6. A farmer has a water reservoir that has the shape of a cylinder of 5ft in diameter and 14 ft in height. Water is pumped from it at a constant rate of 0.25ft^3/min. Find the rate at which the water level is lowering, when the water already has a level of 10ft.

$V = 0.25 \frac{ft^3}{in}$ $V = \pi r^2 h$ $\frac{dV}{dt} = \pi 2 r h$ $0.25 = \pi 2(2.5)(10)\left[\frac{dr}{dt}\right]$ $0.25 = \pi 50\left[\frac{dr}{dt}\right]$ $\boxed{\frac{dr}{dt} = \frac{-1}{25\pi}}$

Procedure at the back of the 2nd page

7. A cube is increasing its size at a rate of 3 cm/min. Determine the rate of change of the volume and the rate of change of the surface area when the edge measures 6 cm.

$\frac{dr}{dt} = 3\frac{cm}{min}$ $V = L^3$ $\frac{dV}{dt} = 3L^2$ $\frac{dV}{dt} = 3L^2\left[\frac{3cm}{in}\right]$ $\frac{dV}{dt} = 3(6)^2[3]$ $\boxed{\frac{dV}{dt} = 324 \frac{cm^3}{min}}$ $SA = 6L^2$ $\frac{dA}{dt} = 12L\left[\frac{dL}{dt}\right]$ $\frac{dA}{dt} = 12(6)(3)$ $\boxed{\frac{dA}{dt} = 216 \frac{cm^2}{min}}$

8. A spherical balloon is being filled with air in such a way that the radius is increasing at a constant rate of change 0.2cm/sec. Find the rate of change of the volume when the radius is 12cm

$\frac{dr}{dt} = 0.2\frac{cm}{sec}$ $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = \frac{4}{3}\pi 3r^2$ $\frac{dV}{dt} = \frac{4}{3}\pi 3r^2\left[\frac{dr}{dt}\right]$ $\frac{dV}{dt} = \frac{4}{3}\pi 3(12)^2\, 0.2$ $\boxed{\frac{dV}{dt} = 115.2 \frac{cm^3}{sec}}$

9. A cone is increasing in such a manner that its height is always three times its radius. Find the rate at which the radius is changing, at the moment when radius measures 6 cm, knowing that its volume is incrementing at a rate of 60 cm^3 per minutes (Hint: $V = \frac{1}{3}\pi r^2 h$) $h = 3r$

$V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi 2r(3r)$ $r = 6cm$ $\frac{dV}{dt} = 60\frac{cm^3}{min}$ $60\frac{cm^3}{min} = \frac{1}{3}\pi 2(6)[3(6)]$ $60\frac{cm^3}{min} = \frac{1}{3}$ $\boxed{\frac{dr}{dt} = \frac{5}{9\pi}\frac{cm}{min}}$

Procedure at the back of 2nd page

10. A triangle is increasing in such a way that its height is always half of its base. Find the rate at which the area is changing, when the base measures 12 cm, if the base is incrementing at a rate of 3 cm per minute. $h = \frac{b}{2}$ $h = \frac{12}{2}$ $h = 6$ $A = \frac{b}{2}\frac{h}{2}$ ① $A = \frac{b}{2}\left[\frac{b}{2}\right]$ $A = \frac{12}{2}\left[\frac{12}{4}\right]$

$A = \frac{bh}{2}$ $b = \frac{2A}{h}$ $b = 2h$ $A = \frac{b^2}{8} = 18$ $b(3) = 18$

$\frac{db}{dt} = 3\frac{cm}{min}$ $\frac{dA}{dt} = \frac{1}{4}b\left[\frac{db}{dt}\right]$ $\boxed{\frac{dA}{dt} = 18\frac{cm^2}{min}}$ ② $A = \frac{b^2}{4}$

$\frac{dA}{dt} = \frac{1}{4}b$ ③ $A = \frac{2b}{4}\left[\frac{db}{dt}\right]$

ANSWERS

1. $\frac{1}{2\pi} \approx 0.159\ in/\sec$ 2. $\frac{5}{2\pi} \approx 0.796\ in/\min$ 3. $0.4\pi \approx 1.257\ m^2/\sec$

4. $\frac{dV}{dt} = 200\pi \approx 628.32\ cm^3/\sec$, $\frac{dA}{dt} = 40\pi \approx 125.66\ cm^2/\sec$ 5. $\frac{12}{\pi} \approx 3.82\ cm/\min$

6. $-\frac{1}{25\pi} \approx -0.013\ ft/\min$ 7. $\frac{dV}{dt} = 324\ cm^3/\min$, $\frac{dA}{dt} = 216\ cm^2/\min$

