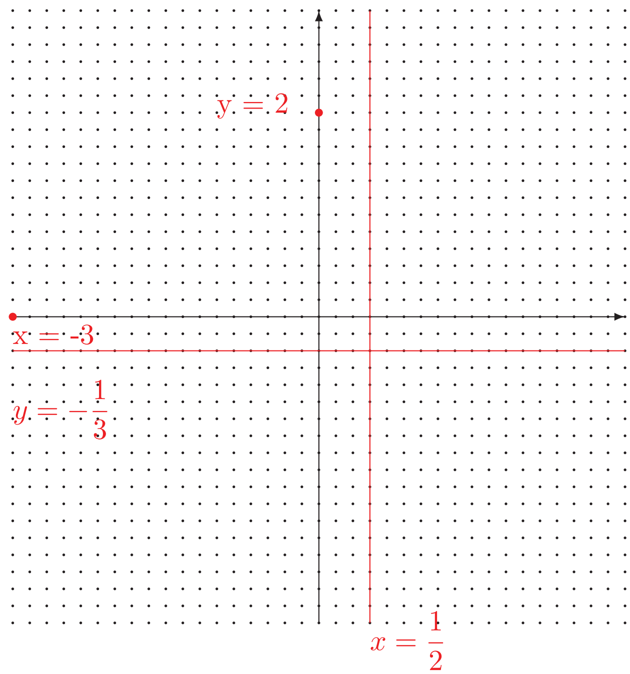
1. $f(x) = \dfrac{2x+6}{-6x+3}$.

   y-intersection: $\mathbf{2}$

   Zeroes $x = \mathbf{-3}$:

   Poles $\mathbf{p : x = \frac{1}{2}}$
   and
   Vertical Asymptotes $\mathbf{x = \frac{1}{2}}$

   $\lim_{x\to\infty} f(x) = -\mathbf{\frac{1}{3}}$
   and
   Horizontal Asymptotes: $\mathbf{y = -\frac{1}{3}}$



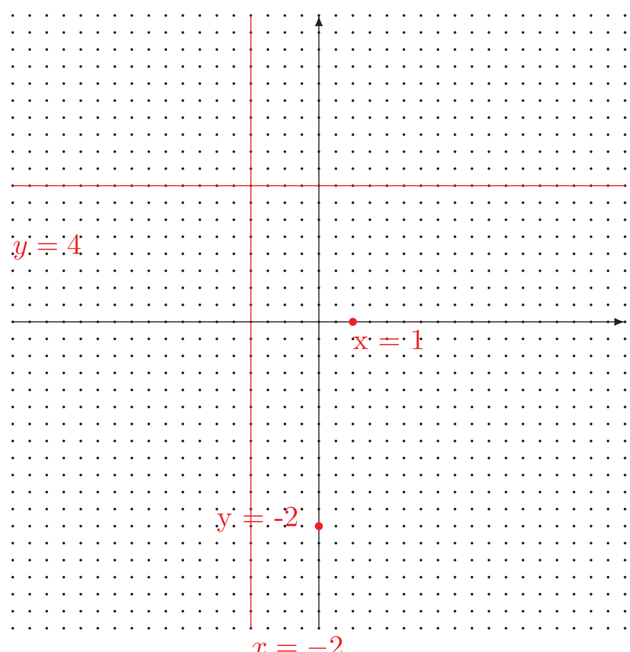
2. $f(x) = \dfrac{4x-4}{x+2}$.

   y-intersection: $\mathbf{-2}$

   Zeroes $x = \mathbf{1}$:

   Poles $\mathbf{p : x = -2}$
   and
   Vertical Asymptotes $\mathbf{x = -2}$

   $\lim_{x\to\infty} f(x) = \mathbf{4}$
   and
   Horizontal Asymptotes: $\mathbf{y = 4}$



