

CALCULUS II
FIRST PARTIAL

QUIZ 1A

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①

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Answer the following problems with complete procedure.

1. Find the approximate value of $(3.04)^3$ (20 pts)

$$f(x+dx) = f'(x) + f(x) \cdot dx$$

$$(3.04)^3 \quad f(3+0.04) = 3(3)^2 + (3)^3 \cdot 0.04$$

$$f' = 3x^2 \quad f(3.04) = 27 + 27 \cdot 0.04$$

$$x_1 = 3.04$$

$$f(3.04) = 27 + 1.08$$

$$x_2 = 3$$

$$f(3.04) = 28.08$$

$$dx = 0.04$$

2. Given the equation $f(x) = x^2 - 2x + 3$ find the line tangent to the curve at $X = a = 0$. (20 pts)

$$f(0) = 3$$

$$f'(x) = 2x - 2$$

$$f'(0) = -2$$

$$y = -2x + 3$$

3. The edge of a cube was found to be 20 cm. with a possible error in measurement of 0.1 cm. Estimate the maximum possible error in computing the volume of the cube (20 pts)

diff

$$dx = 0.1$$

$$dy = f'(x) \cdot dx$$

$$dy = 3l^2 \cdot 0.1$$

$$dy = 3(20)^2 \cdot 0.1$$

$$dy = 1,200 \cdot 0.1$$

$$dy = 120 \text{ cm}^3$$

$$l^3$$

$$3l^2$$



4. A can is going to be modified in such a way that its height will change from 14cms to 14.8 cm but the diameter of the base will remain as 9cm.

a) Find the change in the volume of the can (20 pts)



$$V = \pi r^2 h$$

$$V = \pi (4.5)^2 (14)$$

$$V = 890.64 \text{ cm}^3$$

$$dx = 0.8$$

$$d = 9 \text{ cm}$$

$$V = \pi (4.5)^2 (14.8) \quad r = 4.5$$

$$V = 941.59 \text{ cm}^3$$

$$\Delta V = 50.9 \text{ cm}^3$$

b) Find the approximate change in the volume of the can (20 pts)

-10

$$V' = 2\pi r \cdot dx$$

wrong derivative $dx = 0.8$

$$dy = 2\pi (4.5) \cdot 0.8$$

$V =$

$$dy = 22.62 \text{ cm}^3$$

$$3.14r^2 \cdot h$$

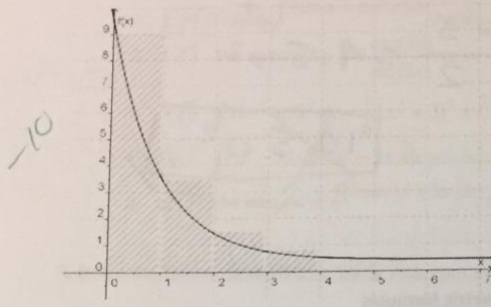
ΔV

dV

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I. Multiple choice. Choose the letter of the right answer (10 points).

1. Choose the sentence that best describes the approximate area below the graph of $f(x)$:



- a) Approximation of the area on the interval $[0, 4]$ using 4 partitions with left-hand calculations.
- b) Approximation of the area on the interval $[1, 5]$ using 4 partitions with right-hand calculations.
- c) Approximation of the area on the interval $[0, 4]$ using 4 partitions with right-hand calculations.
- d) Approximation of the area on the interval $[1, 5]$ using 4 partitions with left-hand calculations.

II. Evaluate the integral using the following values. SHOW THE STEPS OF YOUR PROCEDURE. (5 points each)

$\int_2^4 x dx = 9$ $\int_2^4 x^3 dx = 54$ $\int_2^4 dx = 7$
 $270 + 364 + 42 = 348$
 a. $\int_2^4 (5x^3 + 4x + 6) dx = 5 \int_2^4 x^3 dx + 4 \int_2^4 x dx + 6 \int_2^4 dx = 5(54) + 4(9) + 6(7) = 348$
 b. $\int_2^4 23 dx = 23 \int_2^4 dx = 23(7) = 161$
 c. $\int_5^3 x^3 dx = 0$
 d. $\int_4^2 x dx = -9$

IV. Procedure. Solve the following problem showing your entire procedure.

1) Approximate the area of a plane regions using left hand, right hand and middle points approximations.

