

Continuity and Discontinuity

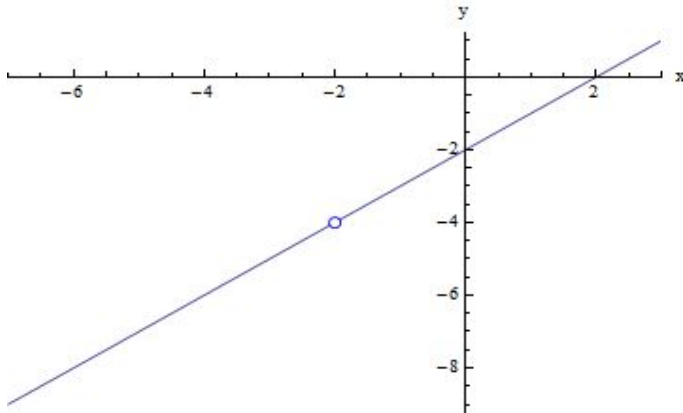
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What's a “discontinuity”?

A discontinuity is defined as a point in which the graph cannot go any further and must “jump”. This happens whenever there is a value that is undefined.

Discontinuities are removable only if the limit towards that point in the graph exists from both sides. Usually it can be fixed by explicitly stating what value covers that hole or factoring it out.



Think of it as a point which inherently does not exist in the realm of the function.

It tends to come in one of four types which differentiate themselves by the limits.

Types of Discontinuities

To illustrate the different types I will introduce you to the **Discontinuity team!**

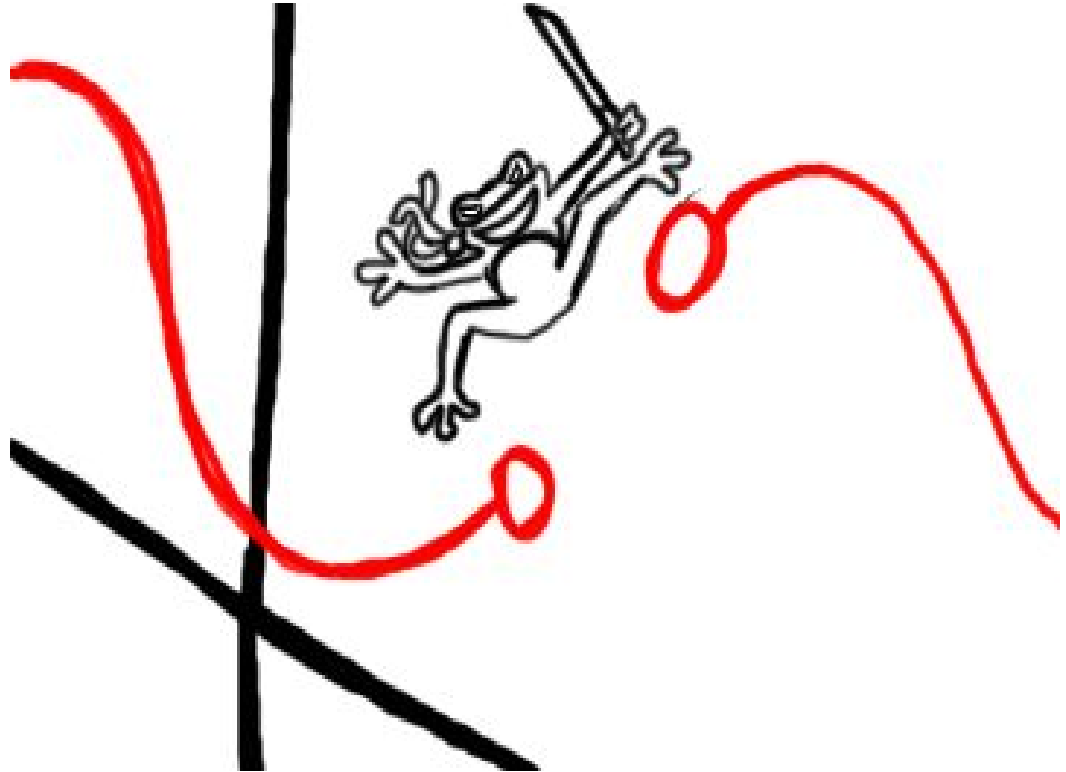


Jump Discontinuity

The Jump Ninja Frog A.K.A Jump Discontinuity

The Jump discontinuity occurs when x approaches from both sides and the limits have different values, creating the illusion of the graph seemingly jumping from one point to another.

