Spherical geometry using GeoGebra
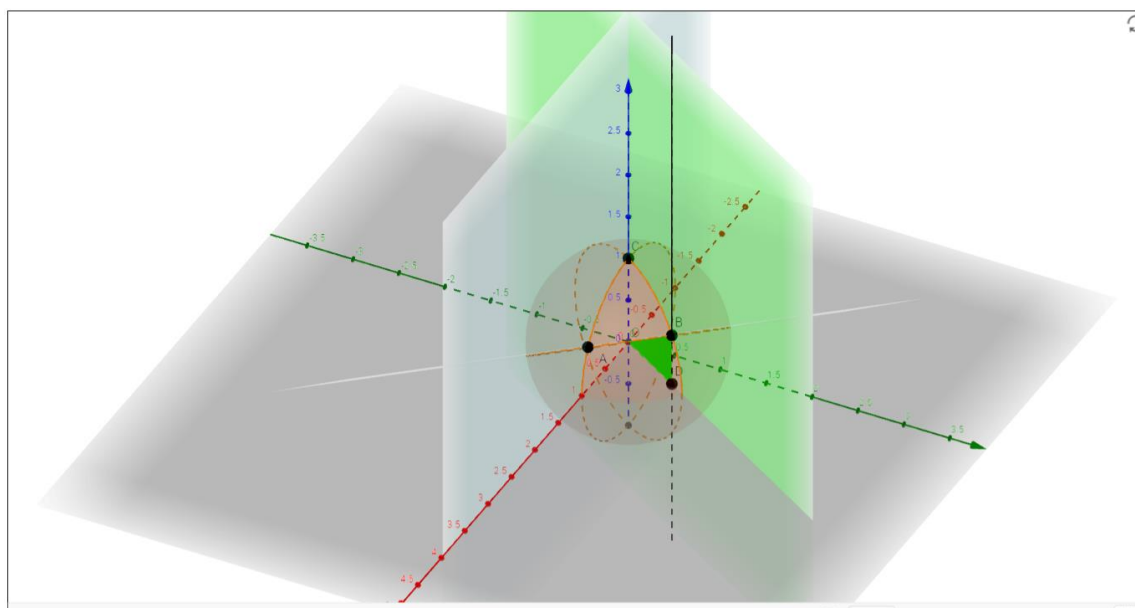
Take a look at the **3-D applet of the unit sphere**. Here is a snapshot:

In the dynamic version, left-click the mouse and twirl the 3-D image. Notice the following:

a) The unit sphere
b) The x-, y- and z-axes (red, green and blue)
c) The x-y plane (z = 0) in grey
d) The x-z plane in blue
e) The origin, O
f) The points A, B & C on the sphere
g) A is in the x-z plane
h) B is any point on the sphere

i) C = (0,0,1) is on the z-axis
j) BD is vertical (parallel to the z-axis)
k) D is in the x-y plane
l) The (green) plane containing OBD
m) The triangle OBD
n) The plane containing OAB
o) Three great circles
p) Various lunes

Now use the navigation bar to see the order of construction:

1. Axes & x-y plane
2. Unit sphere
3. x-z plane
4. Great circle in [3]
5. Point, A, on [4]
6. Point, B, on [2]
7. Vertical line thru B

8. Intersection, D, of [7] with x-y plane
9. Plane thru OBD
10. Triangle OBD
11. C = (0, 0, 1)
12. Great circle in [9]
13. Plane thru OAB
14. Great circle in [13]

To follow the **spherical cosine law** on **Mathematics@CUHK** (Chinese University of Hong Kong), note:
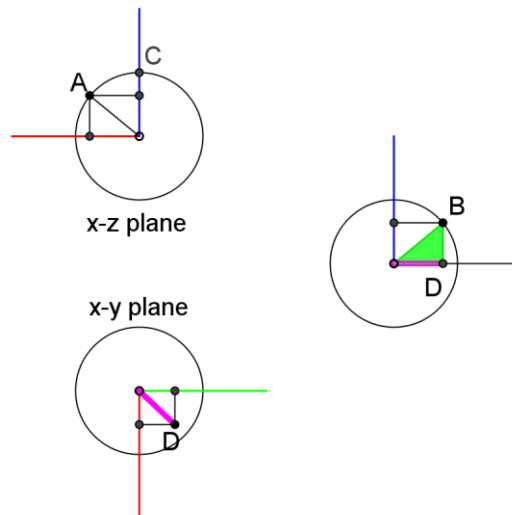
$$A = [\sin(b), 0, \cos(b)]$$

$$|OD| = \sin(a)$$

$$D = \sin(a)[\cos(\gamma), \sin(\gamma), 0] = [\sin(a)\cos(\gamma), \sin(a)\sin(\gamma), 0]$$

$$B = [\sin(a)\cos(\gamma), \sin(a)\sin(\gamma), \cos(a)]$$

$$\cos(c) = A.B = \sin(a)\sin(b)\cos(\gamma) + \cos(a)\cos(b) \quad *$$

Here we have written γ for the angle at C (to avoid confusing c and C), while A.B is the dot product.

x-z plane

x-y plane

We can use the spherical cosine law (*) to find the distance between two points, A & B, on the earth. Let a be the distance from B to C (the North Pole), and b the distance from A to C, then the distance (along a geodesic) from A to B is c, the cosine of which is given by *. Here, γ is the difference in longitude between A and B.

Example: Calculate the distance from Dublin to Hong Kong:

radius (km)			degrees		radians			distance (km)	
6371	E	N	longitude	latitude	longitude	latitude	cos(c)		
				from North		from North			
			diff from dub	Pole	diff from dub	Pole		calculated	web
Dublin	-6.2603	53.3498	0.0000	36.6502	0.0000	0.6397	1.0000	0.00	
Hong Kong	114.1095	22.3964	120.3698	67.6036	2.1008	1.1799	0.0267	9837.71	9844.22

Note that it is easiest to carry out the calculations on the unit sphere and then multiply the result by the radius of the earth. What are a, b and γ here?

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