Transformation of Functions - Solutions

$f(x) = x^2$, $g(x) = \sin x$, $h(x) = x^2 - 1$, $p(x) = x^2 - 2x$.

1. (i)

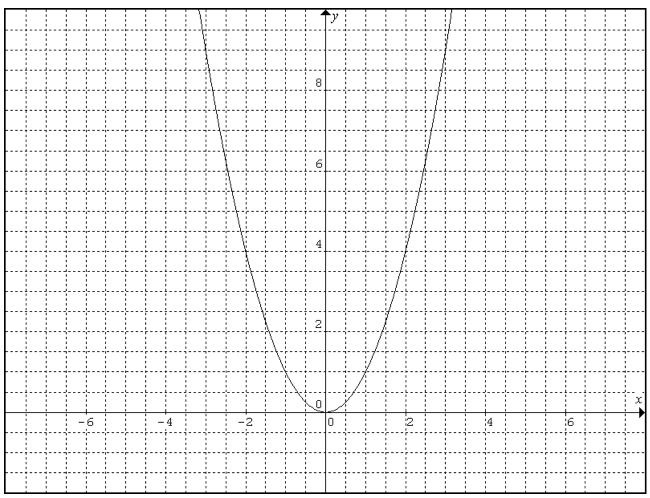
(a) $f(x) + 2 = \underline{x^2 + 2}$

(b) $f(x) + 3 = \underline{x^2 + 3}$

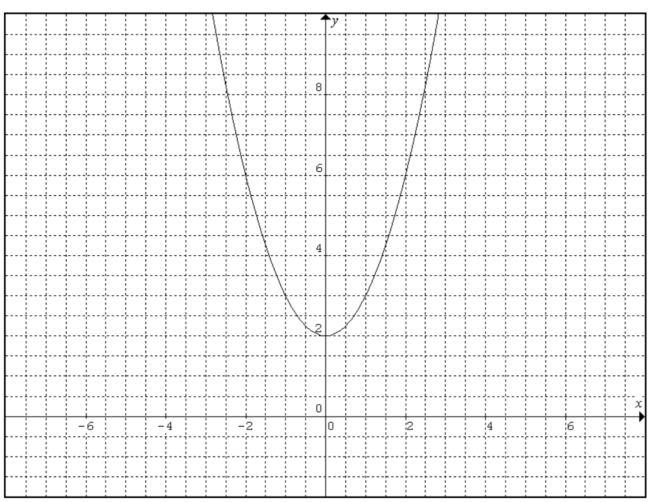
(c) $f(x) + c = \underline{x^2 + c}$

(ii)

