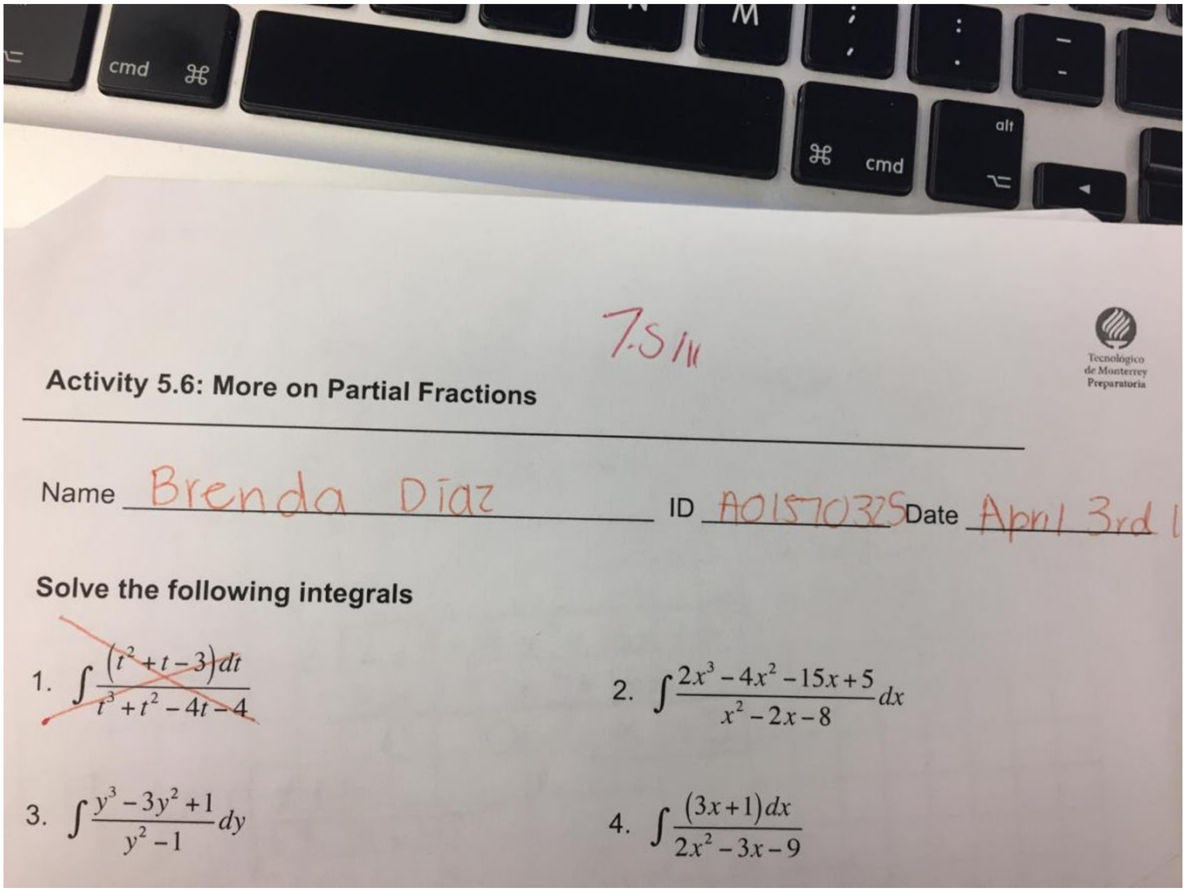
I think that this activity from the third partial is really significant because it is about the partial fractions and this method happen whenever is impossible to solve an antiderivative normally so there are more steps to do. I remember writing in my notebook all the steps and in total there are 10 steps in order to solve these problems, I like organizing these methodologies like that.