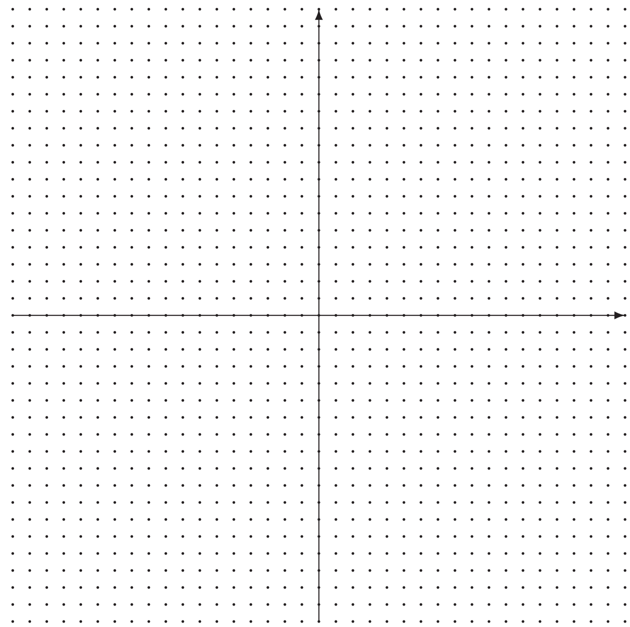
3. $f(x) = \dfrac{1-2x}{2x+3}$.

y-intersection: $\mathbf{\frac{1}{3}}$

Zeroes $x = \mathbf{\frac{1}{2}}$:

Poles $\mathbf{p : x = -\frac{3}{2}}$
and
Vertical Asymptotes $\mathbf{x = -\frac{3}{2}}$

$\lim_{x\to\infty} f(x) = \mathbf{-1}$
and
Horizontal Asymptotes: $\mathbf{y = -1}$



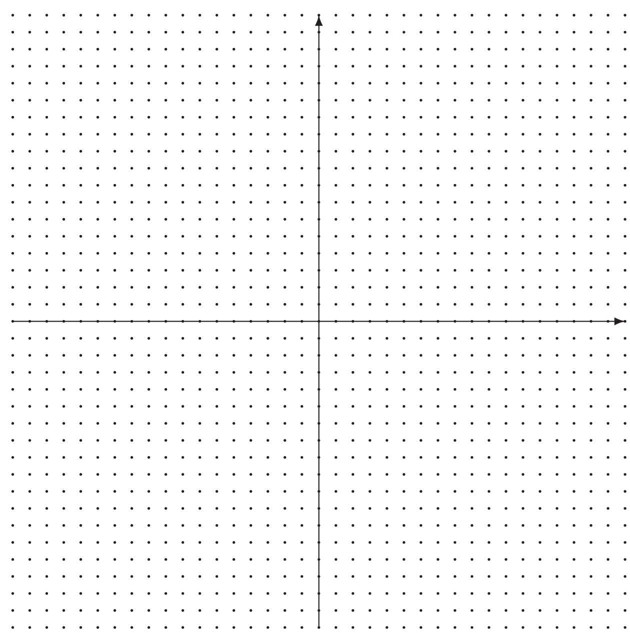
4. $f(x) = \dfrac{4-3x}{x+7}$.

y-intersection: $\mathbf{\frac{4}{7}}$

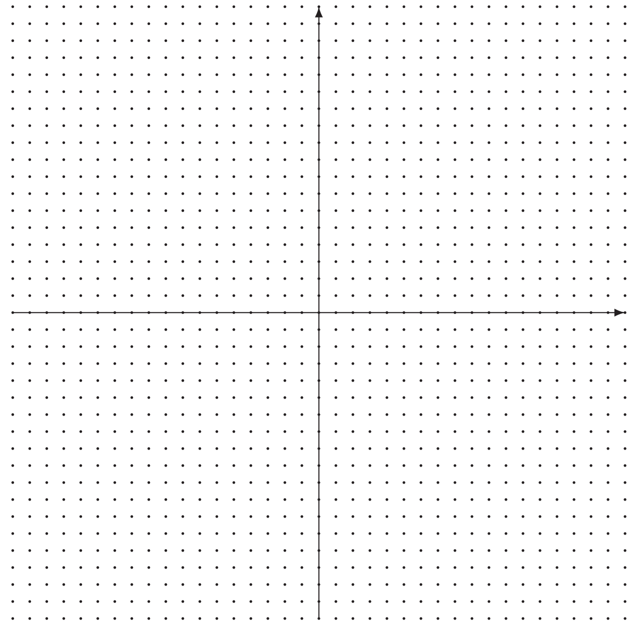
Zeroes $x = \mathbf{\frac{4}{3}}$:

