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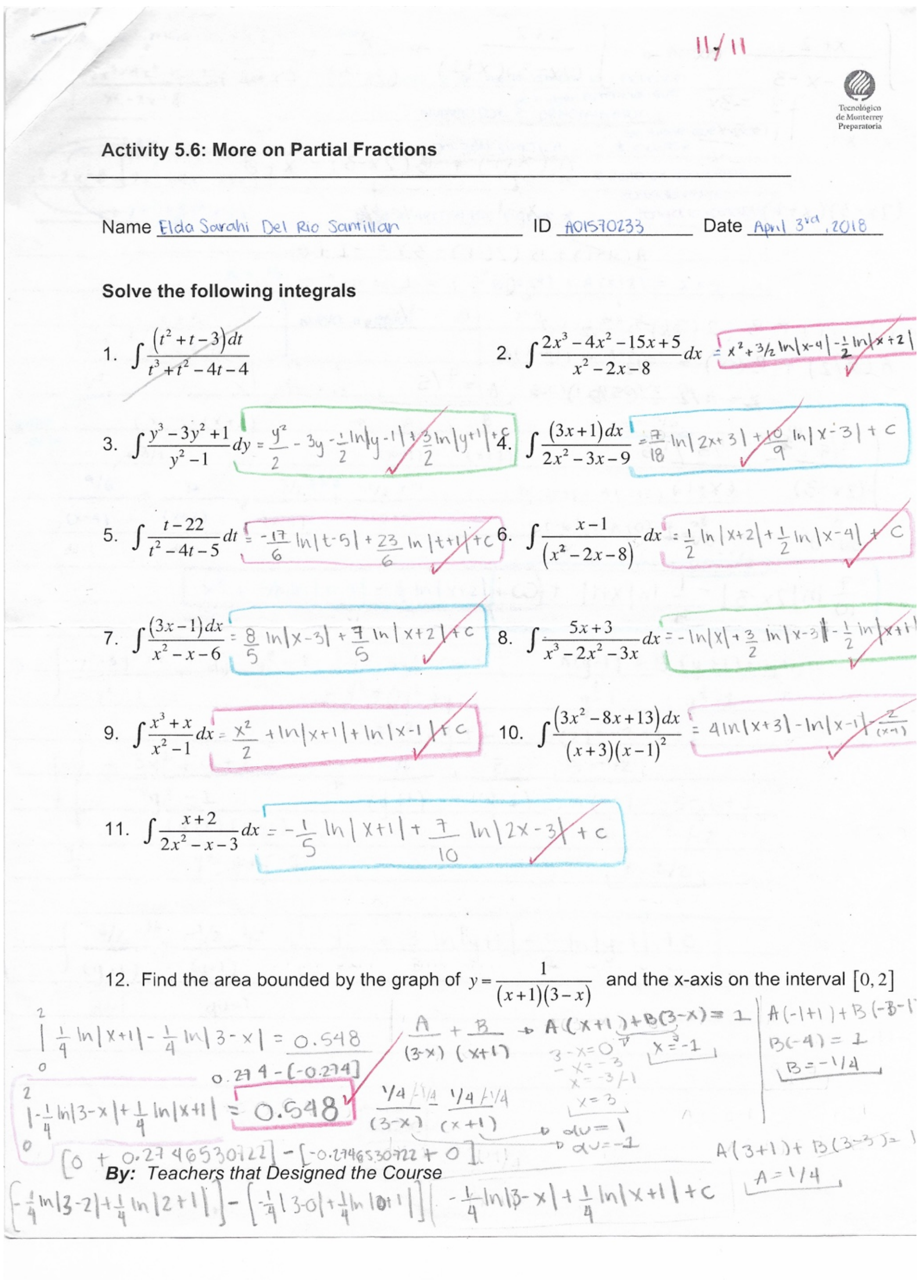
Final ePortfolio
Calculus II

11. Significant Activity 3 Third Partial

Activity 5.6: More on Partial Fractions

I have to confess it was a difficult one. I have been in shock when I saw the procedure at the first time. It is significant because it proves me that I can do whatever I purpose to, with effort and dedication. It took time to me to know how it works but at now I can solve any problem that involves or that need this procedure.



