Activity - Change of Basis – Defining Coordinate Grids

Goals:

- Make the visual connection between the basis vectors and the coordinate grid they define.
- Express any vector given in the Standard Cartesian Coordinate system (defined by the basis vectors $\mathbf{e_1} = \begin{bmatrix}1\\0\end{bmatrix}$ and $\mathbf{e_2} = \begin{bmatrix}0\\1\end{bmatrix}$) as a vector in any **uv**-grid system which is defined by the basis vectors **u** and **v.**
- Understand how the matrix constructed with columns of basis vectors can be used to translate points from the **xy**-representation to a **uv**-representation of the plane.

Representing same vector and location with different basis vectors

1. In the **xy**-grid (Standard coordinate system), **P** = (4, 4) is the terminal point of the vector $\overrightarrow{\mathbf{OP}} = \begin{bmatrix}4\\4\end{bmatrix}$ in standard position. Using precise notation $\overrightarrow{\mathbf{OP}} = \begin{bmatrix}4\\4\end{bmatrix}_{e_1,e_2}$ means $\overrightarrow{\mathbf{OP}} = \begin{bmatrix}4\\4\end{bmatrix}$ is defined in terms of the basis vectors $\mathbf{e_1} = \begin{bmatrix}1\\0\end{bmatrix}$ and $\mathbf{e_2} = \begin{bmatrix}0\\1\end{bmatrix}$.

2. Use a **uv**-grid with basis $\mathbf{u} = \begin{bmatrix}3\\1\end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix}-2\\2\end{bmatrix}$ to determine the multiples of the vectors **u** and **v** that define a linear combination of $\overrightarrow{\mathbf{OP}}$.

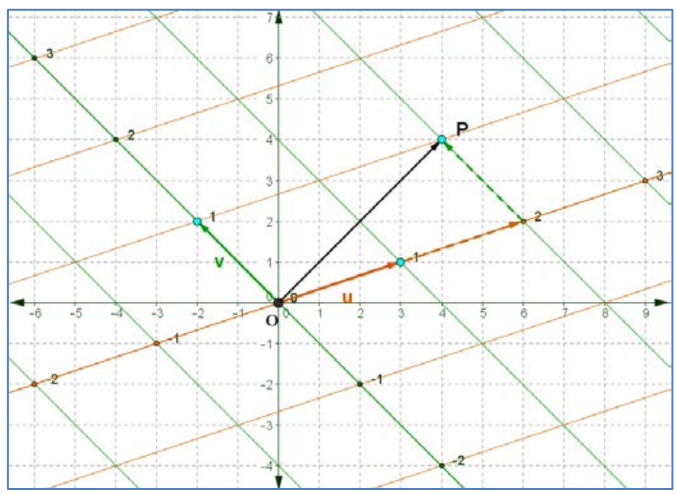
Use the **uv**-grid in the graph to answer the following

If c_1 is the multiple of **u,** then $c_1 =$ __________

If c_2 is the multiple of **v,** then $c_2 =$ __________

Using your observation of the values of $c_1$and c_2, verify that the following equation is true.

$$c_1\mathbf{u} + c_2\mathbf{v} = \begin{bmatrix}4\\4\end{bmatrix}$$



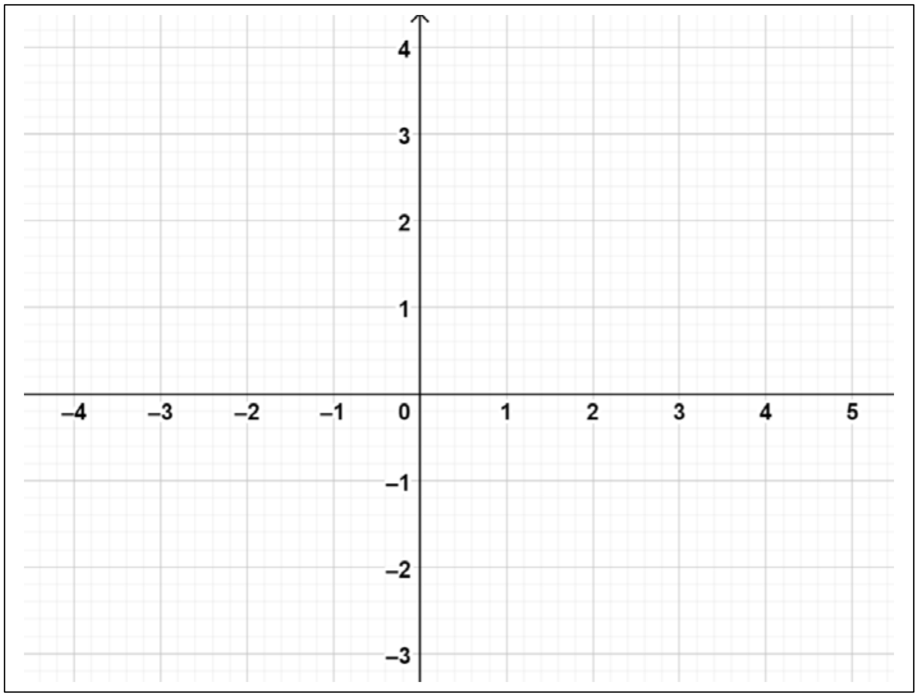
Algebraically, determine c_1 and c_2, from:

