

Continuity at a Point  
By: Lucy Solís



Name Hannia Larissa Gómez W. Group 401 Date 31/08/17

**A function is continuous at  $x = c$  if there is no interruption in the graph of  $f(x)$  at  $x = c$ . Continuity can be destroyed by a hole, an asymptote, a break or a point that is undefined**

**When the discontinuity is because of an undefined point the discontinuity is known as removable.**

Examples of discontinuities

<http://www.mathwarehouse.com/calculus/continuity/what-are-types-of-discontinuities.php>

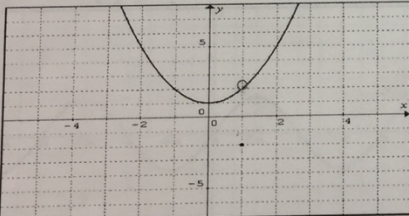
There are three conditions for a function to be continuous at  $x = c$ :

- 1)  $f(c)$  is defined
- 2)  $\lim_{x \rightarrow c} f(x)$  Exists
- 3)  $\lim_{x \rightarrow c} f(x) = f(c)$

I. With your teacher discuss the continuity at the given point

1)

$$y = \begin{cases} x^2 + 1 & \text{if } x \neq 1 \\ -2 & \text{if } x = 1 \end{cases}$$

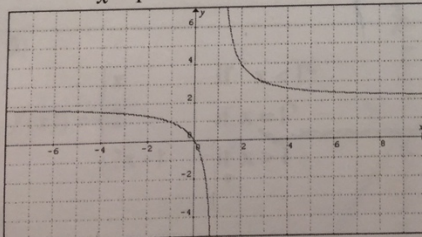


at  $x = 1$

Continuous except at  $x = 1$   
Discontinuous at  $x = 1$   
Removable.

2)

$$f(x) = \frac{2x}{x-1}$$

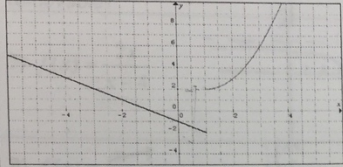


At  $x = 1$

Discontinuous at  $x = 1$   
Continuous except at  $x = 1$   
Unremovable.

3)

$$f(x) = \begin{cases} (x-1)^2 + 2 & \text{if } x \geq 1 \\ -x-1 & \text{if } x < 1 \end{cases}$$



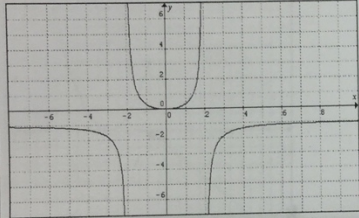
At  $x=1$

Discontinuous at  $x=1$   
and  $x=-2$

Non-removable.

4)

$$f(x) = \frac{-x^2}{x^2 - 4}$$

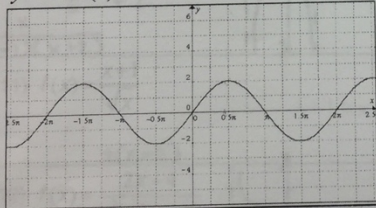


At  $x=2$

Discontinuous at  
 $x=2$   $x=-2$

Non-removable.

5)  $0.01$   
 $y = 2\sin(x)$

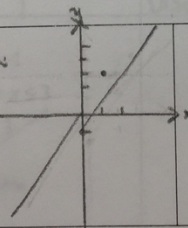


At  $x = 0.5\pi$

Continuous at

6)

$$f(x) = \begin{cases} x & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2x-1 & \text{if } x > 1 \end{cases}$$