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Tecnológico de Monterrey Preparatoria

Activity 5.6: More on Partial Fractions

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Solve the following integrals

1. $\int \frac{(t^2+t-3)dt}{t^3+t^2-4t-4}$

2. $\int \frac{2x^3-4x^2-15x+5}{x^2-2x-8}dx = x^2 + 3/2 \ln|x-4| - \frac{1}{2}\ln|x+2|$

3. $\int \frac{y^3-3y^2+1}{y^2-1}dy = \frac{y^2}{2} - 3y - \frac{1}{2}\ln|y-1| + \frac{3}{2}\ln|y+1| + c$

4. $\int \frac{(3x+1)dx}{2x^2-3x-9} = \frac{7}{18}\ln|2x+3| + \frac{10}{9}\ln|x-3| + c$

5. $\int \frac{t-22}{t^2-4t-5}dt = -\frac{17}{6}\ln|t-5| + \frac{23}{6}\ln|t+1| + c$

6. $\int \frac{x-1}{(x^2-2x-8)}dx = \frac{1}{2}\ln|x+2| + \frac{1}{2}\ln|x-4| + C$

7. $\int \frac{(3x-1)dx}{x^2-x-6} = \frac{8}{5}\ln|x-3| + \frac{7}{5}\ln|x+2| + c$

8. $\int \frac{5x+3}{x^3-2x^2-3x}dx = -\ln|x| + \frac{3}{2}\ln|x-3| - \frac{1}{2}\ln|x+1|$

9. $\int \frac{x^3+x}{x^2-1}dx = \frac{x^2}{2} + \ln|x+1| + \ln|x-1| + c$

10. $\int \frac{(3x^2-8x+13)dx}{(x+3)(x-1)^2} = 4\ln|x+3| - \ln|x-1| - \frac{2}{(x-1)}$

11. $\int \frac{x+2}{2x^2-x-3}dx = -\frac{1}{5}\ln|x+1| + \frac{7}{10}\ln|2x-3| + c$

12. Find the area bounded by the graph of $y = \frac{1}{(x+1)(3-x)}$ and the x-axis on the interval $[0, 2]$

$\Big|_0^2 \frac{1}{4}\ln|x+1| - \frac{1}{4}\ln|3-x| = 0.548$

$0.274 - [-0.274]$

$\Big|_0^2 -\frac{1}{4}\ln|3-x| + \frac{1}{4}\ln|x+1| = \boxed{0.548}$

$[0 + 0.274653072 2] - [-0.2746530722 + 0]$

$\left[-\frac{1}{4}\ln|3-2| + \frac{1}{4}\ln|2+1|\right] - \left[-\frac{1}{4}|3-0| + \frac{1}{4}\ln|0+1|\right]$