Poles $\mathbf{p : x = -7}$
and
Vertical Asymptotes $\mathbf{x = -7}$

$\lim_{x\to\infty} f(x) = \mathbf{-3}$
and
Horizontal Asymptotes: $\mathbf{y = -3}$



5. $f(x) = \dfrac{18}{(x-3)^2}$.



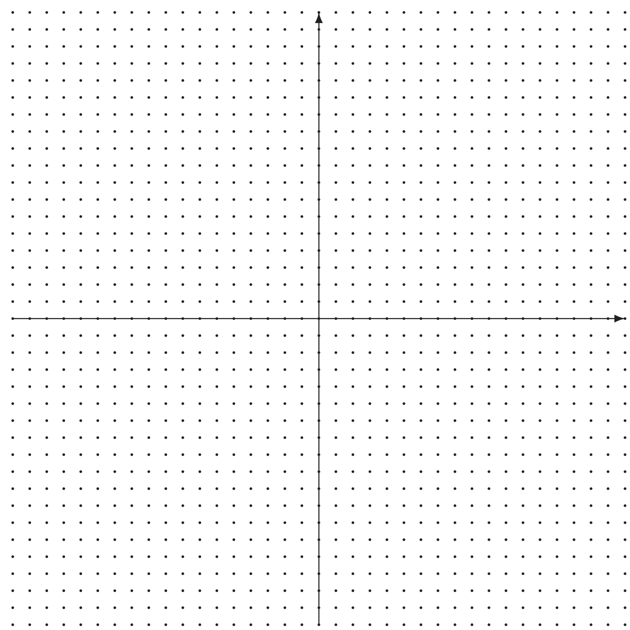
y-intersection: $\mathbf{2}$

Zeroes $x =$ **none**:

Poles $\mathbf{p : x = 3}$
and
Vertical Asymptotes $\mathbf{x = 3}$

$\lim_{x \to \infty} f(x) = \mathbf{0}$
and
Horizontal Asymptotes: $\mathbf{y = 0}$

6. $f(x) = -\dfrac{4}{(x-2)^2}$.



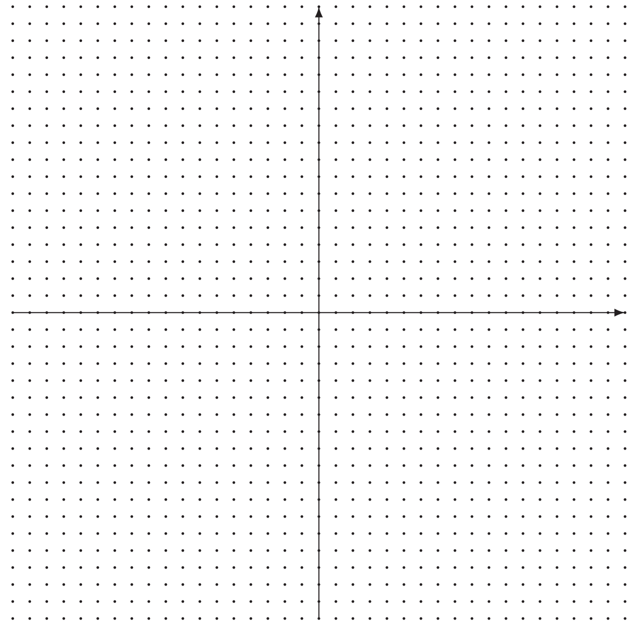
y-intersection: $\mathbf{-1}$

Zeroes $x =$ **none**:

Poles $\mathbf{p : x = 2}$
and
Vertical Asymptotes $\mathbf{x = 2}$

$\lim_{x \to \infty} f(x) = \mathbf{0}$
and
Horizontal Asymptotes: $\mathbf{y = 0}$

7. $f(x) = \dfrac{x-3}{x^2-1}$.



