Calculus I

Second Partial Project "Motion: Position, Velocity and Acceleration"

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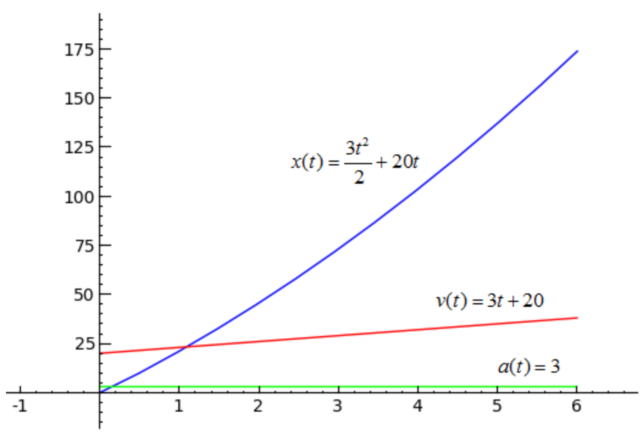
Introduction

Motion is described as the movement of an object; position, velocity and acceleration are three vector quantities that describe this action. Position is given as a function of **x** with respect to time ***x(t)***. The velocity is obtained by derivating the position function ***v(t) = x'(t)*** and the function of acceleration is the second derivative of position, which means, acceleration is the derivative of velocity ***a(t) = v'(t)***. For example:

1. Position: $x(t) = 3t^2 - 4t - 2$ in m.
2. Velocity: $v(t) = 6t - 4$ in $m/_s$ or $x'(t) = 6t - 4$ in $m/_s$.
3. Acceleration: $a(t) = 6$ in $m/_{s^2}$ or $x''(t) = 6$ in $m/_{s^2}$.

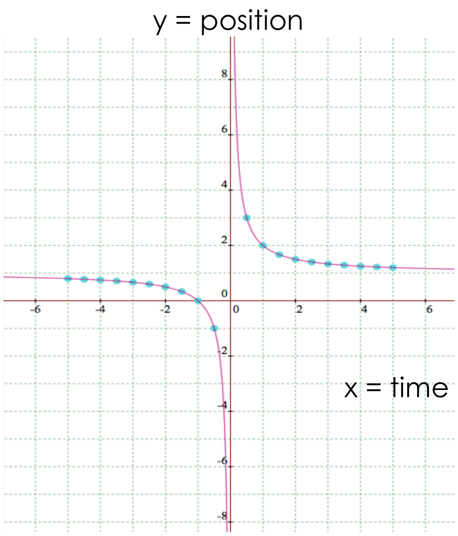
An example of these functions graphed on a Cartesian Plane, could be the following:



Therefore, the objective of this investigation is to prove and show understanding of derivatives and functions that model equations, at the same time, to use mathematical concepts and analysis in order to solve specific problems.

Graphs

Number 1: time vs f(t)



This graph shows a positive rational function with a vertical asymptote of x=0 as there are no transformations in this axis then a horizontal asymptote can be appreciated at y = 1.

Functions:

· Position: $f(t) = \left(\frac{x}{x^2}\right) + 1$

· Velocity: $f(t) = \left(\frac{x}{x^2}\right) + 1$

$$v(t) = (x^{-1}) + 1$$

$$v(t) = -1(x^{-1}) + 1$$

$$v(t) = -1(x^{-1-1})$$

$$v(t) = -1(x^{-2})$$

$$v(t) = -\frac{1}{x^2}$$

· Acceleration: $v(t) = \left(\frac{-1}{x^2}\right)$

$$a(t) = -1(x^{-2})$$

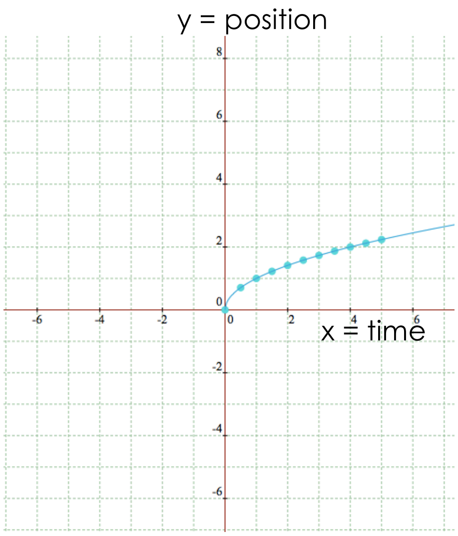
$$a(t) = 2(x^{-2})$$

$$a(t) = 2(x^{-2-1})$$

$$a(t) = 2(x^{-3})$$

$$a(t) = \frac{2}{x^3}$$

Number 2: time vs g(t)



This graph is from a positive square root function and only shows positive square root values because this are the only ones included in the function forming a half-parabola.