(The jump frog rides the graphs until it detects a point which needs to be severed. It cuts the graph and **jumps** to the next point)



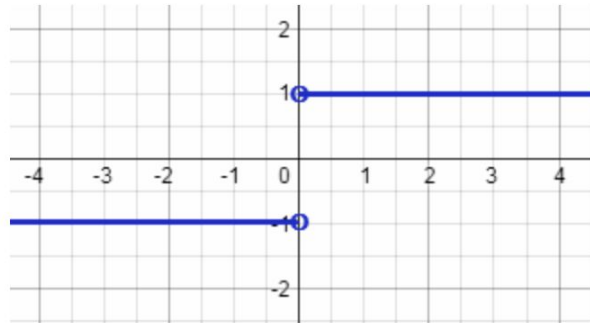
Example

Example 1 : Let us suppose a function f is given in the following way -

$$f(x) = \frac{x}{|x|} = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

Solution: The value of this function is not defined at $x = 0$. For all values of x which are bigger than 1, the function gives 1. On the other hand, for all values of x that are smaller than 1, the function returns -1.

Hence the graph of the function looks as below:



We can easily say that it is a joint discontinuity by simply looking at the graph.

Infinite Discontinuity:

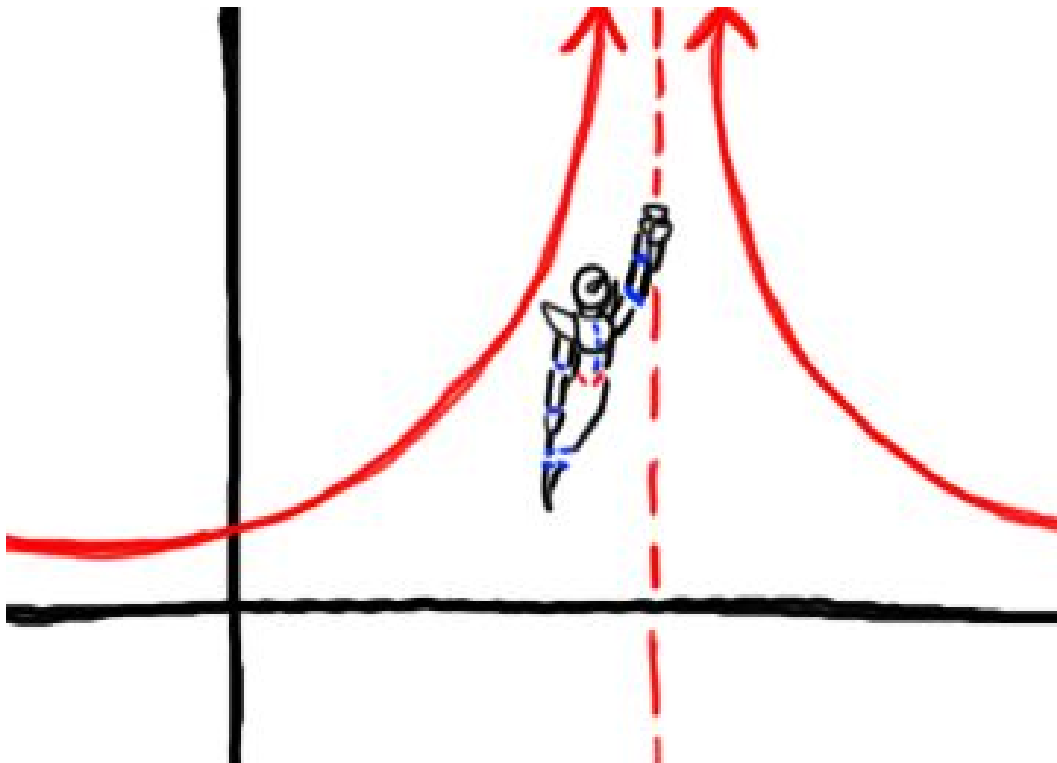
THE INFINITE WARRIOR

A.K.A

INFINITE DISCONTINUITY

An infinite discontinuity occurs when the graph's limit is either infinite (positive or negative) or when both parts of the graph stretch towards different directions infinitely (meaning the limit does not exist.)