$f(x) = 9 - x^2$ on $[3, 5]$ 4 rectangles (20 points)

left

x	3	3.5	4	4.5	5
f(x)	0	-3.25	-7	-11.25	-16
Δx	0.5	0.5	0.5	0.5	0.5
Area	0	-1.625	-3.5	-5.625	-8

Area (Left hand) = $-10.75 u^2$
 $= \frac{2}{7}$ Area (Right hand) = $-18.75 u^2$
 (0.5)

right

x	3.25	3.75	4.25	4.75
f(x)	-1.56	-5.063	-9.06	-13.56
Area	-0.78	-2.5315	-4.53	-6.78

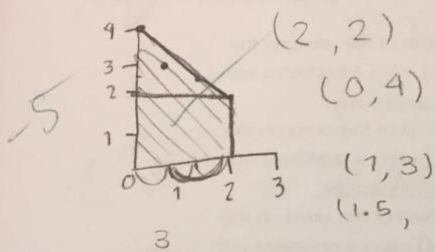
middle = $-14.6215 u^2$
 $\frac{10.5625}{14.063}$
 $\frac{18.06}{22.56}$

2) Give the graph (remember to shade the corresponding area) whose area is given by the following definite integral. Then use a geometric formula to evaluate the integral (by finding the area) (15 points each)

$$\int_0^2 (4-x) dx$$

Graph

Procedure by geometric formulas



$$\begin{aligned} 2 \times 2 &= 6 \text{ u}^2 + \\ \frac{2 \times 3}{2} &= 4.5 \text{ u}^2 \end{aligned}$$

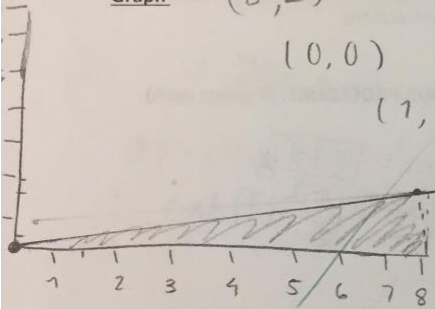
$$\boxed{10.5 \text{ u}^2}$$

3) $\int_0^8 \frac{x}{4} dx$

Graph

(8, 2)

Procedure by geometric formulas



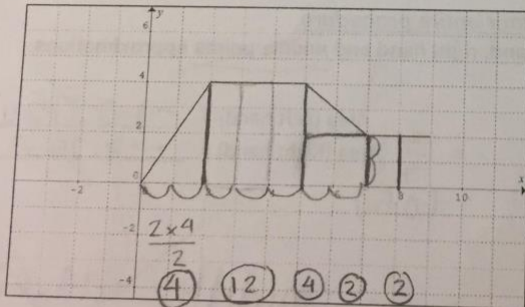
$$\frac{8 \times 2}{2} = \boxed{8 \text{ u}^2}$$

(4, 1)

3, 0.75

(1, 0.25) (3, 0.75)
(2, 0.5)

3) Based on the following graph evaluate the given definite integrals (5 points each):



1. $\int_0^3 f(x) dx$

$$\frac{2 \times 4}{2} = 2 \times 2$$

$$4 + 4 = 8$$

3. $\int_0^7 f(x) dx$

$$4 + 2 = 6$$

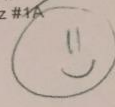
2. $\int_4^6 f(x) dx$

$$-8$$

4. $\int_0^8 f(x) dx$

$$24$$

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I. Determine if the following propositions are True (T) or False (F) (5 points each):

- $\tan(x) = \frac{\sin}{\cos}$
 $u = 3x$
 $du = 3dx$
 \sec / \csc
 $\frac{1}{\sin(3x)} = \frac{\sin(x)}{\sin(3x)}$
- (F) Having $\int (\sin x + \cos x) dx$ is the same as having $\int (\sin x) dx + \int (\cos x) dx$
 - (F) The answer for $\int 6 \frac{\csc(3x)}{\sin(3x)} dx$ is $-2 \cot(3x) + C$
 - (F) $\int x(x^2 + 3)^2 dx = \frac{1}{6}(x^2 + 3)^3 + C$
 - (F) $\int (x^2 - 3) \tan(x^2 - 3x) dx = -\ln |\cos(x^2 - 3x)| + C$
 - (F) The integral of $\int (2 \sin 3x + 3x) dx$ is $-6 \sin 3x + 3 + C$
- $u = x^2 + 3$
 $du = 2x$
 $\frac{1}{2} \frac{(x^2 + 3)^3}{2 \cdot 3}$
 $\int (2 \sin 3x + 3x) dx = -\frac{2}{3} \cos(3x) + \frac{3x^2}{2} + C$

II. Solve the following exercises, show ALL your procedure and frame your final answer. (15 points each).

If the equation of acceleration of an object is $a(t) = \frac{3}{t-4}$ and the velocity at $t = 5$ is 8 m/s, then find the equation that determines the velocity of the object at any time 't'.

