1. Observe and Explore

Module - 13

Circle

13-.1 Introduction :

Study of circle play very important role in the study of geometry as well as in real life. Path traced by satellite, preparing wheels of vehicle, gears of machines, rings, drawing designs etc. Many mathematics concepts get cleared and understands with the study of circle. It influences the mathematics that is taught and enhances students learning.

13.2. Observe and Explore. (Step – 1)

Task – 1. Different shapes of video. (Time – 5 Min.)

The aims of showing this video is to create interest of the student. by showing different shapes, teachers will ask the questions to the students.

Activities – 1.

1. Students you have seen different figures in video. Can you tell me what is the shape of the last figure which you have seen?
2. Teacher will ask so many questions on video clips and involve students in ''guided discussion'' that lead to concept of circle.

Activities – 2. Video clipe

Teacher will take students on the play ground and do the activities as shown in video clips. At the end teacher will ask the questions leading to the previous knowledge.

Teacher : What did you find in this clip?

Pushkar : Sir, I had seen wheel of cycle, car, auto riskshaw, bullock cart.

Teacher : How was their shape?

Pushkar : Sir, their shape are circuler.

Teacher : What you have seen in the ring of cycle?

Pushkar : I had seen spokes.

Teacher : All the spokes are of equal length?

Minu : Yes sir, All the spokes are of equal length.

Teacher : Equal length is called radius and mid point is called centre of the circle.

Minu : Does It mean that all the radius of the circle are equal?

Teacher : Yes, Absolutely right.

Rudraksh : If I will join two spokes. Will it be the diameter? or chord of the circle?

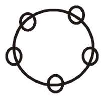
Teacher : Yes, the chord passing through the centre of the circle is called diameter.

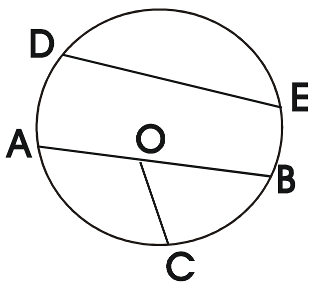
Minu : It means that diameter is doubled than the radius?

Pushkar : No sir, I say that the radius is half of the diameter.

Teacher : Don’t argue. Both of you are correct.
(Teacher will explain circle after all discussion)

Task 3 : Worksheet from geogebra applet.

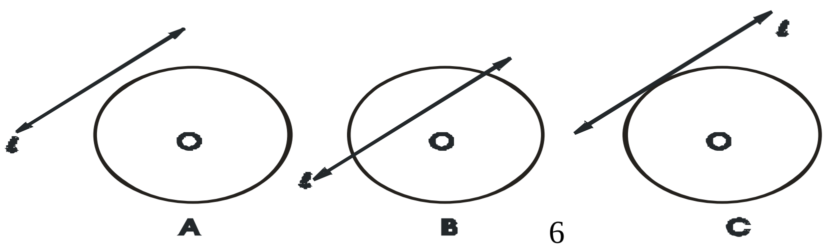




- Centre of circle
- Radius
- Diameter
- Chord

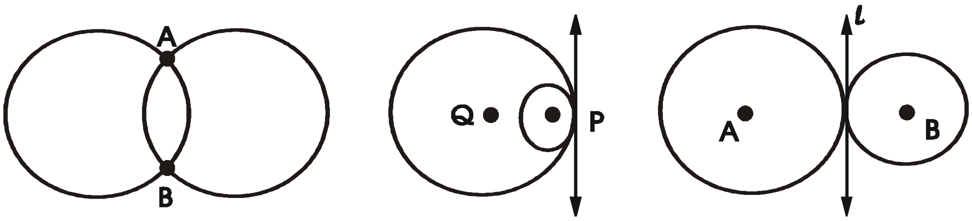
With the help this work sheet teacher will show all the related concept and at the end of the discussion finally lead to conclusion.

Task 4 : Slide (1) concept of Tangent. (P.P.t)



Teacher will explain concept of tangent with the help of a slide and define the tangent.

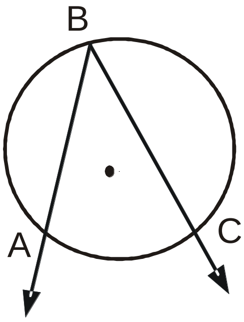
Slide – 2



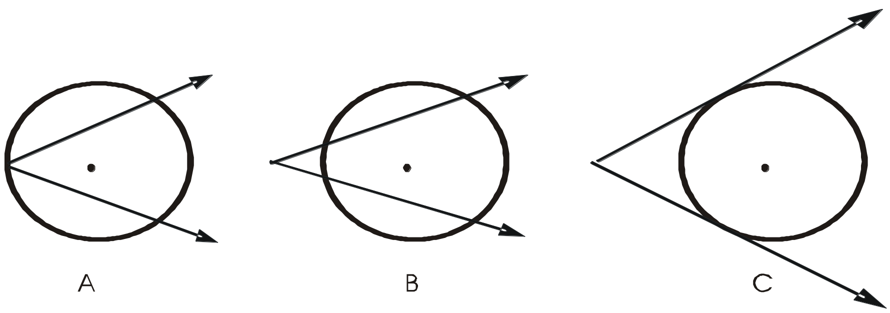
Teacher will explain the concept of touching circle with the help of slide and further define it.