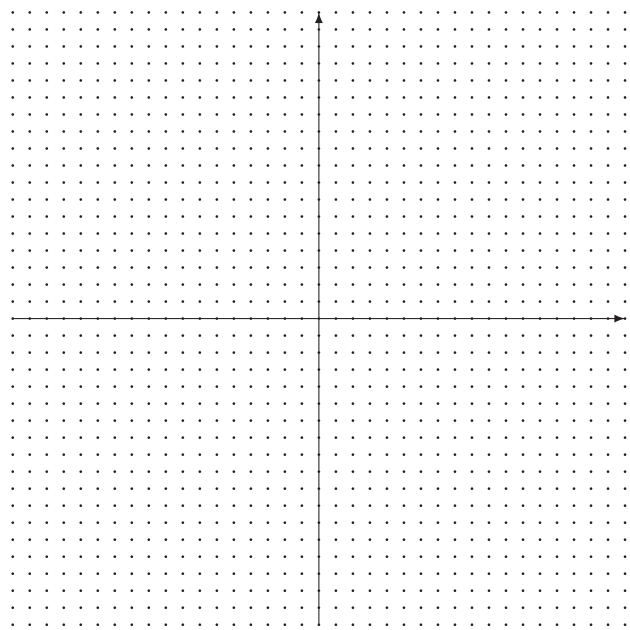
y-intersection: $\mathbf{3}$

Zeroes $x = \mathbf{3}$:

Poles $\mathbf{p : x = \pm 1}$
and
Vertical Asymptotes $\mathbf{x = \pm 1}$

$\lim_{x\to\infty} f(x) = \mathbf{0}$
and
Horizontal Asymptotes: $\mathbf{y = 0}$

8. $f(x) = \dfrac{x+4}{x^2-4}$.



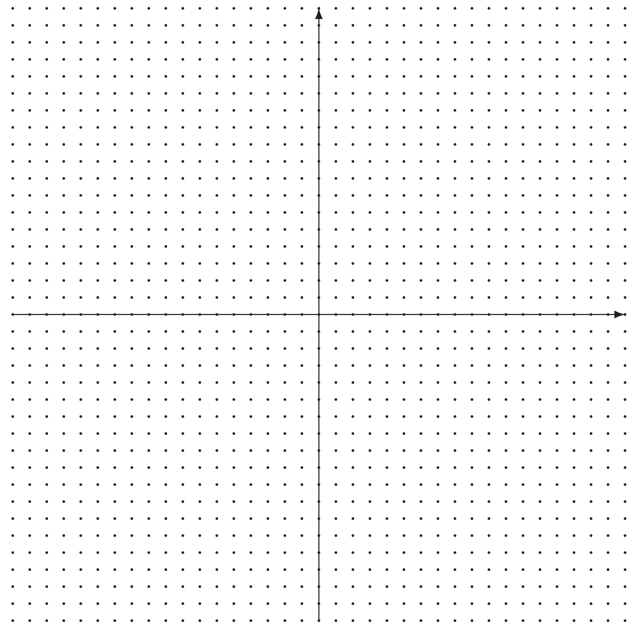
y-intersection: $\mathbf{-1}$

Zeroes $x = \mathbf{-4}$:

Poles $\mathbf{p : x = \pm 2}$
and
Vertical Asymptotes $\mathbf{x = \pm 2}$

$\lim_{x\to\infty} f(x) = \mathbf{0}$
and
Horizontal Asymptotes: $\mathbf{y = 0}$

9. $f(x) = \dfrac{x-2}{x^2-x-6}$.