$\frac{A}{(3-x)} + \frac{B}{(x+1)} \to A(x+1) + B(3-x) = 1$

$3-x=0$, $-x=-3$, $x=-3/-1$, $x=3$

$x=-1$

$A(-1+1) + B(-3-1)$

$B(-4) = 1$

$B = -1/4$

$A(3+1) + B(3-3) = 1$

$A = 1/4$

$\frac{1/4}{(3-x)} - \frac{1/4}{(x+1)}$

$du = 1$

$du = -1$

$-\frac{1}{4}\ln|3-x| + \frac{1}{4}\ln|x+1| + c$

***By:** Teachers that Designed the Course*

5.6: More on Partial Fractions

April 3rd, 2018

$$\int \frac{2x^3-4x^2-15x+5}{x^2-2x-8}\,dx$$

ahora si se puede dividir la fracción

$\boxed{\text{NUMERADOR} > \text{DENOMINADOR}}$ de mayor potencia que

$\boxed{\text{DIVISIÓN SINTÉTICA}}$ se tiene que hacer — * clave *

Reducir *Rellenar con ceros cuando no se tenga la variable*

$$\begin{array}{r|l} & 2x \quad \leftarrow \text{pte entera} \\ x^2-2x-8 & 2x^3-4x^2-15x+5 \\ & -2x^3+4x^2+16x \\ \hline & x+5 \quad \rightarrow \text{pte dec} \end{array}$$

* se invierten los signos *

$$\int 2x + \frac{x+5}{x^2-2x-8}$$ Nueva integral

$2x \rightarrow x^2$ integral de 2x

$\begin{array}{c|c} x & 4 = -4x \\ x & 2 = 2x \end{array}$

$(x-4)(x+2) \rightarrow \dfrac{A}{(x-4)} + \dfrac{B}{(x+2)}$

$B(x-4) + A(x+2) = x+5$

$x=4 \qquad x=-2$

$B(4-4) + A(4+2) = 4+5$

$B(0) + A(6) = 9$

$\underline{A = 9/6}$

$B(-2-4) + A(-2+2) = -2+5$

$B(-6) + A(0) = 3$

$B = 3/-6 \rightarrow \underline{B = -1/2}$

$$\frac{9/6}{(x-4)} + \frac{-1/2}{(x+2)} \rightarrow \frac{3/2 \div 3/2}{(x-4)} + \frac{-1/2 \div -1/2}{(x+2)}$$

$9/6 \rightarrow 3/2$

$du=1 \qquad du=1$

$$\boxed{x^2 + \tfrac{3}{2}\ln|x-4| - \tfrac{1}{2}\ln|x+2| + C}$$

3. $$\int \frac{y^3-3y^2+1}{y^2-1}\,dy$$

$$\begin{array}{r|l} & y \\ y^2-1 & y^3-3y^2+0y+1 \\ & -y^3+0y^2+y \\ \hline & -3y^2+y+1 \end{array}$$

$A(y-1) + B(y+1) = -3y^2+y+1$

$y=1 \qquad y=-1$

$B(2) = -3+1+1$

$\underline{B = -1/2}$

$A(-2) = -3-1+1$

$A = -3/-2$

$\underline{A = 3/2}$

$$\int y + \frac{3y^2+y+1}{y^2-1} \rightarrow \frac{A}{(y+1)} + \frac{B}{(y-1)}$$

$(y+1)(y-1)$

$y \rightarrow \dfrac{y^2}{2}$

$y^2 - \cancel{y} + \cancel{y} - 1$

$$\int \frac{3/2 \div 3/2}{(y+1)} + \frac{-1/2 \div -1/2}{(y-1)} = \boxed{\frac{y^2}{2} + \frac{3}{2}\ln|y+1| - \frac{1}{2}\ln|y-1| + C}$$

$du=1 \qquad du=1$

4. $\int \frac{(3x+1)}{2x^2-3x-9}\,dx \rightarrow \int \frac{(3x+1)}{(2x+3)(x-3)}\,dx \rightarrow \int \frac{A}{(2x+3)} + \frac{B}{(x-3)}$

$2x \mid 3 = 3x$
$x \mid -3 = -6x$
$-3x$

$A(x-3) + B(2x+3) = 3x+1$

$x = 3 \qquad x = -3/2$

$\rightarrow A(3-3) + B(2(3)+3) = 3(3)+1 \quad \rightarrow \quad B = 10/9 = 1.11$