Slide – 3 : Inscribed angle.

You can explore it using geogebra applet worksheet included in the slide. The concept of inscribed angle get cleared to student with the help of a worksheet.



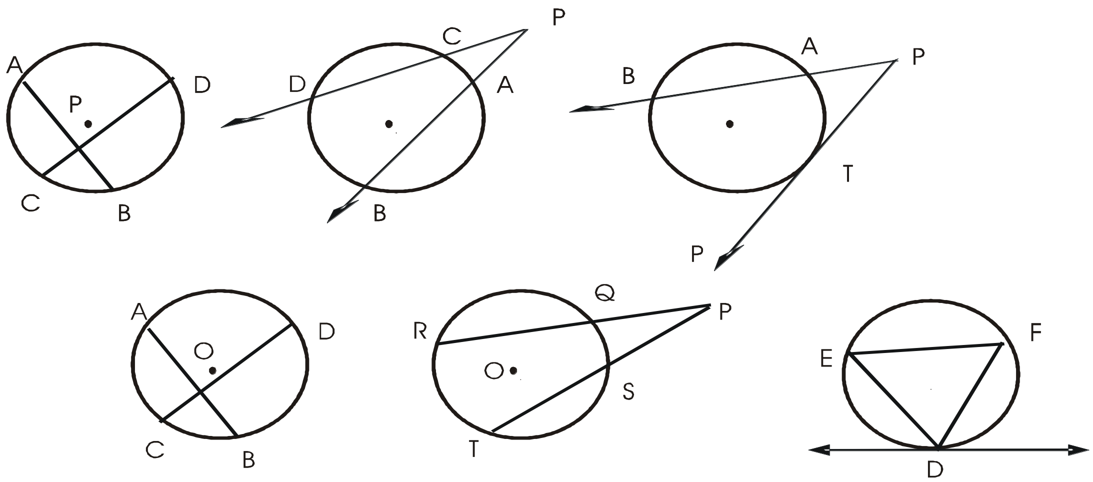
Slide 4 : Intercepted arc.



Teacher ask the questions to the student with the help of above slide and explains the concept of intercepted arc.

Slide – 5 : Geogebra worksheet.

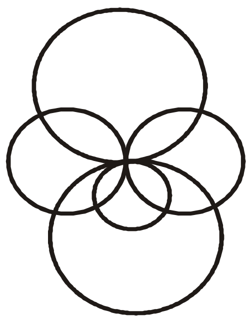
Teacher will explain the property related to Inscribed angle and intercepted arc in the slide below.



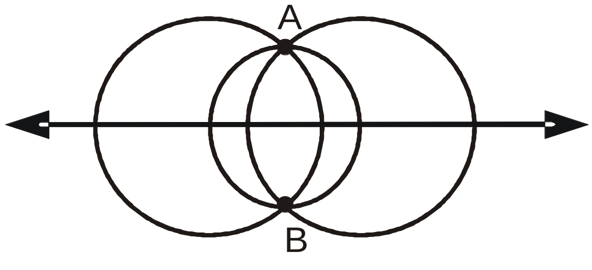
Slide – 6

Geogebra work sheet is also included in this slide.

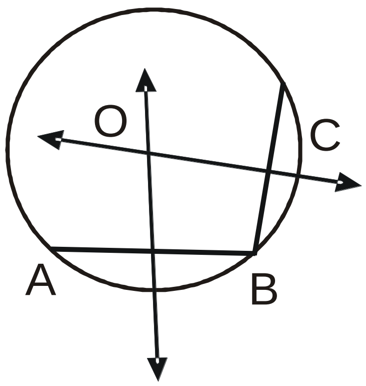
1) Circles passing through one point.



2) Circles passing through two distinct point.



3) One and only one circle is possible through three non collinear point.



1.3 Step - II

Define and Prove

Geogebra Worksheet

1. Radius drawn from point of contact is perpendicular to tangent.
2. A line is perpendicular to radius at its outer end is tangent to the circle.
3. Tangent drawn from same external point to same circle are equal.
4. If two circles touch each other then their point of contact lies on the line joining their centre.
5. Measure of inscribed angle is half of the intercepted arc.
6. Diameter subtends a right angle at a point on the circle.
7. Congruent chords subtends congruent angle ant the centre of a circle.
8. Opposite angles of a cyclic quadrilateral are supplementary.
9. Congruent chords are equidistant from the centre of a circle.
10. Chords subtending congruent angle at the centre are congruent.
11. Equidistant chords are congruent.
12. Segment joining the centre of circle and mid point of chord is perpendicular to the chord. (Proofs of all these theorems teacher uses geogebra book OR by placing the paper of different colour cut it with the help of scissor at proper place and proves the theorem)

3. Apply and Evaluate

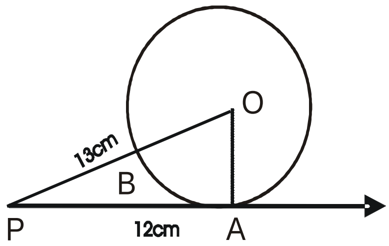
Problem

1) A point P is 13 cm from the centre of a circle. The length of the tangent drawn from P to the circle is 12 cm, find the distance of point P from the nearest point of the circle.