Functions:

· Position: $g(t) = \sqrt{x}$

· Velocity: $g(t) = \sqrt{x}$

$$v(t) = (x)^{\frac{1}{2}}$$

$$v(t) = \frac{1}{2}\left(x^{\frac{1}{2}}\right)$$

$$v(t) = \frac{1}{2}\left(x^{\frac{1}{2}-1}\right)$$

$$v(t) = \frac{1}{2}\left(x^{-\frac{1}{2}}\right)$$

$$v(t) = \frac{1}{2(x)^{\frac{1}{2}}}$$

$$v(t) = \frac{1}{2\sqrt{x}}$$

· Acceleration: $v(t) = \frac{1}{2\sqrt{x}}$

$$a(t) = \frac{1}{2}\left(x^{-\frac{1}{2}}\right)$$

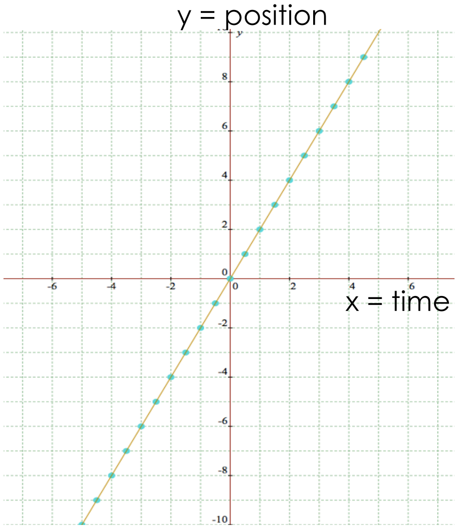
$$a(t) = -\frac{1}{4}\left(x^{-\frac{1}{2}}\right)$$

$$a(t) = -\frac{1}{4}\left(x^{-\frac{1}{2}-1}\right)$$

$$a(t) = -\frac{1}{4}\left(x^{-\frac{3}{2}}\right)$$

$$a(t) = \frac{1}{4\sqrt{x^3}}$$

Number 3: time vs h(t)



The third graph features a simple positive linear function that passes through the origin.

Functions:

· Position: $h(t) = 2x$

· Velocity: $h(t) = 2x$

$v(t) = 2x^1$

$v(t) = 2x^1$

$v(t) = 2x^{1-1}$

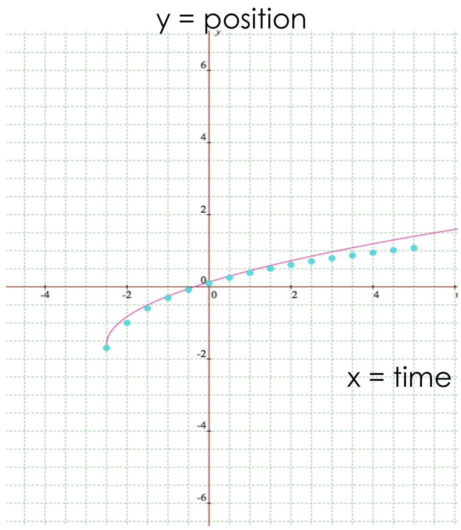
$v(t) = 2x^0$

$v(t) = 2$

· Acceleration: $v(t) = 2$

$a(t) = No\ aceleration$

Number 4: time vs F(t)



This graph shows a positive square root function since only the values of the square roots of positive numbers are part of the function, therefore forming a half-parabola shape.

Functions:

· Position: $F(t) = \sqrt{1.2x + 3} - 1.6$

· Velocity: $F(t) = \sqrt{1.2x + 3} - 1.6$

$$v(t) = (1.2x + 3)^{\frac{1}{2}} - 1.6$$

$$v(t) = \frac{1}{2}(1.2x + 3)^{\frac{1}{2}}\ (1.2)$$

$$v(t) = \frac{1}{2}(1.2x + 3)^{\frac{1}{2}-1}\ (1.2)$$

$$v(t) = \frac{1}{2}(1.2x + 3)^{-\frac{1}{2}}\ (1.2)$$

$$v(t) = \frac{1.2}{2}(1.2x + 3)^{-\frac{1}{2}}$$

$$v(t) = \frac{1.2}{2(1.2x+3)^{\frac{1}{2}}}$$

$$v(t) = \frac{1.2}{2\sqrt{1.2x+3}}$$

· Acceleration: $v(t) = \frac{1.2}{2\sqrt{1.2x+3}}$

$$a(t) = \frac{1.2}{2}\ (1.2x + 3)^{-\frac{1}{2}}$$

$$a(t) = \frac{1.2}{2}\ \left[(1.2x + 3)^{-\frac{1}{2}}\right]$$

$$a(t) = \frac{1.2}{2}\ \left[-\frac{1}{2}(1.2x + 3)^{-\frac{1}{2}}(1.2)\right]$$

$$a(t) = \frac{1.2}{2}\ \left[-\frac{1}{2}(1.2x + 3)^{-\frac{1}{2}-1}(1.2)\right]$$