$A(0) + B(9) = 10 \qquad B = 10/9$

$\rightarrow A(-3/2 - 3) + B(2(-3/2)+3) = 3(-3/2)+1$

$A(-9/2) + B(0) = -7/2 \rightarrow A = -7/2 \div (-9/2) \qquad A = 7/9$

$\frac{7}{18} \int \frac{7/9 \times 18/7}{(2x+3)} + \frac{10}{9} \int \frac{10/9 \div 10/9}{(x-3)} \rightarrow \boxed{\frac{7}{18} \ln|2x+3| + \frac{10}{9} \ln|x-3| + C}$

$du = 2 \qquad du = 1$

5. $\int \frac{t-22}{t^2-4t-5}\,dt \rightarrow \int \frac{t-22}{(t-5)(t+1)} \rightarrow \int \frac{A}{(t-5)} + \frac{B}{(t+1)}$

$t \mid -5 = -5t$
$t \mid 1 = t$
$-4t$

$A(t+1) + B(t-5) = t-22$

$t = -1 \qquad t = 5$

$A(-1+1) + B(-1-5) = -1-22 \qquad B = -23/-6 \rightarrow 3.83$

$A(0) + B(-6) = -23 \qquad B = 23/6$

$A(5+1) + B(5-5) = 5-22 \qquad A = -17/6 \rightarrow 2.83$

$A(6) + B(0) = -17 \qquad A = -17/6$

$\int \frac{-17/6}{(t-5)} + \frac{23/6}{(t+1)} \qquad \div -17/6 \qquad \div 23/6$

$du = 1 \qquad du = 1 \qquad \left(\frac{-17}{6}\right) + \left(\frac{23}{6}\right)\int$

$\boxed{-\frac{17}{6} \ln|t-5| + \frac{23}{6} \ln|t+1| + C}$

$$\int \frac{x-1}{(x^2-2x-8)}\,dx \rightarrow \int \frac{x-1}{(x-4)(x+2)}\,dx \rightarrow \int \frac{A}{(x-4)} + \frac{B}{(x+2)}$$

$x \quad -4 = -4x$
$x \quad 2 = 2x$
$-2x$

$A(x+2) + B(x-4) = x-1$

$x=-2 \qquad x=4$

$B(-2-4) = -2-1 \qquad A(4+2) = 4-1$

$B = -3/-6 \qquad \underline{B = 1/2} \qquad A = 3/6 \rightarrow \underline{A = 1/2}$

$$\int \frac{1/2}{(x-4)} + \frac{1/2}{(x+2)} \Rightarrow \boxed{\frac{1}{2}\ln|x-4| + \frac{1}{2}\ln|x+2| + C}$$

7. $$\int \frac{(3x-1)}{x^2-x-6}\,dx \rightarrow \int \frac{(3x-1)}{(x-3)(x+2)}\,dx \rightarrow \int \frac{A}{(x-3)} + \frac{B}{(x+2)} \rightarrow A(x+2)+B(x-3) = 3x-1$$

$x=-2 \qquad x=3$

$x \quad -3 = -3x$
$x \quad 2 = 2x$
$-x$

$B(-2-3) = 3(-2)-1 \rightarrow 1.4 \qquad A(3+2) = 3(3)-1$

$B = -7/-5 = \underline{7/5 = B} \qquad \underline{A = 8/5} \rightarrow 1.6$

$$\int \frac{8/5}{(x-3)} + \frac{7/5}{(x+2)} \rightarrow \boxed{\frac{8}{5}\ln|x-3| + \frac{7}{5}\ln|x+2| + C}$$

$du=1 \qquad du=1$

8 $$\int \frac{5x+3}{x^3-2x^2-3x}\,dx \checkmark \rightarrow \int \frac{5x+3}{x(x-3)(x+1)} \rightarrow \int \frac{A}{x} + \frac{B}{(x-3)} + \frac{C}{(x+1)}$$

$x(x-3)(x+1)$ * $A(x-3)(x+1) + B(x)(x+1) + C(x)(x-3) = 5x+3$

$x^2-3x\,(x+1)$ OJO AQUÍ = se multiplica

No confundirse con el procedimiento que se hace cuando el num > denominador

$x^3 + x^2 - 3x^2 - 3x$

El que es $\frac{\quad}{(\;)(\;)^2} = \frac{A}{x} + \frac{B}{(\;)} + \frac{C}{(\;)^2}$