$$c_1\mathbf{u} + c_2\mathbf{v} = \begin{bmatrix}4\\4\end{bmatrix}$$

In the **uv**-grid, with basis $\mathbf{u} = \begin{bmatrix}3\\1\end{bmatrix}$, $\mathbf{v} = \begin{bmatrix}-2\\2\end{bmatrix}$, state

$\overrightarrow{\mathbf{OP}}_{\mathbf{uv}} =$ _________ with terminal point $\mathbf{P}_{\mathbf{u,v}}$ = ______

3. Plot and label point $\mathbf{B} = (\mathbf{3}, -\mathbf{2})$ in the **xy-**grid system. State $\overrightarrow{\mathbf{OB}} =$ __________ **S**ketch, and label it.



4. Sketch the **uv-**grid system with basis vectors $\mathbf{u} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$.

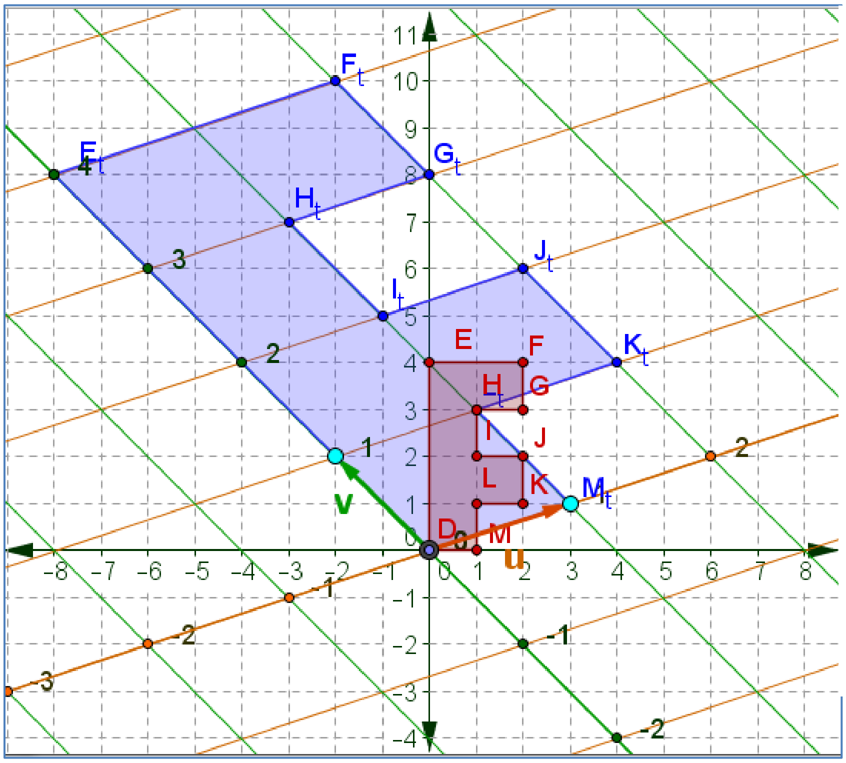
   Algebraically, find $\mathbf{B}_{\mathbf{u},\mathbf{v}}$ using $\mathbf{B}_{\mathbf{u},\mathbf{v}} =$ ___________ Plot and label this **uv**-point.

   $$c_1\mathbf{u} + c_2\mathbf{v} = \overrightarrow{\mathbf{OB}}.$$

   Express $\overrightarrow{\mathbf{OB}}$ in terms of components in the **uv-**grid $\overrightarrow{\mathbf{OB}}_{\mathbf{uv}} =$ __________

5. In the **xy**-Cartesian System point **F** = (2, 4). Draw and label the vector $\overrightarrow{\mathbf{OF}}$ in standard position.