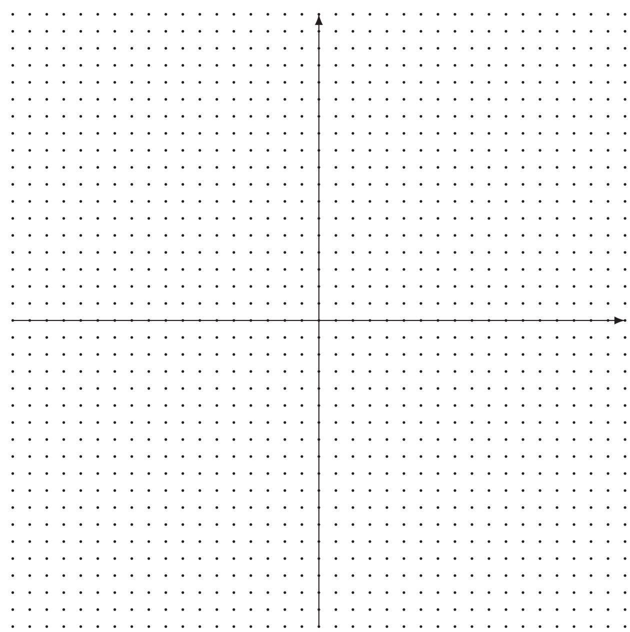
y-intersection: $\mathbf{\frac{1}{3}}$

Zeroes $x = \mathbf{2}$:

Poles $\mathbf{p : x = -2, 3}$
and
Vertical Asymptotes $\mathbf{x = -2, 3}$

$\lim_{x\to\infty} f(x) = \mathbf{0}$
and
Horizontal Asymptotes: $\mathbf{y = 0}$

10. $f(x) = \dfrac{x+1}{x^2+2x-3}$.



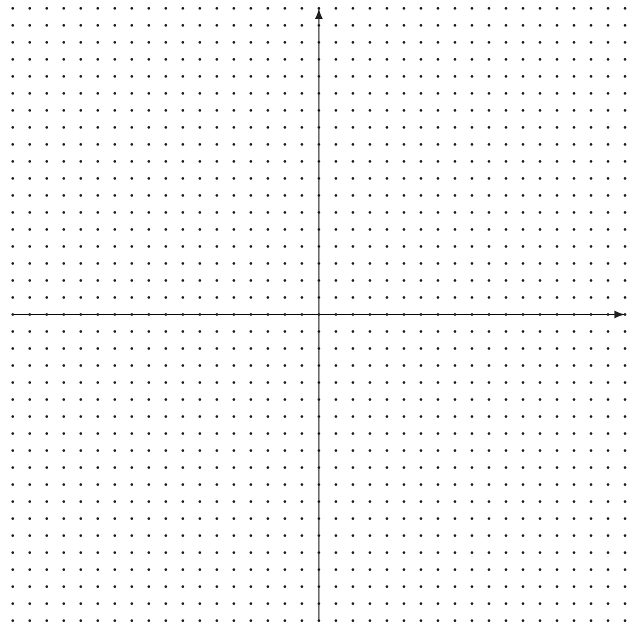
y-intersection: $-\mathbf{\frac{1}{3}}$

Zeroes $x = \mathbf{-1}$:

Poles , $\mathbf{p : x = -3, 1}$
and
Vertical Asymptotes $\mathbf{x = -3, 1}$

$\lim_{x\to\infty} f(x) = \mathbf{0}$
and
Horizontal Asymptotes: $\mathbf{y = 0}$

11. $f(x) = \dfrac{3x+6}{x^2+2x-8}$.



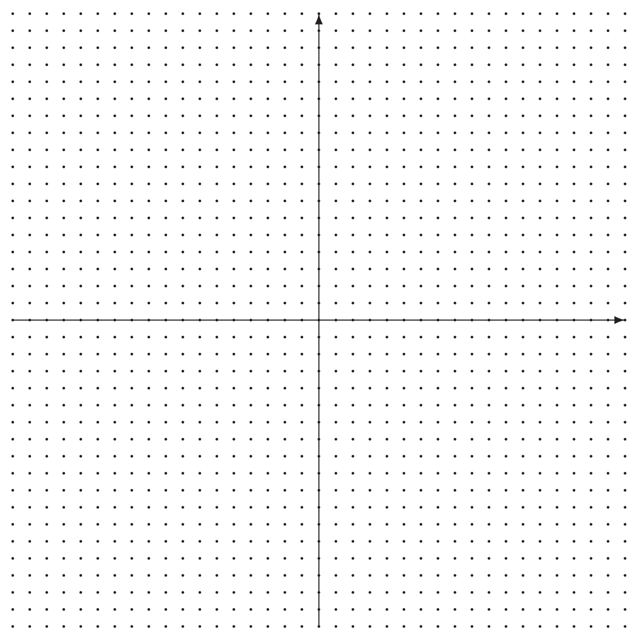
y-intersection: $-\mathbf{\frac{3}{4}}$

Zeroes $x = \mathbf{-2}$:

Poles $\mathbf{p : x = -4, 2}$
and
Vertical Asymptotes $\mathbf{x = -4, 2}$

$\lim\limits_{x\to\infty} f(x) = \mathbf{0}$
and
Horizontal Asymptotes: $\mathbf{y = 0}$

12. $f(x) = \dfrac{2x-4}{x^2+x-2}$.



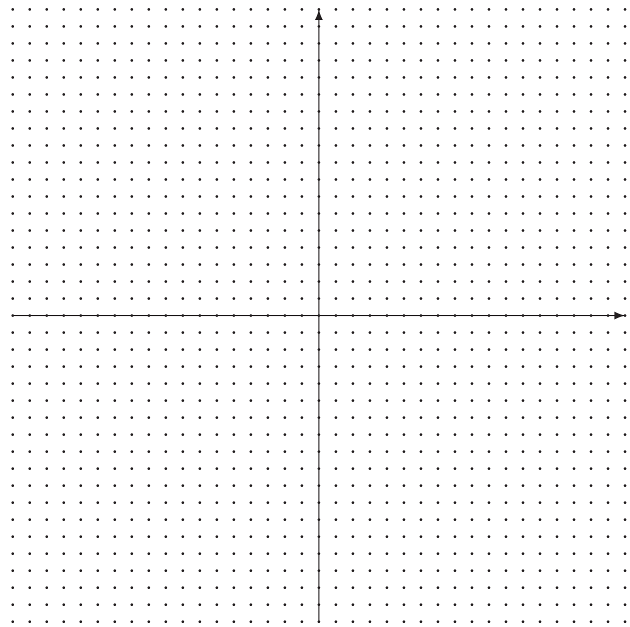
y-intersection: $\mathbf{2}$

Zeroes $x = \mathbf{2}$:

Poles $\mathbf{p : x = -2, 1}$
and
Vertical Asymptotes $\mathbf{x = -2, 1}$

$\lim\limits_{x\to\infty} f(x) = \mathbf{0}$
and
Horizontal Asymptotes: $\mathbf{y = 0}$

13. $f(x) = \dfrac{(x-1)(x+2)}{(x+1)(x-3)}$.



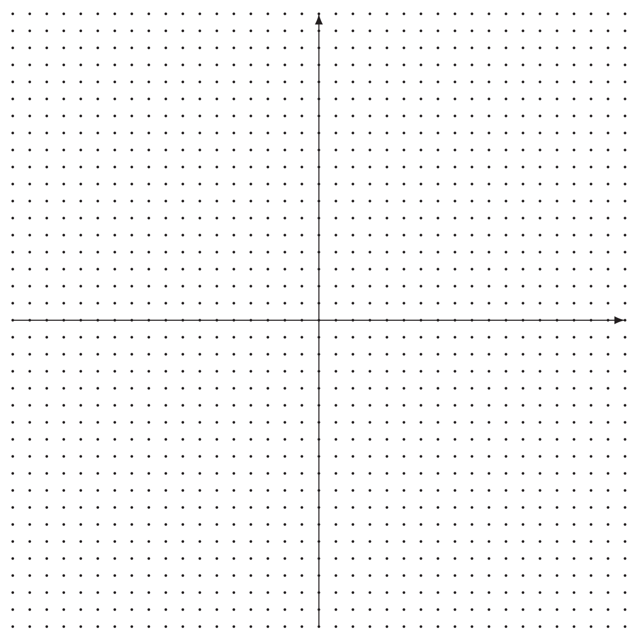
y-intersection: $\mathbf{\dfrac{2}{3}}$

Zeroes $x = \mathbf{-2, 1}$:

Poles $\mathbf{p : x = -1, 3}$
and
Vertical Asymptotes $\mathbf{x = -1, 3}$

$\lim\limits_{x \to \infty} f(x) = \mathbf{1}$
and
Horizontal Asymptotes: $\mathbf{y = 1}$

14. $f(x) = \dfrac{2x(x+2)}{(x-1)(x-4)}$.