$x=1$

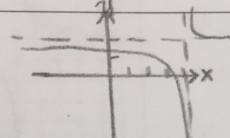
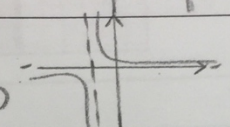
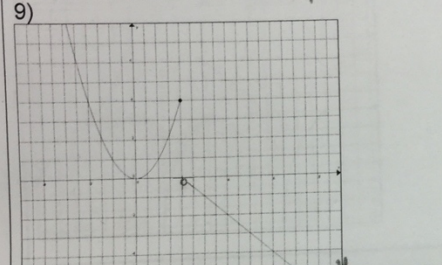
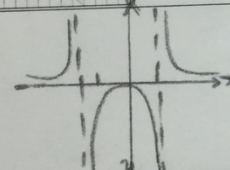
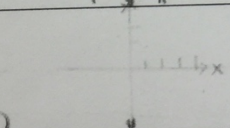
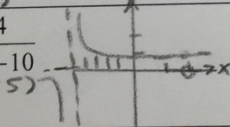
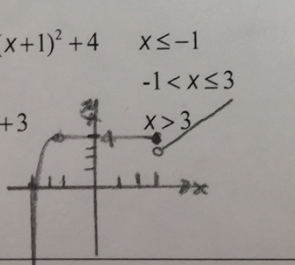
Discontinuous at  $x=1$

Non-removable.

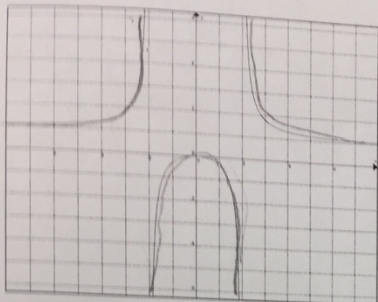
Function  
Continuous Discontinuous  
Removable Non-R.



II. Find the x-values (if any) at which f is not continuous. Which of the discontinuities are removable?

<p>7) <math>f(x) = \frac{2x}{x-4}</math></p> 	<p>Discontinuous at <math>x=4</math> Non-removable.</p>
<p>8) <math>f(x) = \frac{x+1}{x^2-1}</math> <math>\frac{1}{x+1} \cdot \frac{x+1}{(x-1)(x+1)}</math></p> 	<p>Discontinuous at <math>x=-1</math> Non-removable.</p>
<p>9)</p> 	<p><math>x=2</math> Discontinuous at <math>x=2</math> Non-Removable.</p>
<p>10)</p> <p><math>f(x) = \frac{x}{x^2+x+2}</math> <math>\frac{x}{(x+1)(x+2)}</math></p> 	<p>Continuous. and <math>x=0</math> Non-Removable</p>
<p>11) <math>f(x) = \frac{x+1}{\sqrt{x}}</math></p> 	<p>Discontinuous at <math>x=(-\infty, 0]</math> Non-removable.</p>
<p>12) <math>f(x) = \frac{2x-4}{x^2+3x-10}</math> <math>\frac{2(x-2)}{(x-2)(x+5)}</math></p> 	<p>Discontinuity at <math>x=-5</math> Non-removable. Discontinuity at <math>x=2</math> Removable</p>
<p>13)</p> <p><math>f(x) = \begin{cases} -(x+1)^2+4 &amp; x \leq -1 \\ 4 &amp; -1 &lt; x \leq 3 \\ x+3 &amp; x &gt; 3 \end{cases}</math></p> 	<p>Discontinuous at <math>x=3</math> Non-removable</p>

14)



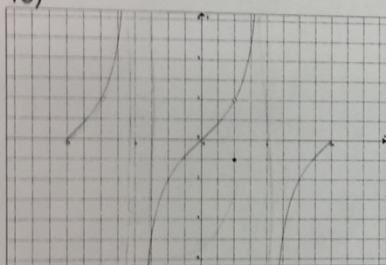
Discontinuity at  $x = -2$  and  $x = 2$

Non-removable

Discontinuous at  $x = 1$

Removable

15)



Discontinuity at  $x = \frac{\pi}{2}$  removable

Continuous  $\neq$  at  $x = -\pi$   
 $x = \pi$

Non-removable

$1/x = 1$

Discontinuous at  $x = 1$

Continuous except at  $x = 1$

Removable