(2) $\int \frac{2x^3-4x^2-15x+5}{x^2-2x-8}\,dx$

$$\begin{array}{r} 2x \\ x^2-2x-8 \,\overline{)\,2x^3-4x^2-15x+5} \\ -2x^3+4x^2+16x \\ \hline 0 \quad 0 \quad \boxed{x+5} \end{array}$$

$\int 2x + \int \frac{x+5}{x^2-2x-8}\,dx$

$\frac{\cancel{2}x^2}{\cancel{2}}$

x	-4
x	2

$\frac{A}{(x-4)} + \frac{B}{(x+2)}$

$A(x+2)+B(x-4) = Ax+2A+Bx-4B$

$Ax + Bx = x$

$A+B=1$

$A = 1 - B$

$A = \frac{2}{2} + \frac{1}{2}$

$A = \frac{3}{2}$

$2A - 4B = 5$

$2(1-B)-4B=5$

$2-2B-4B=5$

$-6B = 5-2$

$B = \frac{3}{-6}$ $\quad B = -\frac{1}{2}$

$\frac{\frac{3}{2}}{(x-4)} - \frac{\frac{1}{2}}{(x+2)}$

$x^2 + \frac{3}{2}\ln|x-4| - \frac{1}{2}\ln|x+2| + C$

don't forget

$$* \; \frac{y^2}{2} - 3y$$

③ $\int \frac{y^3 - 3y^2 + 1}{y^2 - 1}\, dy$

$$\begin{array}{r|l} & y - 3 \\ \hline y^2 - 1 & y^3 - 3y^2 + 0 + 1 \\ & -y^3 - 0 + y \\ \hline & -3y^2 + y + 1 \\ & +3y^2 - 0 - 3 \\ \hline & \boxed{y} - 2 \end{array}$$

$$\int y - \int 3 + \int \frac{y-2}{y^2-1}$$

$$\frac{y-2}{y^2-1} = \frac{A}{y-1} + \frac{B}{y+1}$$

$$A(y+1) + B(y-1)$$

$$Ay + A + By - B$$

$$Ay + By = y$$

$$A + B = 1$$

$$A = 1 - B$$

$$A = \frac{2}{2} - \frac{3}{2}$$

$$A = -\frac{1}{2}$$

$$A - B = -2$$

$$(1-B) - B = -2$$

$$1 - 2B = -2$$

$$-2B = -3$$

$$B = \frac{3}{2}$$

$$\frac{-\frac{1}{2}}{y-1} + \frac{\frac{3}{2}}{y+1}$$

$$-\frac{1}{2}\ln|y-1| + \frac{3}{2}\ln|y+1|$$

$$\frac{y^2}{2} - 3y - \frac{1}{2}\ln|y-1| + \frac{3}{2}\ln|y+1| + C$$

④ $\int \frac{3x+1}{2x^2 - 3x - 9}\, dx$

$$\begin{array}{c|c|c} 2x & -6 & 3 \\ 1x & 3 & -3 \end{array}$$

$$\frac{A}{(2x+3)} + \frac{B}{(x-3)}$$

$$3x + 1 = A(2x+3) + B(x-3)$$

$$3x + 1 = x(2A + B) + 3A - 3B$$

$$\frac{10}{9}\ln|x-3| + \frac{7}{9}\ln|2x+3| + C$$

⑤ $\int \frac{t-22}{t^2-4t-5} = \frac{t-22}{t^2-4t-5} = A(t+1)+B(t-5)$

$1 = A + B$

$A = -\frac{17}{6}$

$-22 = A - 5B$
$-22 = (1-B) - 5B$
$-23 = -6B$
$\frac{23}{6} = B$

$\boxed{\frac{23}{6}\ln|t+1| - \frac{17}{6}\ln|t-5| + C}$

⑥ $\int \frac{x-1}{(x-2)(x+2)} = x-1 = A(x+2)+B(x-2)$

$1 = A + B$
$A = \frac{1}{4}$

$-1 = 2A - 2B$
$-1 = 2(1-B) - 2B$
$-1 = 2 - 4B$
$\frac{3}{4} = B$

$\boxed{\frac{1}{4}\ln|x-2| + \frac{3}{4}\ln|x+2| + C}$ 1/2

⑦ $\int \frac{3x-1}{(x-3)(x+2)} = 3x-1 = A(x+2)+B(x-3)$

$3 = A + B$
$A = \frac{8}{5}$

$-1 = 2A - 3B$
$-1 = 2(3-B) - 3B$
$-7 = -5B$
$B = \frac{7}{5}$

$\boxed{\frac{8}{5}\ln|x-3| + \frac{7}{5}\ln|x+2| + C}$