Solution :

PA $\perp$ OA

In Rt Δ OAP, By pytha. Theo.



$OA^2 = Op^2 - Ap^2$

$= 13^2 - 12^2$

$OA^2 = 25$

$OA = 5$

Nearest point of circle from point P is B.

PB = OP – OB

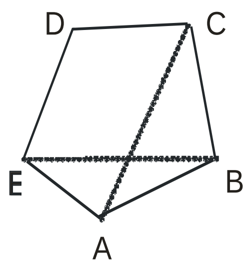
= 13 -5 ………………… (OA = OB = r)

PB = 8 cm

2) Prove that any four vertices of a regular pentagon lie on a circle.

Solution :

Join AC And BE



$\angle$ ABC = $\angle$ EAB ………….. Angles of regular Polygon.

BC = AE …………… sides of regular Polygon.

AB = AB ………. Common side.

Δ ABC $\cong$ Δ ABE …………… SAS test

$\angle$ ACB = $\angle$ AEB ……………. C.A.C.t.

Points A, B, C, E lies on a circle. By Theorem two congruent angle on the same side of segment.

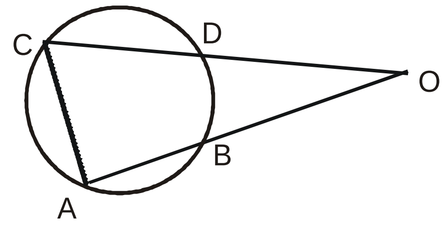
3) Consider the figure as shown.

Chord AB and CD of the circle Are produced to meet at 'O'

Prove that Δ ODB ~ Δ OAC.

Given CD = 2cm, DO = 6 cm And BO = 3cm. Calculate AB.

Also find $\frac{A(\square CABD)}{A(\Delta CAD)}$



Solution :

In Δ OBD and Δ OCA

$\angle OBD = \angle ACO$ ……… $\because$ ABDC is cyclic, Exterior angle of cyclic quadrilateral .

Also $\angle AOC = \angle BOD$ ……………….. Common angle

Δ OBD ~ Δ OCA ………………… A. A. Test

$\frac{AO}{DO} = \frac{CO}{BO} \quad \therefore \quad \frac{AO}{6} = \frac{8}{3}$ …….. (Do = 6, Bo = 3 (Co = CD+DO = 2+6 = 8)

AO = 16 cm

AB = AO – BO = 16-3 = 13 cm

$\frac{ar(\Delta DBO)}{ar(\Delta CAO)} = \frac{(DO)^2}{(AO)^2} = \frac{(6)^2}{(16)^2} = \frac{(3)^2}{(8)^2} = \frac{9}{64}$

64 ar (ΔDBO) = 9 ar (ΔCAO)

64 { ar (ΔCAO) - ar ($\square$CABD)} = 9 ar (Δ CAO)

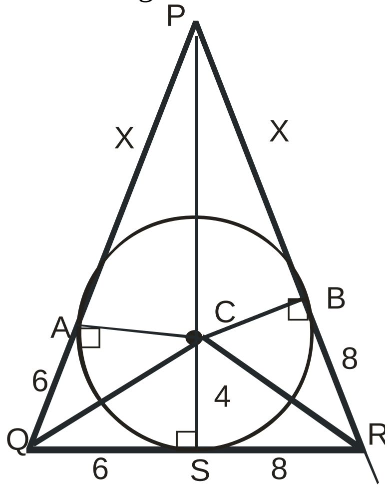
55 ar (ΔCAO) = 64 ar ($\square$CABD)

$\frac{Ar(\square CABD)}{Ar(\Delta CAO)} = \frac{55}{64}$

Que 4 A) Δ PQR is drawn to circumscribed a circle of radius 4cm Seg QR divides by point ‘S’ in two parts RS & QS of length 8cm of 6 cm resp. Find side PQ & PR

Solution :

Answer Fig.



Let PA = PB = x

$a = x + 6$

$b = x + 8$

$c = 6 + 8 = 14$

$$s = \frac{a + b + c}{2}$$

$$= \frac{x + 6 + x + 8 + 6 + 8}{2}$$

$$= \frac{2x + 28}{2}$$

$$\boxed{S = x + 14}$$

$$A(\Delta PQR) = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{(x + 14)(x + 14 - x - 6)(x + 14 - x - 8)(x + 14 - 14)}$$

$$= \sqrt{(x + 14)\, 8 \times 6 \times X}$$

$$A(\Delta PQR = \sqrt{48x(x + 14)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now,

$$A(\Delta PQR) = A(\Delta CQR) + A(\Delta PCR) + A(\Delta PCQ)$$

$$= \tfrac{1}{2} \times 14 \times 4 + \tfrac{1}{2} \times (x+8) \times 4 + \tfrac{1}{2} \times (x + 6) \times 4$$

$$= 14 \times 2 + (x + 8) \times 2 + (x + 6) \times 2$$

$$= 28 + 2x + 16 + 2x + 12$$

$$A(\Delta PQR) = 4x + 56 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From 1 & 2

$$\sqrt{(x + 14) \times 48x} = 56 + 4x$$

Squaring both. Sides.