This means $\overrightarrow{\mathbf{OF}} = \begin{bmatrix} \quad \end{bmatrix}_{\boldsymbol{e_1},\boldsymbol{e_2}}$



Find the translated point $\mathbf{F_t}$ on the slanted polygon corresponding to the point **F**.

$\mathbf{F_t}$ = ____________

Draw and label the vector $\overrightarrow{\mathbf{OF_t}}$ in standard position.

Write $\overrightarrow{\mathbf{OF_t}} = \begin{bmatrix} \quad \end{bmatrix}_{\boldsymbol{e_1},\boldsymbol{e_2}}$

(in the Standard Cartesian system)

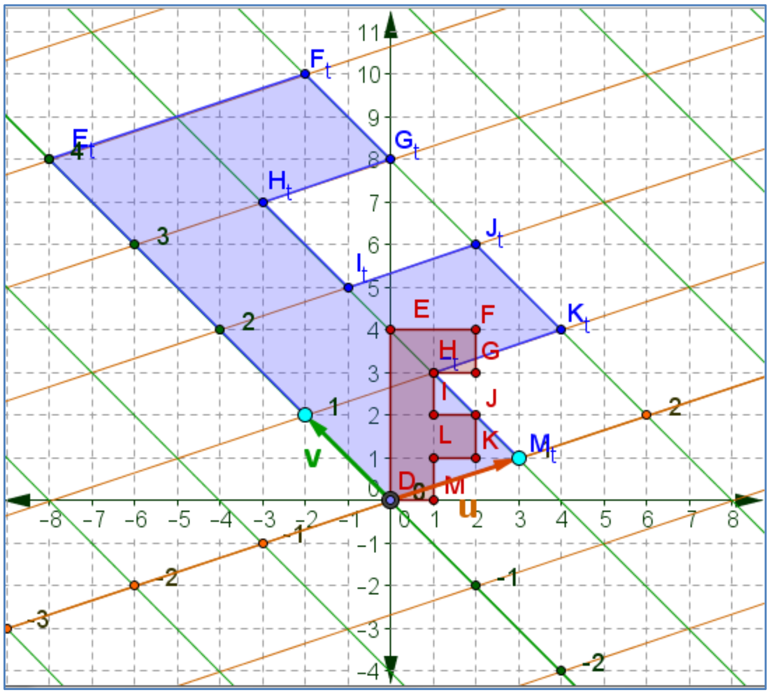
Write $\overrightarrow{\mathbf{OF_t}} = \begin{bmatrix} \quad \end{bmatrix}_{\mathbf{u},\mathbf{v}}$ (in **uv**-grid system with basis vectors $\mathbf{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$).

The matrix $\mathbf{M} = \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$ has columns consisting of **u** and **v.** <u>Note</u>: vectors **u**, **v**, $\overrightarrow{\mathbf{OF}}$ and $\overrightarrow{\mathbf{OF_t}}$ are written using the Standard Coordinate System. Perform matrix multiplication to show the following is true.

$$\mathbf{M} \cdot \overrightarrow{\mathbf{OF}} = \overrightarrow{\mathbf{OF_t}} .$$

In the **xy**-Cartesian System point **E** = _______ Draw and label the vector $\overrightarrow{\mathbf{OE}}$ in standard position.

This means $\overrightarrow{\mathbf{OE}} = \begin{bmatrix} \quad \end{bmatrix}_{e_1,e_2}$



Find the translated point $\mathbf{E_t}$ on the slanted polygon corresponding to the point **E**.

$\mathbf{E_t}$ = ____________

Draw and label the vector $\overrightarrow{\mathbf{OE_t}}$ in standard position.

Write $\overrightarrow{\mathbf{OE_t}} = \begin{bmatrix} \quad \end{bmatrix}_{e_1,e_2}$ (in the Standard Cartesian system).

Write $\overrightarrow{\mathbf{OE_t}} = \begin{bmatrix} \quad \end{bmatrix}_{\mathbf{u},\mathbf{v}}$ (in **uv**-grid system with basis vectors $\mathbf{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$)

The matrix $\mathbf{M} = \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$ has columns consisting of **u** and **v.** <u>Note:</u> vectors **u**, **v,** $\overrightarrow{\mathbf{OE}}$ and $\overrightarrow{\mathbf{OE_t}}$ are written using the Standard Cartesian Coordinate System. Perform matrix multiplication to show the following is true.

$$\mathbf{M} \cdot \overrightarrow{\mathbf{OE}} = \overrightarrow{\mathbf{OE_t}} .$$