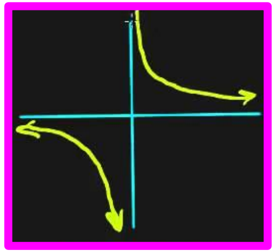

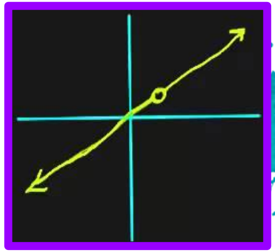

1st PARTIAL PROJECT

Continuity and discontinuity

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What is a Discontinuity?

Functions may be discontinuous at finite sets of points, at countable sets of points and on uncountable proper subsets of their domain.

Discontinuities can be classified as *jump*, *infinite*, *removable*, *endpoint*, or *mixed*.

- Jump Discontinuities: both one-sided limits exist, but have different values.
- Infinite Discontinuities: both one-sided limits are infinite.
- Endpoint Discontinuities: only one of the one-sided limits exists.
- Mixed: at least one of the one-sided limits does not exist.

- Removable discontinuities are characterized by the fact that the limit exists.
- Removable discontinuities can be "fixed" by re-defining the function.
- The other types of discontinuities are characterized by the fact that the limit does not exist. Specifically,

4 Types of Discontinuities

- Type 1 : Asymptotic Discontinuity

Present when you see the graph approaching a point but never touching the point. The same thing is happening on the other side as well. From both sides, it looks like the graph almost touches the point. But because the function never touches the point, it is a discontinuity in the graph.

- **Type 2: Point Discontinuity**

A function is said to have a point of discontinuity at x = a or the graph of the function has a hole at x = a, if the original function is undefined for x = a, whereas the related rational expression of the function in simplest form is defined for x = a.

- **Type 3: Jump Discontinuity**

The graph of function seems like taking a jump or step from one connected point to another. Jump discontinuity occurs when the function jumps from one particular point to some other point. This actually happens when both left hand limit and right hand limit exist, but they are equal.

- **Type 4: Infinite Discontinuity**

Exists when one of the one-sided limits of the function is infinite or one of the other three varieties of infinite limits. If the two one-sided limits have the same value, then the two-sided limit will also exist. Graphically, this situation corresponds to a vertical asymptote.

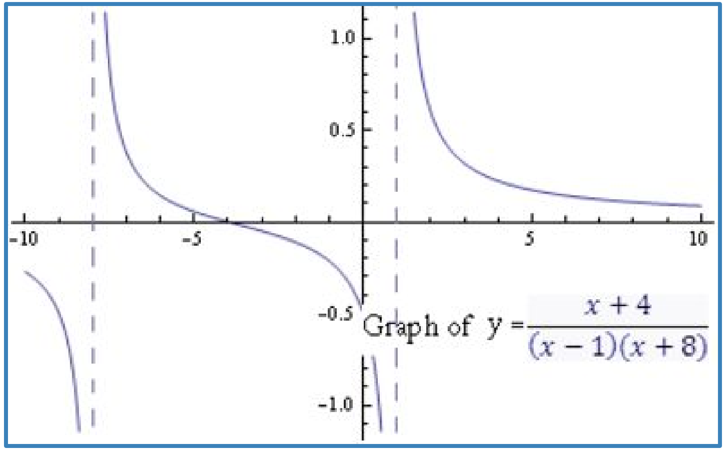
Examples

For each type of

DISCONTINUITY

Type 1: Asymptotic Discontinuity

In the function $f(x) = \dfrac{x+4}{(x-1)(x+8)}$ we can see the domain which is limited to real numbers except 1 and -8.
We can see in the graph that the function behaves strangely at the holes in the domain.
Values of the function approaching x=1:
The graph shows that the "y" value seems to approach - from the left as the "x" value approaches x=1, marked by the dotted line.



Values of y = (x+4)/(x-1)(x+8)

x	y
0.25	-0.687
0.5	-1.059
0.75	-2.171
1.0	undefined
1.25	2.270
1.5	1.158
1.75	0.786

Specifically, this type of discontinuity is called an asymptotic discontinuity. In general, asymptotes occur when a function approaches infinity at a specific value of *x* or *y*. We look for asymptotes at points where the denominator is zero, so that the value of the function gets larger.

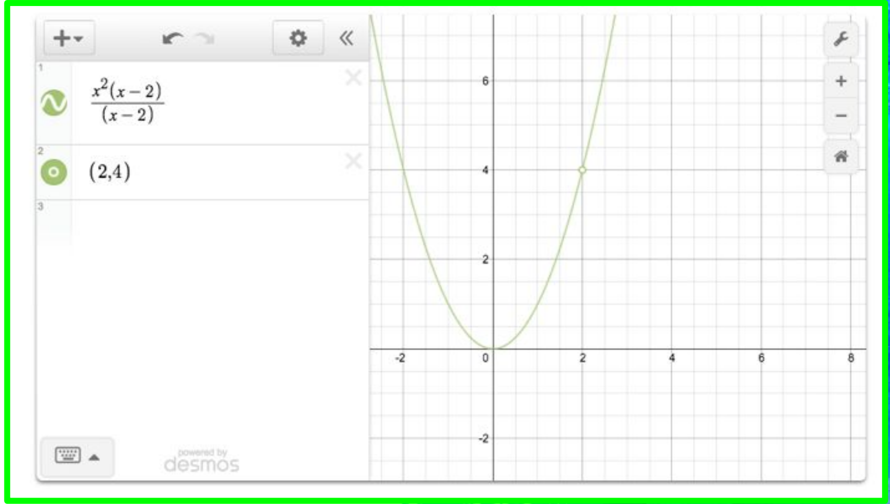
Type 2: Point Discontinuity

Also called removable discontinuities or singularities. Sometimes we come across functions that are defined differently for a certain point. Consider:

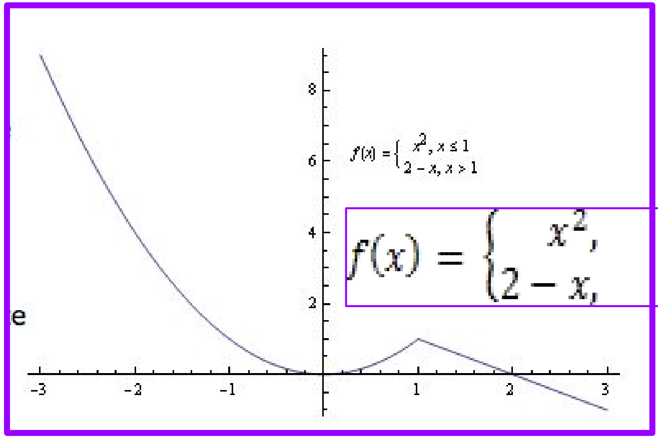
$$f(x) = \begin{cases} 1, & x = 3 \\ x^2, & \text{all other real } x - \text{values} \end{cases}$$

We defined the value of the function to be 1 at the point *x=3*, yet the rest of the function is dictated by *f(x)= x^2.* The graph is continuous except for the tiny hole in the curve at x=1. It is discontinuous at a single point, and this discontinuity is called a <u>point discontinuity.</u>

Another example

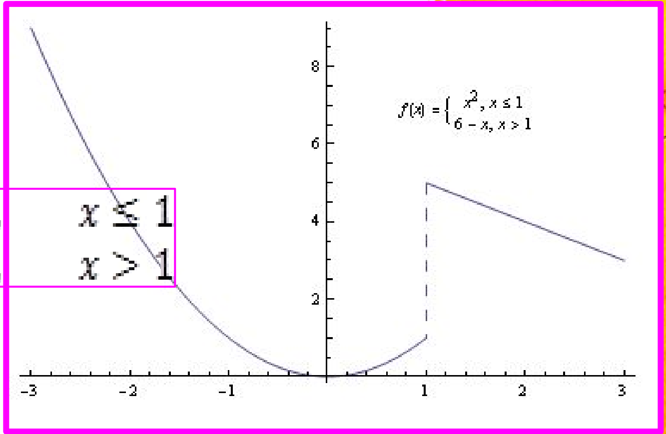


Type 3: Jump Discontinuity



$$f(x) = \begin{cases} x^2, & x \le 1 \\ 2 - x, & x > 1 \end{cases}$$

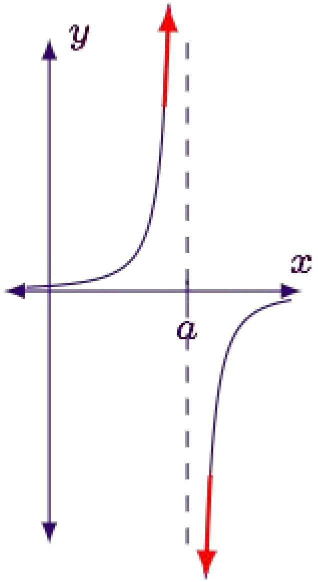
$$f(x) = \begin{cases} x^2, & x \le 1 \\ 6 - x, & x > 1 \end{cases}$$

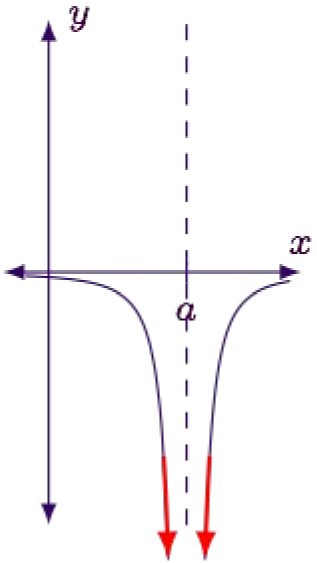


For all intents and purposes, we can consider it two separate functions - one that is defined for x less than or equal to 1 and another function that is defined for x greater than 1. It is useful to think of it this way when we eventually use integration, differentiation, and other such mathematical tools. However, as it is written, f is a single function, called a **piecewise function**, since it is defined piece-by-piece. Note that the function adheres to our definition of being continuous.

The two pieces now have a different value at x = 1, and we can see in the graph that our function f seems to "jump" from one branch to the other. Note this this jump makes the function discontinuous. We refer to this as a **jump discontinuity**.

Type 4: Infinite Discontinuity





→ The arrows on the function indicate it will grow infinitely large as x approaches a. Since the function doesn't approach a particular finite value, the limit does not exist. This is an **infinite discontinuity.**

→ The following two graphs are also examples of infinite discontinuities at x=a. Notice that in all three cases, both of the one-sided limits are **infinite.**

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