$$(x + 14) \times 48x = (56 + 4x)^2$$

$$(x + 14) \times 48x = 4^2 (14 + x)^2$$

$$48x = \frac{16 (x+14)(x + 14)}{(x + 14)}$$

$$48x = 16 (x + 14)$$

$$48x = 16x + 224$$

$$48x - 16x = 224$$

$$32x = 224$$

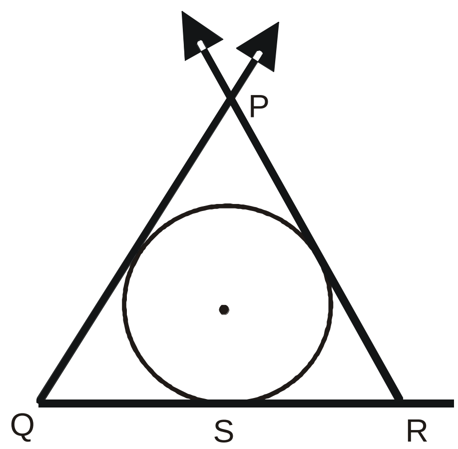
$$x = \frac{224}{32}$$

$$\boxed{\mathbf{x = 7}}$$

$\therefore$ PQ = x + 6 = 7 + 6 = 13 cm.

PR = x + 8 = 7 + 8 = 15 cm.

Questions figure :

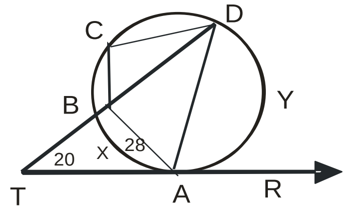


5) TAR is a Tangent to the Circle at A, If

$\angle BTA = 20^0$ and $\angle BAT = 28^0$

Find 1) $\angle DAR$

1) $\angle BCD$



Solution

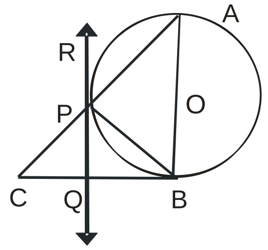
m$\angle$ ABD	=	$\angle$BTA + $\angle$BAT ………Measure of the exterior angle of a triangle.
	=	$20^0 + 28^0$
	=	48^0
$\angle$DAR	=	m $\angle$ABD ……….. Angles in alternate segment
	=	48^0
m$\angle$DAR	**=**	**48^0**

m (arc BXA)	=	2m $\angle$ BAT ……………. Tangent secant theo.
	=	2 x 28
	=	56^0
m (arc DYA	=	2 $\angle$DAR ………………. Tangent secant theo
	=	2 x 48 = 96^0
BCD	=	½ (arc BXA + arc DYA) ……… (Inscribed angle Theorem)
	=	½ (56 + 96)
	=	½ (152)
$\angle$BCD	=	76^0

Que 6) In ΔABC, $\angle B = 90^0$

A circle with AB as a diameter is drawn to intersect side AC in point P. PQ is a Tangent to the circle at point P.

Prove that PQ bisects side BC.



Solution

AB is diameter & $\angle B = 90^0$ ……….. given.

$\therefore$ Line BC is tangent

Line PQ is tangent ……………….. given

$\therefore$ PQ = BQ ……………….. (1) tangent seg.

$\therefore$ $\angle QPB = \angle QBP = x$ (say) …………. Base angles of iso triangle

$\angle APB = 90^0$ ……………….. subtended by diam.

$\therefore$ $\angle QPB + \angle RPA = 90^0$

$X + \angle RPA = 90^0$

$\therefore$ $\angle RPA = 90^0 - X$

$\angle RPA \cong \angle CPQ$ ------------ (Vertically opp. Angle)

$\therefore$ $\angle CPQ = 90^0 - x$ ………….. (2)

$\angle PQC$ is an exte. Angle of ΔPQB.

$\therefore$ $\angle PQC = x + x = 2x$

In ΔPCQ

$\angle PCQ + \angle CPQ + \angle PQC = 180^0$

$\angle PCQ + 90 - x + 2x = 180^0$

$\angle PCQ + 90 + x = 180^0$

$\angle PCQ = 180 - (90 + x)$

$\angle PCQ = 180 - 90 - x$

$\angle PCQ = 90 - x$ ……………. (4)

From 2 and 4

$\angle CPQ \cong \angle PCQ$

∴ CQ = PQ ………….. (5) side opp. To cong. Angles.

From 1 & 5

BQ = CQ,

∴ Tangent PQ bisect the seg BC.

Step – 3

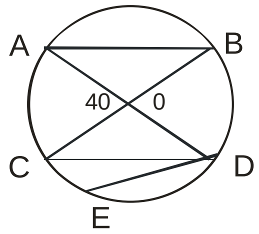
Apply and Evaluate Unit Circle

Que 7) If AB || Seg CD and O is the centre of circle, $\angle AOC = 40^0$

m (arc CE) = 50^0

m (arc ED) = 60^0

Find $\angle CED$ and $\angle COD$



Ans : $\angle AOC + \angle COD$ = 180 ……….. Linear Pair angle

40 + $\angle COD$ = 180

$\angle COD$ = 180 – 40

$\angle COD$ = 140

$\angle COD \cong \angle AOB = 140$ …………. (V – O- A)

∴ m $\angle CED$ = ½ m (arc CAD) … (Inscribed angle theo.)