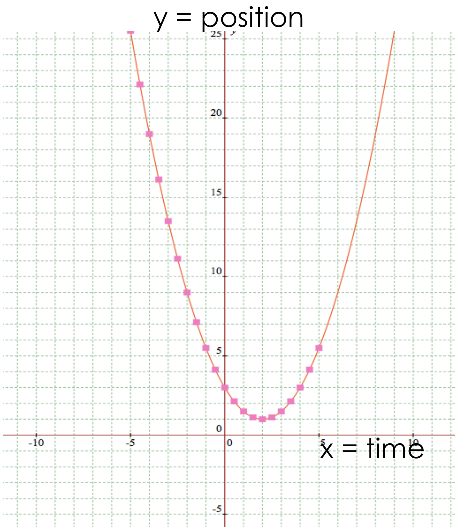
$$a(t) = \frac{1.2}{2}\ \left[-\frac{1}{2}(1.2x + 3)^{-\frac{3}{2}}(1.2)\right]$$

$$a(t) = \frac{1.2}{2}\ \left[-\frac{1.2}{2}(1.2x + 3)^{-\frac{3}{2}}\right]$$

$$a(t) = -\frac{1.44}{4}(1.2x + 3)^{-\frac{3}{2}}$$

$$a(t) = -\frac{1.44}{4\sqrt{(1.2x+3)^3}}$$

Number 5: time vs G(t)



This graph shows a positive quadratic function, since its shape is a parabola which opens up. One can clearly see characteristic elements of this type of function: there is a vertex at (2,1) and a y-intercept at y=3.

Functions:

· Position: $G(t) = 0.5x^2 - 2x + 3$

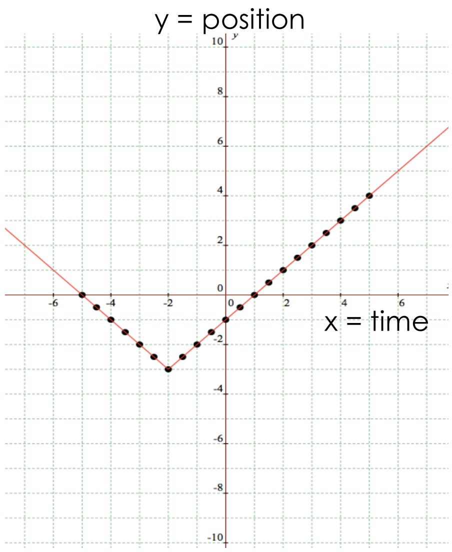
· Velocity: $G(t) = 0.5x^2 - 2x + 3$

$v(t) = x - 2$

· Acceleration: $v(t) = x - 2$

$a(t) = 1$

Number 6: time vs H(t)



This graph shows an absolute value function because no negative y-values appear on the graph, and as a result a V-shape is obtained. There is an inflection point at (-2, -3). To the right of it, the graph has a slope of 1 and to the left of it the graph has a slope of -1.

Functions:

· Position: $H(t) = |x + 2| - 3$

· Velocity: $H(t) = \lceil x + 2 \rceil - 3$

$v(t) = 1$

· Acceleration: $v(t) = 1$

$a(t) = \textit{ No acceleration.}$

Reflection:

Laura Londoño:

The main objective of the partial project was to develop competences to represent data from real life situations using tables, graphs, diagrams and equations putting into practice all topics seen in class.

Despite, elaborating this work was challenging due to the fact the second step after plotting the data in the Cartesian Plane, was to find the exact function that passes through all the points. Therefore, we started by testing parent functions and from that, transformations were included. At the end, when the equation of position was obtained, we derivate in order to get the velocity and acceleration of the graph; in order words

José Luis Obregón:

By analyzing the scatter plot graph including different points from each function, it was easier for the team to find the equation that belongs to each. Using previous knowledge and changing values in order to obtain the exact graph that passes through all the points or trying to do so. In one specific case, the values are not fulfilled completely, considering that the graph is an estimation from the given points.

Andrés Piñones:

I consider the project was a very useful practice to reinforce derivatives from the most recent topics learned in calculus to obtain position, acceleration and velocity from a given function as well as graphing functions, describing their transformations and identifying factors that define which type of function is being presented by remembering topics learned before. The use of graphing software ease the process.

Alan Garza:

In my opinion the project was good because it enabled us to remember several types of functions, as well as apply calculus concepts such as derivatives of position and speed to those graphs. It also proved a challenge, because at first, we struggled to obtain the graphs and equations but nonetheless this provided more learning about graphing softwares.

References:

1. Chegg Study. (2003-2017). Definition of Motion, Position, Velocity and Acceleration. October 10th, 2017, de Chegg Study. Retrieved from: http://www.chegg.com/homework-help/definitions/motion-position-velocity-and-acceleration-29

2. Sparknotes. (2017). Position, Velocity and Acceleration in One Dimension. October 10th, 2017, de Sparknotes.com. Retrieved from: http://www.sparknotes.com/physics/kinematics/1dmotion/section2.rhtml

3. Stapel, E. (n.d.). Graphing Rational Functions: Introduction. Retrieved October 10, 2017, from http://www.purplemath.com/modules/grphrtnl.htm