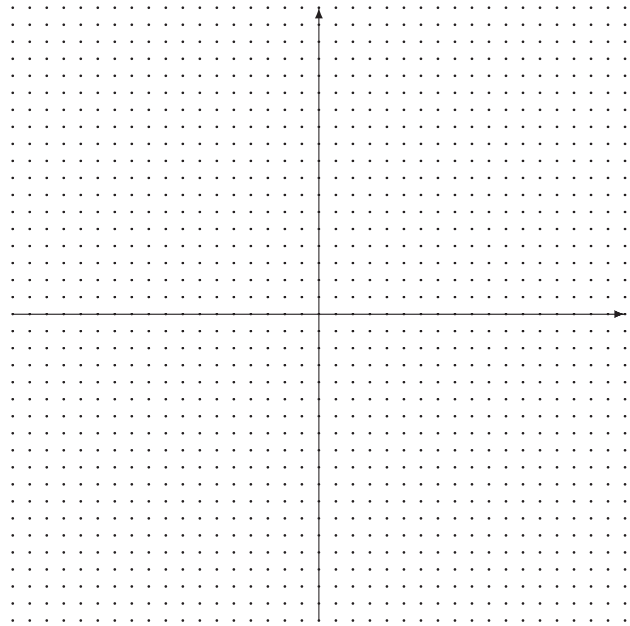
y-intersection: $\mathbf{0}$

Zeroes $x = \mathbf{-2, 0}$:

Poles $\mathbf{p : x = 1, 4}$
and
Vertical Asymptotes $\mathbf{x = 1, 4}$

$\lim\limits_{x \to \infty} f(x) = \mathbf{2}$
and
Horizontal Asymptotes: $\mathbf{y = 2}$

15. $f(x) = \dfrac{x^2 - 2x + 1}{x^2 + 2x + 1}$.



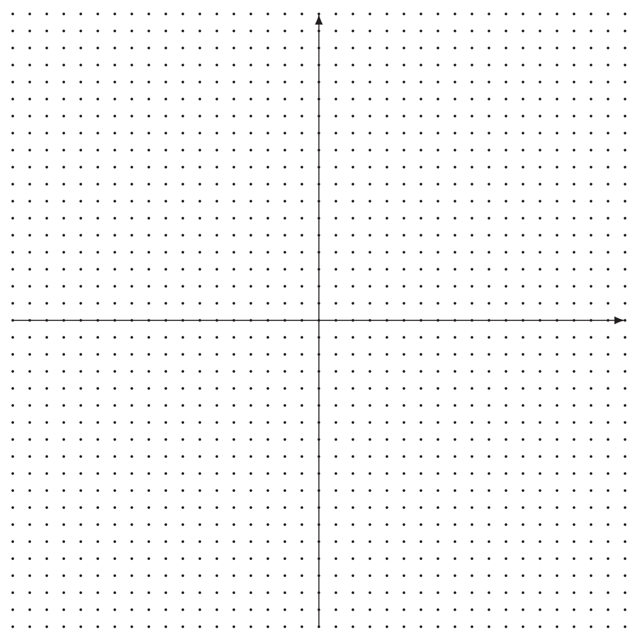
y-intersection: $\mathbf{1}$

Zeroes $x = \mathbf{1}$:

Poles $\mathbf{p : x = -1}$
and
Vertical Asymptotes $\mathbf{x = -1}$

$\lim\limits_{x\to\infty} f(x) = \mathbf{1}$
and
Horizontal Asymptotes: $\mathbf{y = 1}$

16. $f(x) = \dfrac{2x^2 + 10x - 12}{x^2 + x - 6}$.