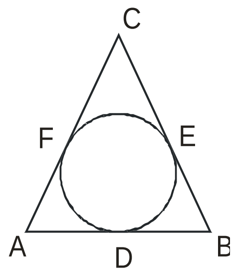
m (arc AC) = m (arc BD) = 40^0

m $\angle CED$ = ½ [40 + 140 + 40]

= ½ [220]

m$\angle CED$ = 110

8) A circle is inscribed in a Δ ABC having side 8cm, 10cm and 12cm as shown in the figure. Find AD, BE and CF.



Ans :

AB = 8cm, BC = 10cm, AC = 12cm

AD = AF
BD = BE ……. (1) By theorem.
CF = CE

AD + BD + CF = AF + BE + CE Adding

AD + BE + CE = AF + CF + BE

AD + BC = AC + BE

AD - BE = 12 – BC

AD – BE = 12 – 10

AD – BD = 2

AD – (AB – AD) = 2

AD + AD – AB = 2

2AD = 2 + AB

2AD = 2 + 8

2AD = 10

AD = 5

AD + BD = AB

5 + BD = 8

BD = 8-5

BD = 3

BD = BE = 3

AF + FC = AC

AD + CF = 12

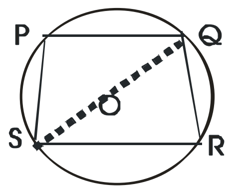
5 + CF = 12

CF = 12 – 5

CF = 7

9) In fig. □PQRS is cyclic trapezium

If PQ || SR, Then prove that PS ≅ QR.



Proof : - Draw chord QS

In □ PQRS, PQ||SR and QS is the transversal

∴ ∠PQS ≅ RSQ ………. (1) (Alternate angle)

Now, inscribe angle ∠ PQS intercepted the arc PS

m ∠ PQ = ½ m (arc PS) ……. (2) (by inscribed angle Theorem)

Now, m ∠ RSQ = ½ m (arc QR) …... (3) (by inscribed angle Theorem)

From 1, 2 & 3

m (arc PS) = m (arc QR)

arc PS ≅ arc QR

Chord PS ≅ Chord QR ……… (by definition of minor arc)

∴ PS ≅ QR.

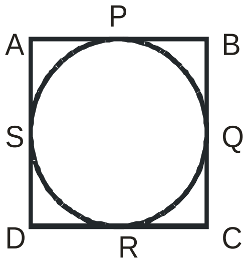
10) If all the sides of parallelogram touch a circle, show that the parallelogram is rhombus.

Given :

□ ABCD is a parallelogram

Pont P,Q,R,S are touching

Point of the circle.



To prove : □ABCD is rhombus.

Proof : □ABCD is parallelogram …………. Given.

∴ AB = DC

AD = BC …………….. (1)

AS = AP
SD = DR
BQ = BP
QC = CR
— By theorem

Adding Both side.

AS + SD + BQ + QC = AP + DR + BP + CR

AD + BC = AP + BP + DR + RC

AD + BC = AB + DC

AD + AD = AB + AB …………. By (1)

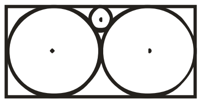
2 AD = 2 AB

AD = AB ……………. (2)

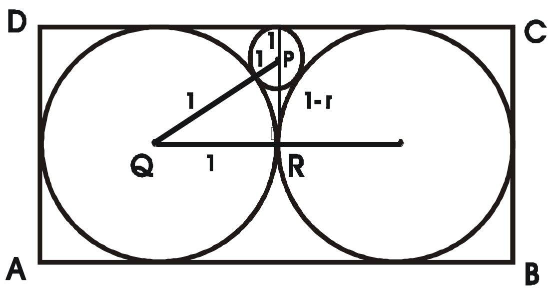
From (1) and (2)

□ ABCD is rhombus.

11) Two circles each with a radius of one unit and tangential to each other are inscribed in a 2 by 4 rectangle. A smaller circle is inscribed in the space between the circles & the longer edge of the rectangle such that it is tangent to both circles and the edge of the rectangle. What is the radius of this smaller circle?



Solution :



Let radius of the smaller circle be r

PQ = 1 + r

PR = 1 – r

QR = 1

In rt ∠ PRQ

$PR^2 + QR^2 = PQ^2$ ………………… by pythagorus theorem

$(1 - r)^2 + 1^2 = (1 + r)^2$

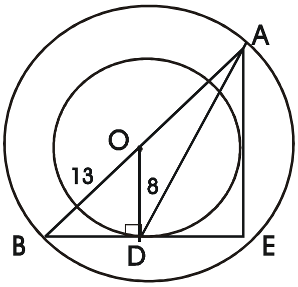
$1 - 2r + r^2 + 1 = 1 + 2r + r^2$

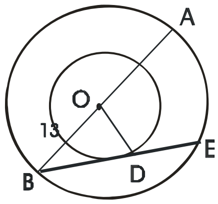
$4r = 1$

$$\boxed{\mathbf{r = \tfrac{1}{4}}}$$

12) The radii of two concentric circle are 13cm & 8cm. AB is a diameter of the bigger circle. BD is a tangent to the smaller circle touching it at D then find length AD.