(The infinite warrior comes from the depths of the negative infinity to trace the vertical and horizontal asymptotes with his blade)



Example

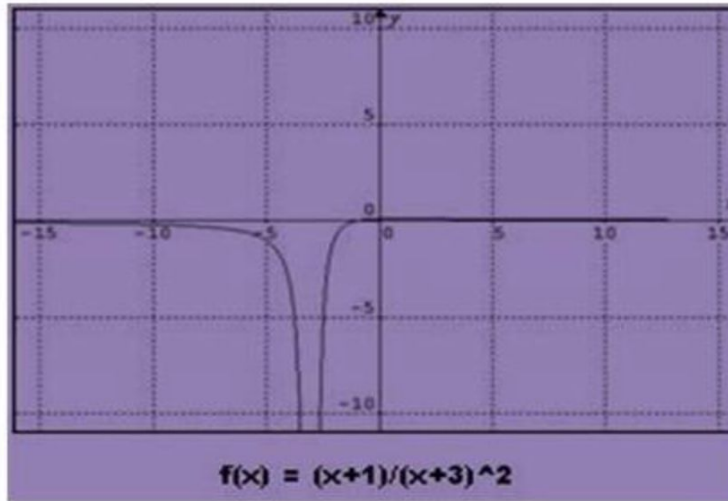
Question 1: $\lim_{x \rightarrow a} \frac{(x+1)}{(x+3)^2}$

Solution:

$$\lim_{x \rightarrow a} \frac{(x+1)}{(x+3)^2} \text{ By plugging-in } x = -3$$

$$= \frac{(-3+1)}{(-3+3)^2}$$

$$= \infty$$

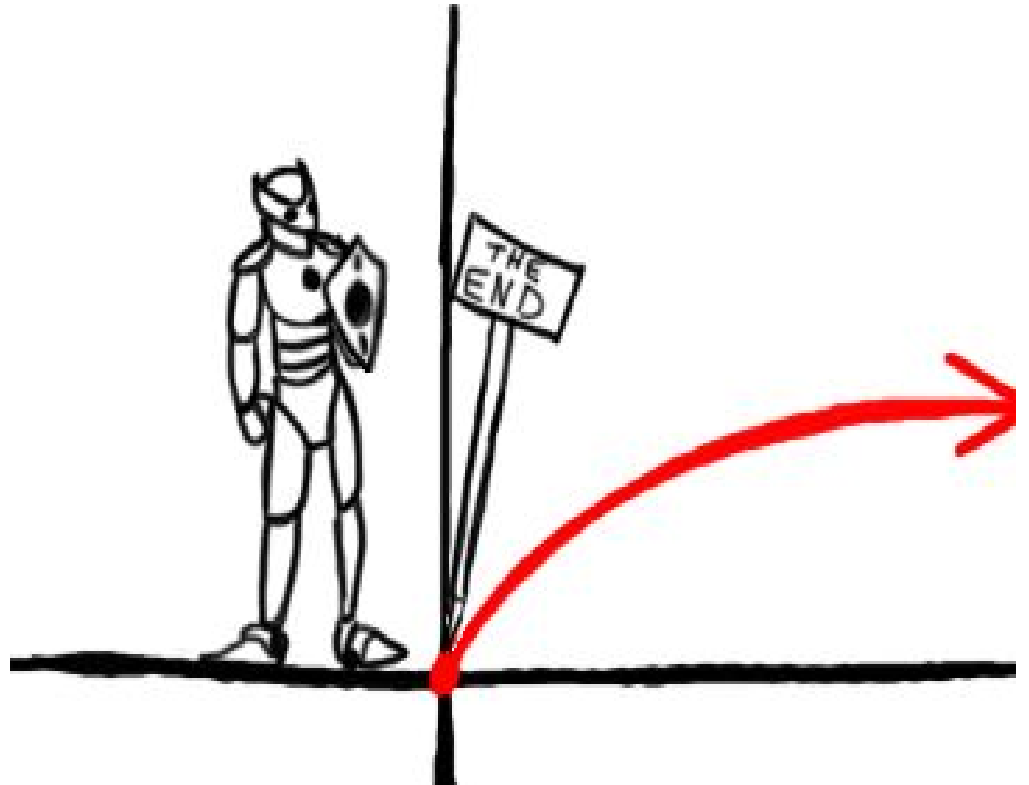


Endpoint discontinuity

The End Knight **A.K.A** **Endpoint discontinuity**

This one occurs whenever the graph only comes from one side and ends at a point of origin. Meaning that the limit from both sides does not exist, because the graph stretches to infinity from one side only.

(The End Knight keeps watch of the endpoint so no further values can be plotted on the other side. Keeping trespassing values at bay.)

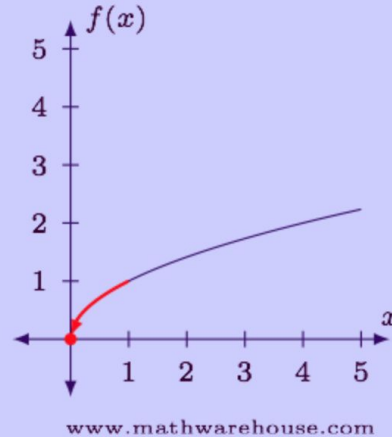


Example

Endpoint Discontinuities

When a function is defined on an interval with a closed endpoint, the limit cannot exist at that endpoint. This is because the limit has to examine the function values as x approaches from both sides.