8. $115.2\pi \approx 361.9 cm^3/\sec$ 9. $\frac{dr}{dt} = \frac{5}{9\pi}[\frac{cm}{\min}] \approx 0.177[\frac{cm}{\min}]$ 10. $\frac{dA}{dt} = 18[\frac{cm^2}{\min}]$

$A = \frac{2b}{4}(3)$ $A = \frac{1}{2}b(3)$ $A = \frac{3b}{2}$ $A = \frac{3(12)}{2}$ $\frac{dA}{dt} = \frac{3b}{2}$ $\frac{dA}{dt} = 18\frac{cm^2}{min}$

10/10

Tecnológico de Monterrey Preparatoria

By: Arq

Related Rates

By: Arq. Monica M. Paniagua, Ing. Ziad Najjar, Lic. Lucy Solis, Lic Carmela de la fuente

Name Laura M. Londoño Hidalgo ID # A01570224 Date 401

Related rates

1. A spherical balloon is being filled at a rate of 50in³/sec, at what rate does the radius increase when the radius is 5in? rate of change = derivative

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2$

$50 = \frac{4}{3}\pi 3r^2\left[\frac{dr}{dt}\right]$

$50 = \frac{4}{3}\pi 3(5)^2\left[\frac{dr}{dt}\right]$

$50 = \frac{4}{3}\pi 3(25)\left[\frac{dr}{dt}\right]$

$50 = 100\pi\left[\frac{dr}{dt}\right]$

$\boxed{\frac{dr}{dt} = \frac{1}{2\pi}\frac{in}{sec}}$ //

2. The area of a circle is increasing at a rate of $\underbrace{20in^2/min}_{rate}$, Find the rate at which the radius is increasing when the radius is 4in.

$A = \pi r^2$

$\frac{dA}{dt} = \pi 2r$

$A = 20\frac{in^2}{min}$

$A = \frac{dA}{dt}\left[\frac{dr}{dt}\right]$

$20 = \pi 2r\left[\frac{dr}{dt}\right]$

$20 = \pi(2)(4)\left[\frac{dr}{dt}\right]$

$20 = 8\pi\left[\frac{dr}{dt}\right]$

$\boxed{\frac{5}{2\pi} = \left[\frac{dr}{dt}\right]}$ //

3. A stone is thrown into a lake and a circular ripple moves out at a constant rate of 0.5 meters/sec. Find the rate at which the circle's area is increasing at r = 0.4 meters.

$A = \pi r^2$ $\frac{dr}{dt} = 0.5\frac{m}{sec}$

$r = 0.4$ $\frac{dA}{dt} = \pi 2r\frac{dr}{dt}$

$\frac{dA}{dt} = \pi 2(0.4)(0.5)$

$\boxed{\frac{dA}{dt} = 0.4\pi\frac{m^2}{sec}}$ //

4. Air is being pumped into a spherical balloon making the radius change at a constant rate of 0.5cm/sec. Find the rate of change of the volume and the rate of change of the surface area when the radius is 10cm ($V = \frac{4}{3}\pi r^3$, $A = 4\pi r^2$)

① $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2$

② $A = 4\pi r^2$

$\frac{dA}{dt} = 4\pi 2r$

$\frac{dr}{dt} = 0.5\frac{cm}{sec}$

$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2\left[\frac{dr}{dt}\right]$

$\frac{dV}{dt} = \frac{4}{3}\pi 3(10)^2(0.5)$

$\frac{dV}{dt} = \frac{4}{3}\pi 3(10)^2(0.5)$

$\boxed{\frac{dV}{dt} = 200\pi\frac{cm^3}{sec}}$ //

$\frac{dA}{dt} = 4\pi 2(10)(0.5)$

$\boxed{\frac{dA}{dt} = 40\pi\frac{cm^2}{sec}}$ //

→ 5. A cone is increasing in size as time goes by in such a way that the volume is changing at a constant rate of 75cm³/min. The height is twice the radius. Determine the rate of change of the height, when the height is 5cm. ($V = \frac{1}{3}\pi r^2 h$)

$\frac{dV}{dt} = 75\frac{cm^3}{min}$

$\frac{dh}{dt} =$

$h = 5cm$

① $V = \frac{1}{3}\pi r^2 h$

$h = 2r$ → $r = \frac{h}{2}$

$V = \frac{1}{3}\pi\left[\frac{h}{2}\right]^2(h)$

$V = \frac{1}{3}\pi\left[\frac{h^2}{4}\right](h)$

$V = \frac{\pi h^3}{12}$

$75 = \frac{\pi(5)^3}{12}\left[\frac{dr}{dt}\right]$

$\frac{12(75)}{(5)^3\pi} = \left[\frac{dr}{dt}\right]$

$\boxed{\frac{dh}{dt} = \frac{12}{\pi}\frac{cm}{min}}$

procedure at the back or 2nd page