Solution :





Join AE & AD

OD ⊥ BE ……………. Tangent radius properly

∠ AEB = 90 ………… Properties of diamenter

In Δ BOD & Δ BAE

∠B ≅ ∠B ………….. common angle

∠ODB ≅ ∠AEB …………….. each 90^0

∴ ΔBOD ~ Δ BAE ………….. A. A. test

$$\frac{BO}{BA} = \frac{OD}{AE}$$ ……….. corr. Side of ~ Δ

$$\frac{13}{26} = \frac{8}{AE}$$

$$AE = \frac{26 \times 8}{13}$$

AE = 16

In rt ∠Δ BDO

$OB^2 = OD^2 + BD^2$ …………….. pythagorous thm.

$13^2 = 8^2 + BD^2$

$BD^2 = 169 - 64$

$BD^2 = 105 \rightarrow BD = \sqrt{105}$ cm

In rt ∠Δ AED

$$AD^2 = AE^2 + DE^2$$

$$= 16^2 + (\sqrt{105})^2$$

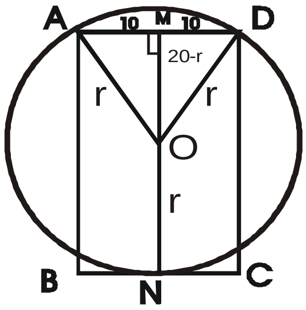
$$= 256 + 105$$

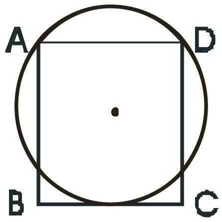
$$AD^2 = 361$$

$$AD = 19 \text{ cm}$$

13) A square with a side length 20 has two vertices on the circle and one side touching the circle. Find the diameter of the circle.

Solution :





Let radius = r

Join OD, OA, ON

Draw OM $\perp$ AD

AM = MD = 10 [$\perp^{er}$ drown from centre to the chord bisect chord]

OM = MN - ON

= 20 – r

In rt $\angle\Delta$ AOM

$AM^2 + MO^2 = AO^2$ ……………. By pythagorous thm

$10^2 + (20- r^2) = r^2$

$100 + 400 – 40r + r^2 = r^2$

$\therefore$ 40 r = 500

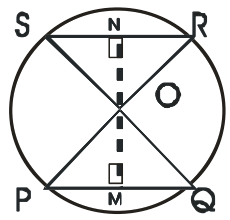
r = $\frac{500}{40}$

r = 12.5 cm

diameter = 2r

= 2 x 12.5 = 25 cm

14) If $\angle POQ \cong \angle ROS$, SR|| PQ. Dimeter of circle Is 26 cm and length of chord is 24 cm then what is perpendicular distance between two chord.



Solution :

PM = ½ PQ …… (Perpendicular drawn from centre of circle To the chord)

PM = ½ x 24 ……….. (Length of chord = 24cm)

PM = 12

OP = OR = ½ PR ……… (length of diameter = 26)

= ½ x 26

= 13

In right ……… angle Δ OPQ, by PGT

$OP^2 = PM^2 + OM^2$

$13^2 = 12^2 + OM^2$

$OM^2 = 13^2 - 12^2$

$OM^2 = 169 - 144$

$OM^2 = 25$

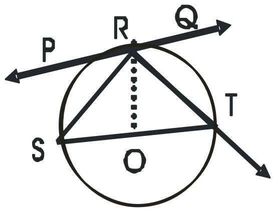
OM = 5

Similarly,ON = 5

O – M – N

MN = OM + ON = 5 + 5 = 10

15) In the fig. PQ is the tangent at a point R of the circle with centre O. If $\angle$ TRQ = 30^0 Find the $\angle$ PRS, $\angle$SRT, $\angle$ROT, $\angle$ORT, $\angle$STA



Ans : m $\angle$ TRQ = 30 ……… (Given)

m $\angle$SRT = 90^0 …….. (Angle in semicircle)

m $\angle$SRQ = m $\angle$SRT + m $\angle$TRQ

= 90 + 30

m $\angle$SRQ = 120^0

m $\angle$PRS = 180 – 120 = 60^0 (Linear pair angle)

m $\angle$TRQ = ½ m (arc RT) ….. Tangent secant theorem.

30 = ½ m (arc RT)

m (arc RT) = 60^0 ………. Def^n of minor arc.

m $\angle$RST = ½ m (arc RT) ……. (inscribe angle theo.)