For example, consider finding $\lim_{x \rightarrow 0} \sqrt{x}$ (see the graph below).



Note that $x = 0$ is the left-endpoint of the function's domain: $[0, \infty)$, and the function is technically not continuous there because the limit doesn't exist (because x can't approach from both sides).

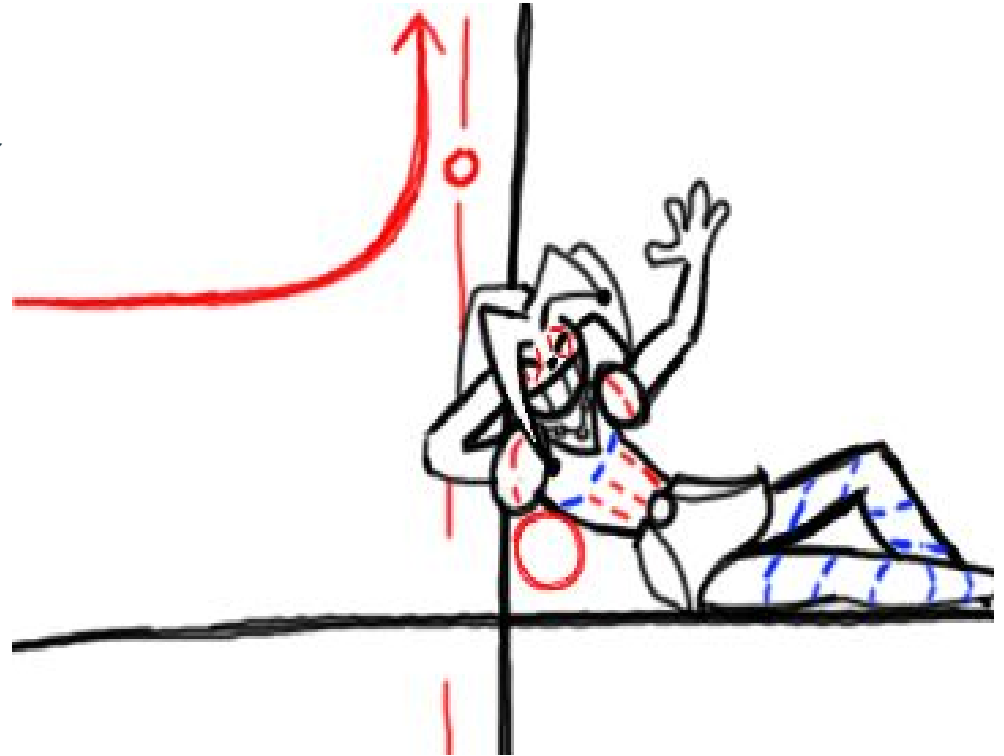
We should note that the function is right-hand continuous at $x = 0$ which is why we don't see any jumps, or holes at the endpoint.

Mixed Discontinuities:

THE MIXED JESTER A.K.A MIXED DISCONTINUITY

The mixed discontinuity happens when there are more than one type of discontinuity inside of the function's graph. (Example: an infinite one mixed with a hole.)

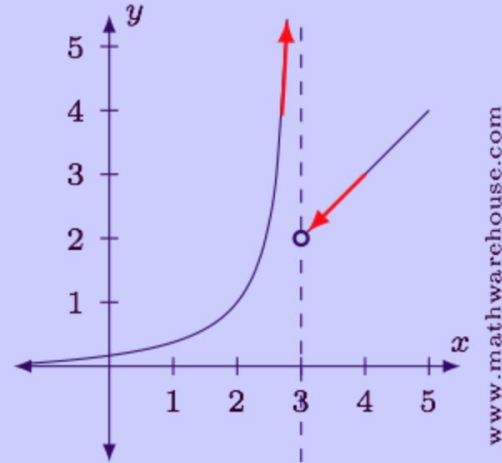
(The Mixed Jester is the god of discontinuity, he likes to play pranks on calculus students and litter their graphs with several discontinuities just for his own amusement)



Example

Mixed Discontinuities

Consider the graph shown below.



A Mixed Discontinuity

The function is obviously discontinuous at $x = 3$. From the left, the function has an infinite discontinuity, but from the right, the discontinuity is removable. Since there is more than one reason why the discontinuity exists, we say this is a *mixed* discontinuity

References:

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