$u = t - 4$
 $du = 1 dx$
 $\frac{3}{t-4}$
 $v(t) = 3 \ln |t-4| + 8$
 $3 \ln |t-4| + C = 8$

III. Find the antiderivative or integral of the following problems. SHOW YOUR ENTIRE PROCEDURE. (15 pts each)

1- $h(x) = 96 \sin^2(2x + \pi) \cos(2x + \pi)$

$u = \sin(2x + \pi)$
 $du = 2 \cos(2x + \pi)$
 $48u^2 = 48 \sin^3(2x + \pi)$
 $H(x) = 16 \left[\frac{\sin^3(2x + \pi)}{3} \right] + C$

$$2- v(t) = \frac{e^{5t}}{3t^2}$$

$$\sqrt{3t^{-2}} e^{5/4}$$

$$-3 \quad u = 5t^{-1} \\ du = -5t^{-2} dx$$

$$-5 \quad | \quad 3 \cdot \frac{5}{3}$$

$$x(t) = \frac{5}{3} e^{5/4} + C$$

$$v(t) = \frac{5}{3t}$$

$$3- \int 3x \cot(6x^2-1) \sin(6x^2-1) dx$$

$$\cot = \frac{\cos}{\sin}$$

$$3x \frac{\cos}{\sin} (6x^2-1) \sin(6x^2-1) dx$$

$$3x \cos(6x^2-1) dx$$

$$u = 6x^2 - 1$$

$$du = 12x dx$$

$$\frac{1}{4} \sin(6x^2-1) + C$$

$$4- \int 7 \sec(3x) \tan(3x) dx$$

$$7 \sec(3x) + C$$

$$u = 3x$$

$$du = 3 dx$$

$$3 \cdot 7$$

$$\frac{7}{3} \sec(3x) + C$$

$$\frac{7}{3}$$

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I. Solve the following integrals. SHOW THE STEPS OF YOUR PROCEDURE. (20 points each)

1. $\int \sin^3(2x) dx$

$\int \sin(2x) \sin^2(2x) dx$

$\int \sin(2x) (1 - \cos^2(2x)) dx$

$\int \sin(2x) dx - \int \sin(2x) \cos^2(2x) dx$

$\frac{-1}{2} \cos(2x) + \frac{1}{2 \cdot 3} \cos^3(2x) = \frac{\cos^3(2x)}{6}$

2. $\int x^6 \cos^2(x^7) dx$

$\frac{1}{2} \int x^6 (1 + \cos(2x^7)) dx$

$\frac{1}{2} \int x^6 dx + \int x^6 \cos(2x^7) dx$

$\frac{1}{2} \left[\frac{x^7}{7} + \frac{1}{14} \sin(2x^7) \right] + C$

3. $\int 9x^4 \tan^3(x^5) dx$

$\int 9x^4 \tan(x^5) \tan^2(x^5) dx$

$\int 9x^4 \tan(x^5) (\sec^2(x^5) - 1) dx$

$\int 9x^4 \tan(x^5) \sec^2(x^5) dx - \int 9x^4 \tan(x^5) dx$

* $\frac{9}{5} \frac{\tan^2(x^5)}{2} - \frac{9 \tan^2(x^5)}{10} + \frac{9}{5} \ln |\cos(x^5)| + C$

DOUBLE ANGLE

$u = \cos 2x$

$du = -2 \sin 2x dx$

$\cos^2(x) = \frac{1}{2}(1 + \cos 2x)$

$\sin^2(x) = \frac{1}{2}(1 - \cos 2x)$

$\tan^2(x) = \sec^2 x - 1$

PYTHAGORIAN

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x = 1 - \sin^2 x$

$\tan^2 x = \sec^2 - 1$

$\cot^2 x = \csc^2 - 1$

4. $\int x^3 \sin^2(x^4) dx$

$\frac{1}{2} \int x^3 (1 - \cos 2(x^4)) dx$

$\frac{1}{2} \int x^3 dx - \int x^3 \cos(2x^4) dx$

$u = 2x^4$
 $du = 8x^3$

$\frac{1}{2} \left[\frac{x^4}{4} - \frac{1}{8} \sin(2x^4) \right] + C$

~~$\frac{x^4}{8} - \frac{\sin(2x^4)}{16} + C$~~

5. $\int \cot^2(5x) dx =$

$\int (\csc^2(5x) - 1) dx$

~~$-\frac{1}{5} \cot(5x) - x + C$~~

$u = 6x$
 $du = 6$

$\cos^4(3x)$
 $\left(\frac{1}{2}\right)^2 \int (1 + \cos 2(3x))^2 dx$
 $1 + 2 \cos 2(3x) + \cos^2(3x)$
 $x + \frac{1}{3} \sin(6x) +$

$\cos^4(3x)$
 $\left(\frac{1}{2}\right)^2 \int (1 + \cos 2(3x))^2 dx$
 $\frac{1}{4} \int 1 + 2 \cos(6x) + \cos^2(6x)$
 $\frac{1}{4} \int x + \frac{1}{3} \sin(6x) +$

BONUS (8 POINTS)

~~$\int \cos^5(3x) dx$~~

$\int \cos(3x) (\cos^2(3x))^2 dx$

$\int \cos(3x) \left(\frac{1}{2}\right)^2 (1 + \cos 2(3x))^2 dx$

$\int 1 + 2 \cos 6x + \cos^2 6x$

$\cos 6x$

$\cos(3x) + \cos(3x) 2 \cos 6x + \cos(3x) \cos^2 6x$
 $\frac{1}{3} \sin(3x) +$