= ½ x 60

m $\angle$RST = 30

m $\angle$STA = m $\angle$RST + m $\angle$SRT ……. Remote interior angle theorem

m $\angle$STA = 30 + 90

m $\angle$STA = 120

m $\angle$PRS = m $\angle$ RTO = 60^0

In ΔORT, OR = OT ……… (Raddi of same circle)

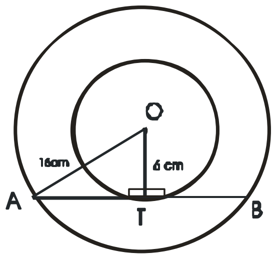
m $\angle$ORT = m$\angle$OTR = 60^0

m $\angle$ORT = 60^0

m $\angle$PRS = 60^0, m $\angle$SRT = 90^0 , m $\angle$ ROT = 60^0

m $\angle$ORT = 60^0, m $\angle$STA = 120^0

16) In the two concentric circle a chord of larger circle become a tangent to the smaller circle whose radius is 10cm and radius of smaller circle is 6cm. Find the length of tangent to the smaller circle.



Solution :

OT $\perp$ Chord AB ………… (Tangent is perpendicular radius)

AT = TB ……….. (A perpendicular drawn from the centre of Cirle to the chord bisect chord)

OA = 10 cm & OT = 6cm

In right angle Δ OTA,

By PGT

$\therefore$ $OA^2 = OT^2 + AT^2$

$\therefore$ $10^2 = 6^2 + AT^2$

$\therefore$ $10 = 36 + AT^2$

$\therefore$ $AT^2 = 100 - 36$

$\therefore$ $AT^2 = 64$

$\therefore$ $\mathbf{AT = 8}$

$\therefore$ $AB = 2AT$

$\therefore$ $AB = 2 \times 8$

$\therefore$ $AB = 16cm$

Length of chord of circle tangent to the smaller circle = 16cm.

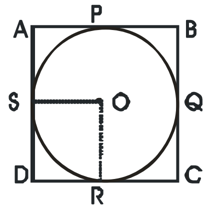
17) A circle touches the side of a quadrilatral ABCD at point P,Q,R,S respectively. Show that the angle subtended at the centre and angle form at opposite vertex are supplementary.

Ans :

Given : In □ ABCD

Point P, Q, R, S are touching

Point of the circle.



To prove : $\angle SOR + \angle SDR = 180^0$

Construction : Joint OS and OR.

Proof : O is the centre of the circle

Radius OS ⊥ tangent AD $\therefore \angle OSD = 90^0$

Radius OR ⊥ tangent DC $\therefore \angle ORD = 90^0$

$\angle OSD + \angle ORD = 90 + 90$

$\angle OSD + \angle ORD = 180$ ………………… (1)

In □ORDS

$\angle SOR + \angle ORD + \angle OSD + \angle SDR = 360$

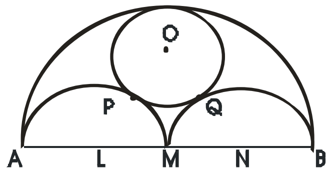
$\angle SOR + \angle SDR + 180 = 360$

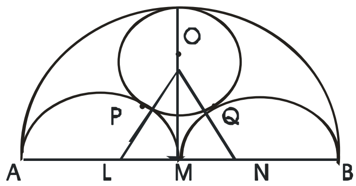
$\angle SOR + \angle SDR = 360 - 180$

$\angle SOR + \angle SDR = 180$

18) Problem : An Exploration

AB is a line tangent and M is the midpoint semicircle are drawn with AM, MB and AB as diameters on the same side of line AB. A circle with centre O and radius r is drawn so that It touches all the three semicircles.





Prove that : $r = {}^{1}/_{6}\,AB$

Solution :

Let L, N be the midpoint of AM, MB resp. Let circle c (o,r) touch the semicircle with centre L,M,N at P,R, Q resp.

Join OL, ON, MR. Points O – Q – N,

O – P – L, R – O – M

Let AB = x

$OL = r + {}^{x}/_{4}$ (since $PL = LM = {}^{x}/_{4}$) and $ON = r + {}^{x}/_{4}$

$\therefore\ \Delta OLN$ is isosceles and M is midpoint of base LN and $OM \perp LN$.

ΔOML is right triangle.

$OL^2 = OM^2 + LM^2$

OR $(r + {}^{x}/_{4})^2 = (RM - r)^2 + ({}^{x}/_{4})^2$

$$(r + {}^{x}/_{4})^2 = ({}^{x}/_{2} - r)^2 + ({}^{x}/_{4})^2$$

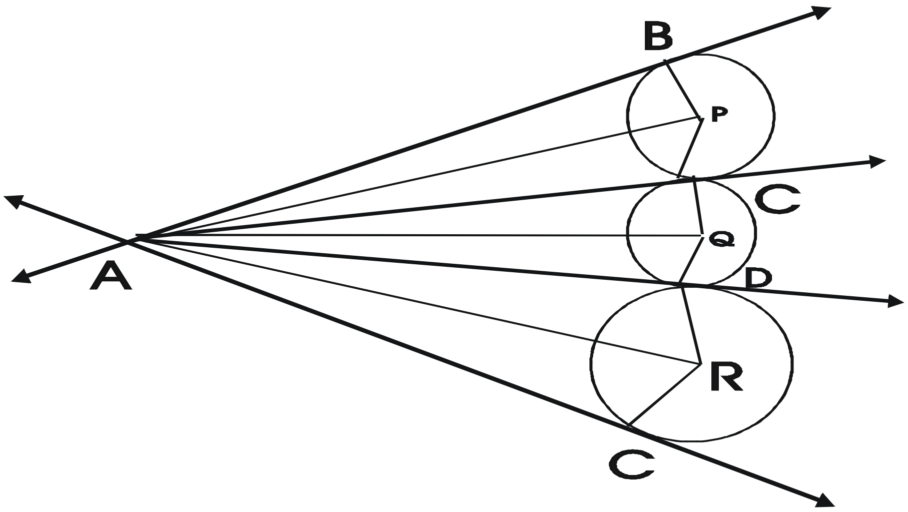
$$r^2 + {}^{rx}/_{2} + {}^{x^2}/_{16} = {}^{x^2}/_{4} + r^2 - rx + {}^{x^2}/_{16}$$