$\underline{x^3 - 2x^2 - 3x} \checkmark$

$A(x-3)(x+1) + B(x)(x+1) + C(x)(x-3) = 5x+3$

$x=3, \; x=-1 \qquad x=0 \quad x=-1 \qquad x=0 \quad x=3$

$A\underbrace{(3-3)}_{0}\underbrace{(3+1)}_{4} + B(3)(3+1) + C(3)(3-3) = 5(3)+3$

$A(0) + B(12) + C(0) = 18 \rightarrow B = 18/12 \rightarrow \underline{B = 3/2}$

$A\underbrace{(-1-3)}_{-4}(-1+1) + B(-1)(-1+1) + C(-1)(-1-3) = 5(-1)+3$

$A(0) + B(0) + C(4) = -2 \rightarrow C = -2/4 \rightarrow \underline{C = -1/2}$

$A(x-3)(x+1) + 3/2(x)(x+1) - 1/2(x)(x-3) = 5x+3$

$3/2x\,(x+1) \qquad -1/2x\,(x-3)$

$Ax^2 - 2x(A) - 3(A) + 3/2x^2 + 3/2x - 1/2x^2 + 3/2x = 5x+3$

$\underbrace{x^2+x-3x-3}$ $\hookrightarrow x^2(A+3/2-1/2) + x(-2A+3) - 3A = 5x+3$

$A(x^2-2x-3)$

$-2A+3 = 5 \qquad -3A = 3$

$-2A = 5-3 \qquad A = 3/-3$

$-2A = 2 \qquad \underline{A = -1}$

$A = 2/-2$

$\underline{A = -1}$

$$\int \frac{-1}{x} = -1 + \frac{3/2}{(x-3)} - \frac{1/2}{(x+1)} = -1/2$$

$du = 1 \quad du = 1 \quad du = 1$

$$8 \quad \boxed{-\ln|x| + \frac{3}{2}\ln|x-3| + \frac{1}{2}\ln|x+1| + C}$$

⑨ $\int \frac{x^3 + x}{x^2 - 1}\, dx$

$\boxed{x}$ PTE ENTERA

$$x^2 - 1 \;\big)\; x^3 + 0 + x$$
$$\cancel{-x^3} + 0 + x$$
$$+2x$$

$$\int x + \frac{2x}{x^2-1} \rightarrow (x+1)(x-1) \rightarrow \int \frac{A}{(x+1)} + \frac{B}{(x-1)}$$

$x^2 \cancel{+x+x} - 1$

$x^2 - 1$ ✓

$\frac{x^2}{2}$

Nueva integral

$$A(x-1) + B(x+1) = 2x$$

$x = 1 \qquad x = -1$

$$A(1-1) + B(1+1) = 2(1)$$
$$A(0) + B(2) = 2 \rightarrow B(2) = 2 \quad B = 2/2 \quad \boxed{B =}$$

$$A(-1-1) + B(-1+1) = 2(-1) \qquad A(-2) = -2$$
$$A(-2) + B(0) = -2 \rightarrow A = -2/-2 \quad \boxed{A = 1}$$

$$\frac{1}{(x+1)} + \frac{1}{(x-1)} \rightarrow = \boxed{\frac{x^2}{2} + \ln|x+1| + \ln(x-1) + C}$$

$du = 1 \qquad du = 1$

⑩ $\int \frac{(3x^2 - 8x + 13)}{(x+3)(x-1)^2}\, dx = \int \frac{3x^2 - 8x + 13}{(x+3)(x-1)^2} + \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$

