

# Continuity at a Point

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**A function is continuous at  $x = c$  if there is no interruption in the graph of  $f(x)$  at  $x = c$ . Continuity can be destroyed by a hole, an asymptote, a break or a point that is undefined**

**When the discontinuity is because of an undefined point the discontinuity is known as removable.**

Examples of discontinuities

<http://www.mathwarehouse.com/calculus/continuity/what-are-types-of-discontinuities.php>

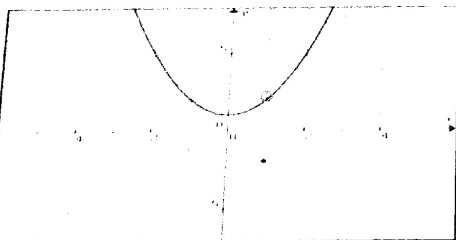
There are three conditions for a function to be continuous at  $x = c$ .

- 1)  $f(c)$  is defined
- 2)  $\lim_{x \rightarrow c} f(x)$  Exists
- 3)  $\lim_{x \rightarrow c} f(x) = f(c)$

I. With your teacher discuss the continuity at the given point

1)

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \neq 1 \\ -2 & \text{if } x = 1 \end{cases}$$

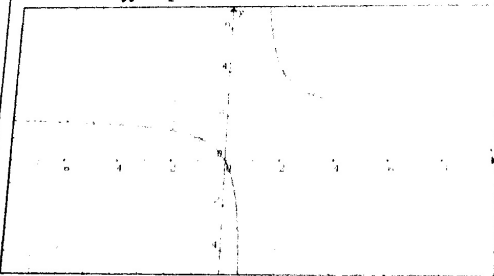


at  $x = 1$

$f(1) = -2$  is defined  
 $\lim_{x \rightarrow 1} f(x) = 2$  exists  
 $\lim_{x \rightarrow 1} f(x) \neq f(1)$

2)

$$f(x) = \frac{2x}{x-1}$$



At  $x = 1$

$f(1)$  is not defined  
 $\lim_{x \rightarrow 1} f(x)$  does not exist  
 $\lim_{x \rightarrow 1} f(x) \neq f(1)$

3)  

$$f(x) = \begin{cases} (x-1)^2 & \text{if } x > 1 \\ x-1 & \text{if } x < 1 \end{cases}$$

At  $x = 1$

4)

$f(x) = \frac{x}{x+4}$

At  $x = 2$

5)

$y = 2\sin(x)$

At  $x = 0.5\pi$

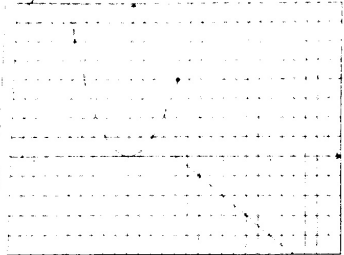
6)

$$f(x) = \begin{cases} x & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2x-1 & \text{if } x > 1 \end{cases}$$

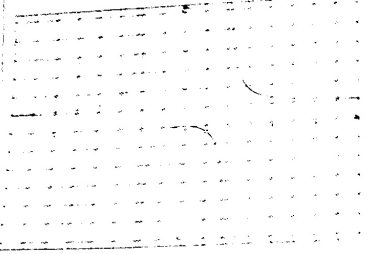
$x = 1$

*Handwritten notes:*  
 At  $x = 1$ ,  $f(x) = 2$   
 The function is continuous at  $x = 1$   
 because  $\lim_{x \rightarrow 1^-} f(x) = 1$ ,  $f(1) = 2$ , and  $\lim_{x \rightarrow 1^+} f(x) = 1$ .

II. Find the x-values (if any) at which f is not continuous. Which of the discontinuities are removable?

<p>7) <math>f(x) = \frac{2x}{x-4}</math></p>	<p><math>x=4</math> removable</p>
<p>8) <math>f(x) = \frac{x-1}{x^2-1}</math></p>	<p><math>x=1</math> removable <math>x=-1</math> not removable</p>
<p>9)</p> 	<p><math>x=1</math> not removable <math>x=2</math> removable <math>x=3</math> not removable</p>
<p>10) <math>f(x) = \frac{x}{x^2+x+2}</math></p>	<p><math>x=-2</math> not removable <math>x=-1</math> not removable</p>
<p>11) <math>f(x) = \frac{x+1}{\sqrt{x}}</math></p>	<p><math>x=0</math> not removable</p>
<p>12) <math>f(x) = \frac{2x-4}{x^2+3x-10}</math></p>	<p><math>x=2</math> removable <math>x=-5</math> not removable</p>
<p>13) <math>f(x) = \begin{cases} -(x+1)^2+4 &amp; x \leq -1 \\ 4 &amp; -1 &lt; x \leq 3 \\ x+3 &amp; x &gt; 3 \end{cases}</math></p>	<p><math>x=3</math> removable <math>x=-1</math> not removable</p>

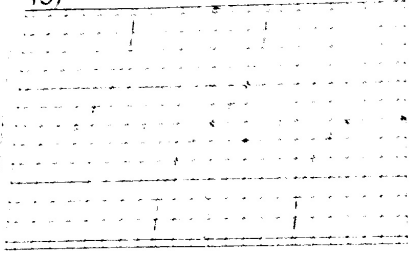
14)



- ✓ continuous on  $(-\infty, 1)$
- ✓ continuous on  $(1, \infty)$

discontinuous at  
 $x=1$ ,  
removable

15)



- ✓ continuous on  $(-\infty, 1)$
- ✓ continuous on  $(1, \infty)$
- ✓ discontinuous at  $x=1$
- ✓ removable discontinuity
- ✓  $\lim_{x \rightarrow 1^-} f(x) = 1$