$${}^{3rx}/_{2} = {}^{x^4}/_{4}$$

$$r = {}^{x}/_{6} = {}^{1}/_{6}\,AB$$

$$\boxed{\mathbf{r = {}^{1}/_{6}\,AB}}$$

19) In the fig. Two circle touches each other externally and third one is touches to the second circle externally. Point A is exterior point, AB, AC,AD and AE are tangents to the respective circles. If AB = 4cm, radii of circles are 3cm, 2cm and 5cm respectively. Find AQ AR and AP.



Solution

AB = AC = 4 …………. (by thm)

AC = AD = 4

AD = AE = 4

In Δ ABP, by PGT

$AP^2 = AB^2 + PB^2$

$AP^2 = 4^2 + 3^2$

$AP^2 = 16 + 9$

$AP^2 = 25$

$\therefore$ **AP = 5**

In right angle Δ ACQ, by PGT

$AQ^2 = AC^2 + QC^2$

$AQ^2 = 4^2 + 2^2$

$AQ^2 = 16 + 4 = 20$

$AQ = \sqrt{20}$

In right angle Δ ADR

$AR^2 = AD^2 + DR^2$

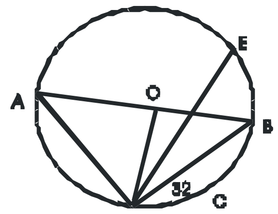
$AR^2 = 4^2 + 5^2$

$AR^2 = 16 + 25$

$AR^2 = 41$

$AR = \sqrt{41}$

20) In fig. AD is a diameter O is the centre of the circle. Diameter AD|| chord BC. $\angle CBD = 32^2$, then find



i) $\angle OBD$

ii) $\angle AOB$

iii) $\angle BED$

Solution : In big AD|| BC.

$\therefore$ $\angle ODB \cong \angle CBD$ ($\therefore$ alternate angles made by transversal)

$\therefore$ $\angle ODB = 32^0$

$\therefore$ $\angle OBD$ = ODB ……... ($\therefore$ opposite angles of congruent sides of iso Δ)

$\therefore$ $\angle$OBD = 32^0 ……….. (1)

In ΔBOD

$\angle BOD + \angle OBD + \angle ODB = 180^0$ …. ($\therefore$ sum of $\angle$ a Δ)

$\angle BOD + 32 + 32 = 180^0$

$\angle BOD = 180 – 64$

$\angle BOD = 116^0$

Now $\angle AOB + \angle BOD = 180^0$ ($\therefore$ linear pair)

$\angle AOB + 116^0 = 180^0$

$\angle AOB = 180 – 116^0$

$\angle AOB = 64^0$ ……………. (2)

m (arc BCD) = m$\angle$BOD ……. (central angle)

m (arc BCD) = 116^0

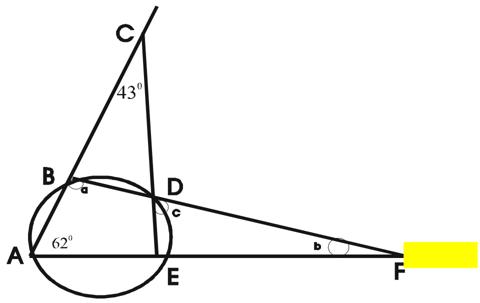
Now $\angle BED$ = ½ m (arc BCD) ………. ($\therefore$inscribed angle)

= ½ x 116^0

$\therefore$ $\angle BED$ = 68^0 ………………. (3)

i) $\angle OBD = 32^0$, ii) $\angle AOB = 64^0$, iii) $\angle BED = 58^0$

21) From figure find value of a,b,c



Solution :

In ΔAEC, m $\angle$A + m$\angle$E + m $\angle$C = 180^0 ………. (angle sum property)

62^0 + m $\angle$AEC + 43^0 = 180^0

m $\angle$AEC = 180 – 105

m $\angle$AEC = 75^0

Now, □ ABDE is cyclic quadrilateral

m$\angle$A = m$\angle$EDF = 62^0 … (Exterior angle of cyclic quadrilateral)

C = 62^0

In □ABDE,

m $\angle$B + m $\angle$AED = 180^0 …. (opp.angle of cyclic quadrilateral)

a + 75 = 180

a = 180 – 75

a = 105^0

In ΔABF

m $\angle$A + m$\angle$B + m$\angle$AFB = 180 …… (Angle sum property)

62 + 105^0 + b = 180^0

167 + b = 180

b = 180- 167

b = 13^0

A = 105^0, b = 13^0, C = 62^0

2. Define and Prove