$x^2 - 2x + 1$

$$A\cancel{(x+3)}(x-1)^2\cancel{(x+3)} + B(x+3)\cancel{(x-1)^2}\cancel{(x-1)} + C(x+3)\cancel{(x-1)^2}\cancel{(x-1)^2}$$

$$\underbrace{A(x-1)^2}_{\text{desarrollarlo}} + B(x+3)(x-1) + C(x+3) = 3x^2 - 8x + 13$$

$$4(x-1)^2 + B(x+3)(x-1) + 2(x+3) = 3x^2 - 8x + 13$$

$x^2 - x + 3x - 3$

$+2x$

$$4x^2 - 8x + 4 + Bx^2 + 2x(B) - 3(B) + 2x + 6$$

$$x^2(4 + B) + x(-6 + 2B) - 3(B) + 10 =$$

$4 + B = 3 \qquad -6 + 2B = -8 \qquad B = \frac{-2}{2}$

$B = 3 - 4 \rightarrow B = -1 \qquad 2B = -8 + 6 \qquad \boxed{B = -1}$

$$\int \frac{4/4}{(x+3)} + \frac{1/1}{(x-1)} + \frac{2}{(x-1)^2} \quad 2(x-$$

$du = 1 \quad du = 1 \quad du = 1$

$$\boxed{4\ln|x+3| - \ln|x-1| - \frac{2}{(x-1)} +}$$

$$\rightarrow A(x-1)^2 + B(x+3)(x-1) + C(x+3) = 3x^2 - 8x + 13$$

$x = 1, \quad x = -3, \quad x = 1 \quad x = -3$

$$A(1-1) + B(1+3)(1-1) + C(1+3) = 3(1)^2 - 8(1) + 13$$
$$C(4) = 3 - 8 + 13$$
$$C(4) = 8 \rightarrow C = 8/4 \quad \boxed{C = 2}$$

$$A(-3-1)^2 + B(-3+3)(-3-1)$$
$$+ C(-3+3) = 3(-3)^2 - 8(-3)$$
$$+ 13$$
$$A(16) = 27 + 24 + 13$$
$$A = 64/16 \quad \boxed{A = 4}$$

(11) $\int \frac{x+2}{2x^2-x-3}\,dx \to \int \frac{x+2}{(2x-3)(x+1)} \to \frac{A}{(2x-3)} + \frac{B}{(x+1)}$

$2x \mid +3 = -3x$

$x \mid 1 = 2x$

$-x$

$A(x+1) + B(2x-3) = x+2$

$x=-1 \qquad x = 3/2$

$(2x-3)(x+1)$

$\cancel{A(-1+1)} + B(2(-1)-3) = -1+2$

$B(-5) = 1$

$B = -1/5$

$A(3/2+1) + B(2(3/2)-3) = 3/2 + 2$

$A(5/2) + B(0) = 7/2$

$A = 7/2 \div (5/2) \to A = 7/5$

$\int \frac{7/5 \cdot \frac{10}{4}}{(2x-3)} - \frac{1/5 \,/\, 1/5}{(x+1)}$

$du = 2 \qquad du = 1$

$$\boxed{\frac{7}{10}\ln|2x-3| - \frac{1}{5}\ln|x+1| + C}$$

12. Conclusion

It wasn’t felt it as a class because I really enjoy learning. I have to confess that at the beginning of the semester I was scared because I was with any friend at the classroom, but there I met some guys who are kind and a quiet funny.

The activities I made, the quizzes I answered, the projects I did, and many others activities really contribute to my critical thinking and I know that I’m able to analyze and solve calculus problems. I am on my way to an engineering, and the things I learnt here are definitely worth it.

I noticed that I got good grades in the activities, but in many quizzes I couldn’t realized why I got not too good grades: perseverance and practice are the keys to have success, that’s my opinion.

In conclusion, I enjoy being part of this class, the teacher is awesome teaching. I didn’t have the chance of being her student at the past, but this semester she had an important role because she was focused in let us know how does Calculus II works.