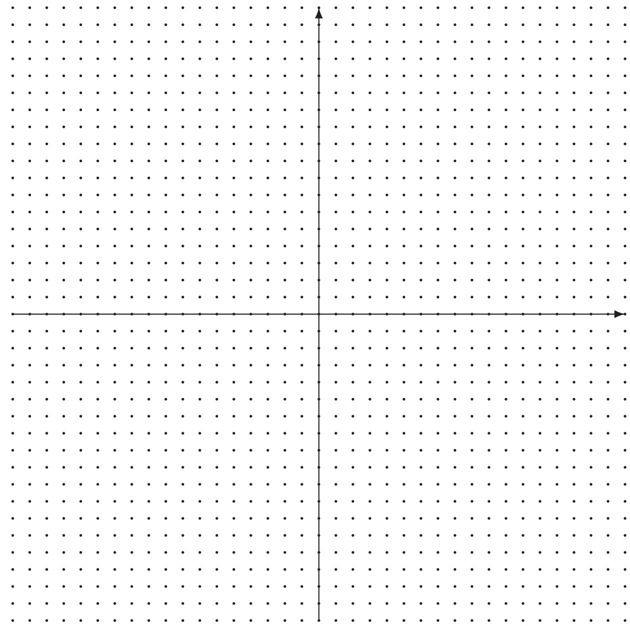
y-intersection: $\mathbf{2}$

Zeroes $x = \mathbf{-6, 1}$:

Poles $\mathbf{p : x = -3, 2}$
and
Vertical Asymptotes $\mathbf{x = -3, 2}$

$\lim\limits_{x\to\infty} f(x) = \mathbf{2}$
and
Horizontal Asymptotes: $\mathbf{y = 2}$

17. $f(x) = \dfrac{2x^2 + 2x - 4}{x^2 + x}$.



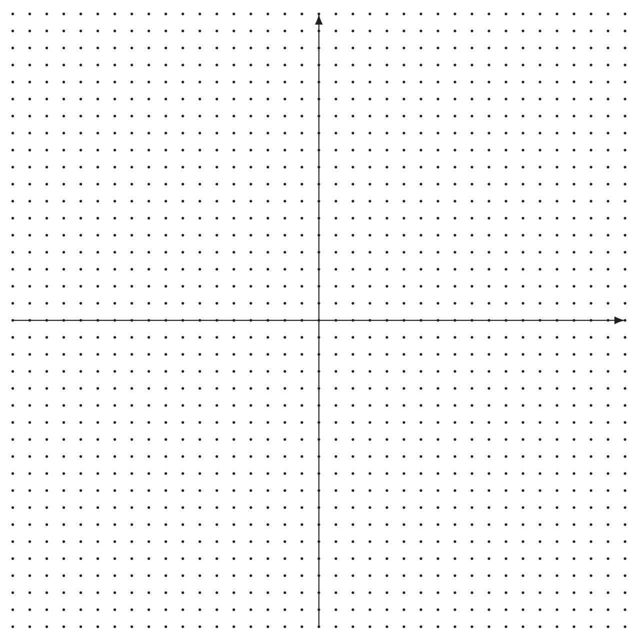
y-intersection: **none**

Zeroes $x = \mathbf{-2, 1}$:

Poles $\mathbf{p : x = -1, 0}$
and
Vertical Asymptotes $\mathbf{x = -1, 0}$

$\lim\limits_{x\to\infty} f(x) = \mathbf{2}$
and
Horizontal Asymptotes: $\mathbf{y = 2}$

18. $f(x) = \dfrac{x^2 + 3x}{x^2 - x - 6}$.



y-intersection: $\mathbf{0}$

Zeroes $x = \mathbf{-3, 0}$:

Poles $\mathbf{p : x = -2, 3}$
and
Vertical Asymptotes $\mathbf{x = -2, 3}$

$\lim\limits_{x\to\infty} f(x) = \mathbf{1}$
and
Horizontal Asymptotes: $\mathbf{y = 1}$