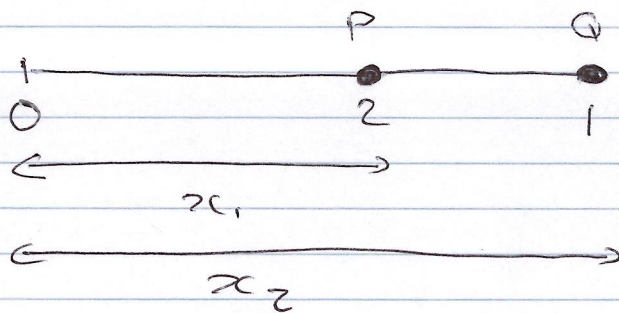


Step IV 2004

Q 11)



$$t=0 \quad x_1 = 0 \quad x_2 = 1 \quad z = 1,$$
$$\dot{x}_1 = 1 \quad \dot{x}_2 = 0 \quad \dot{z} = -1$$

Apply F=ma for P & Q.

$$2 \ddot{x}_1 = -\frac{2}{z^3}$$

$$1 \times \ddot{x}_2 = \frac{2}{z^3}$$

$$\Rightarrow \ddot{x}_1 = -\frac{1}{z^3}$$

$$\ddot{z} = \ddot{x}_2 - \ddot{x}_1 = \frac{3}{z^3} \Rightarrow \frac{d^2 z}{dt^2} = \frac{3}{z^3}$$

using $\frac{d^2 z}{dt^2} = v \frac{dv}{dz} \Rightarrow v \frac{dv}{dz} = 3z^{-3}$

$$\Rightarrow \frac{1}{2} v^2 = -\frac{3}{2} z^{-2} + c$$

as $\dot{z} = v$ $t=0 \Rightarrow \frac{1}{2} = -\frac{3}{2} + c$

$$\Rightarrow \underline{\underline{c=2}}$$

$$\Rightarrow \frac{1}{2} v^2 = -\frac{3}{2} z^{-2} + 2 \Rightarrow v^2 = -3z^{-2} + 4$$

$$\Rightarrow \frac{1}{-3z^{-2} + 4} \left(\frac{dz}{dt} \right)^2 = 1$$

$$\Rightarrow \frac{z^2}{4z^2-3} \left(\frac{dz}{dt}\right)^2 = 1.$$

$$\Rightarrow \frac{z}{\sqrt{4z^2-3}} \frac{dz}{dt} = \pm 1.$$

$t=0 \quad z=1 \quad \& \quad \dot{z}=-1 \Rightarrow$ must be
 -1

$$\frac{z}{\sqrt{4z^2-3}} \frac{dz}{dt} = -1.$$

$$\Rightarrow \frac{1}{4} (4z^2-3)^{\frac{1}{2}} \frac{dz}{dt} = -t + c.$$

$$t=0 \quad z=1 \Rightarrow \frac{1}{4} = c.$$

$$\Rightarrow \frac{1}{4} (4z^2-3)^{\frac{1}{2}} = -t + \frac{1}{4}.$$

$$\Rightarrow \cancel{\frac{1}{4}} \quad 4z^2-3 = (1-4t)^2.$$

$$\Rightarrow 4z^2-3 = 1-8t+16t^2.$$

$$\Rightarrow 4z^2 = 4-8t+16t^2.$$

$$\Rightarrow z^2 = t^2-2t+1 \Rightarrow z = \sqrt{4t^2-2t+1}$$

$$2\ddot{x}_1 = -\frac{2}{z^3} \Rightarrow 2\ddot{x}_1 + \ddot{x}_2 = 0.$$

$$y = 2x_1 + x_2 \Rightarrow \ddot{y} = 0$$

$$t=0 \Rightarrow y=1 \quad \dot{y}=2$$

$$\ddot{y}=0 \Rightarrow \dot{y}=c \Rightarrow \dot{y}=2$$

$$y=2t+c \Rightarrow t=0 \quad y=1 \Rightarrow c=1$$

$$\Rightarrow y=2t+1$$

$$y=2x_1+x_2 \quad z=x_2-x_1$$

$$y-z=3x_1 \Rightarrow x_1=\frac{1}{3}(y-z)$$

$$\Rightarrow x_1=\frac{1}{3}(2t+1-\sqrt{4t^2-2t+1})$$

$$y+2z=3x_2 \Rightarrow x_2=\frac{1}{3}(y+2z)$$

$$\Rightarrow x_2=\frac{1}{3}(2t+1+2\sqrt{4t^2-2t+1})$$

$$x_2=\frac{1}{3}(2t+1)+\frac{2}{3}(4t^2-2t+1)^{\frac{1}{2}}$$

$$\dot{x}_2=\frac{2}{3}+\frac{1}{3}(8t-2)(4t^2-2t+1)^{-\frac{1}{2}}$$

consider $t \rightarrow \infty$ for x_1

~~$$(4t^2-2t+1)^{\frac{1}{2}} = 1 + \frac{1}{2}(4t^2-2t) + \dots$$~~

~~$$= 1 + 2t^2 - 2t + \dots$$~~

$$(4t^2-2t+1)^{\frac{1}{2}} = 2t \left(1 + \frac{1}{2t} + \frac{1}{4t^2} \right)^{\frac{1}{2}}$$

$$= 2t \left(1 + \frac{1}{2} \left(-\frac{1}{2t} + \frac{1}{4+t^2} \right) + \dots \right)$$

$$= 2t \left(1 - \frac{1}{4t} + \frac{1}{8+t^2} + \dots \right)$$

$$= 2t - \frac{1}{2} + \frac{1}{4t} + \dots$$

$$\Rightarrow t \rightarrow \infty \quad x_1 \rightarrow \frac{1}{3} \left(2t + 1 - \left(2t - \frac{1}{2} \right) \right)$$

$$= \frac{1}{2}$$

$$x_2 \rightarrow \frac{2}{3} + \frac{1}{3} \frac{(8t-2)}{2t-\frac{1}{2}}$$

$$= \frac{2}{3} + \frac{4}{3} = \underline{\underline{2}}$$