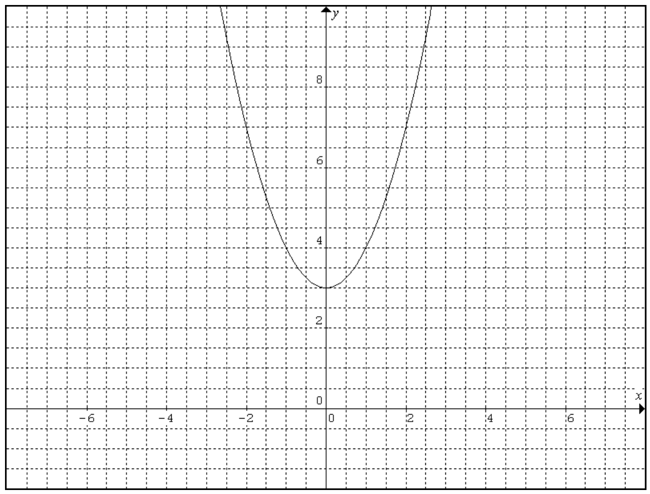
(a) $y = f(x)$



(b) $y = f(x) + 2$



(c) $y = f(x) + 3$

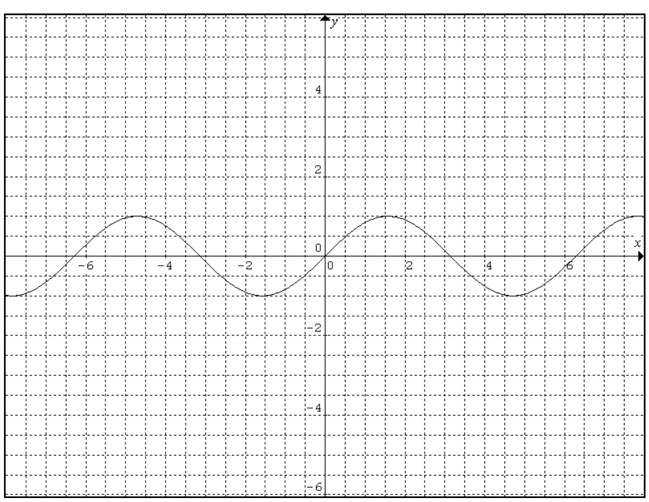


(iii) the transformation of $y = f(x)$ which produces the graphs of $y = f(x) + 2$ and $y = f(x) + 3$, is **<u>a shift of 2 and 3 units respectively in the positive *y*-direction</u>**. The graphical effect of transforming $y = f(x)$ into $y = f(x) + c$ **<u>is a shift (or translation) of *c* units in the positive *y*-direction</u>**. If *c* is negative, the graph shifts in the negative *y*- direction by *c* units..

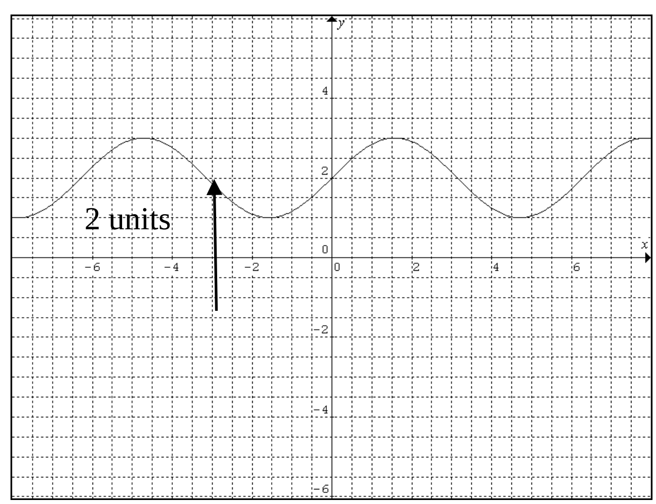
(iv) $g(x) + c$ = $\underline{\sin(x) + c}$.

(v) The graph of *g*(*x*) is also shifted by 2 units in the positive *y*-direction by the transformation *g*(*x*) + 2, so the result in (iii) still applies:

$y = g(x)$



$y = g(x) + c$ where $c = 2$



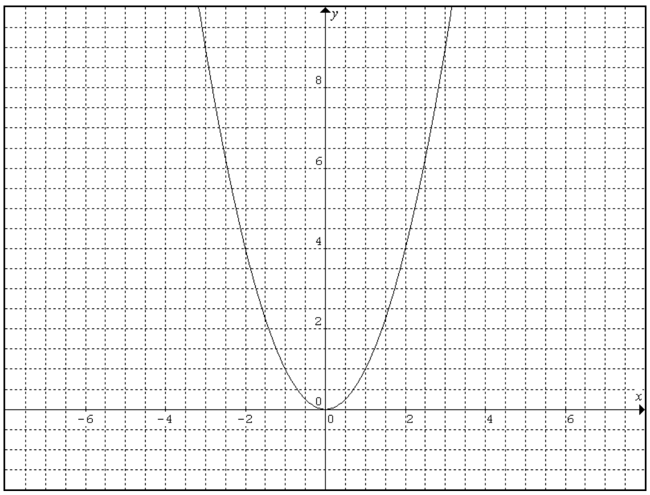
2. (i) (a) $f(x+2) = \underline{(x+2)^2}$

(b) $f(x+3) = \underline{(x+3)^2}$

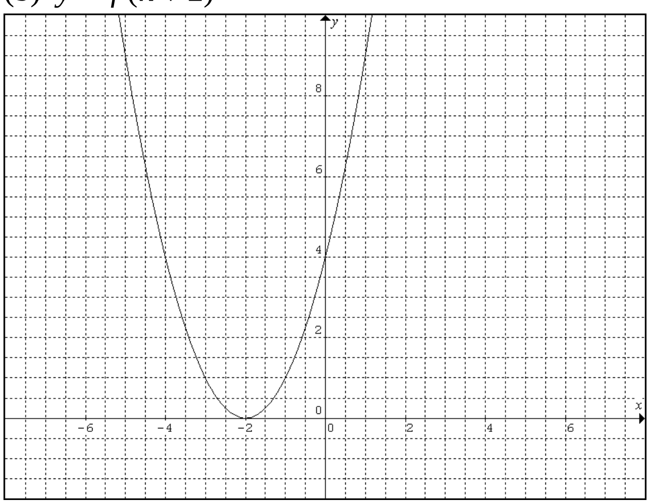
(c) $f(x+c) = \underline{(x+c)^2}$

(ii)

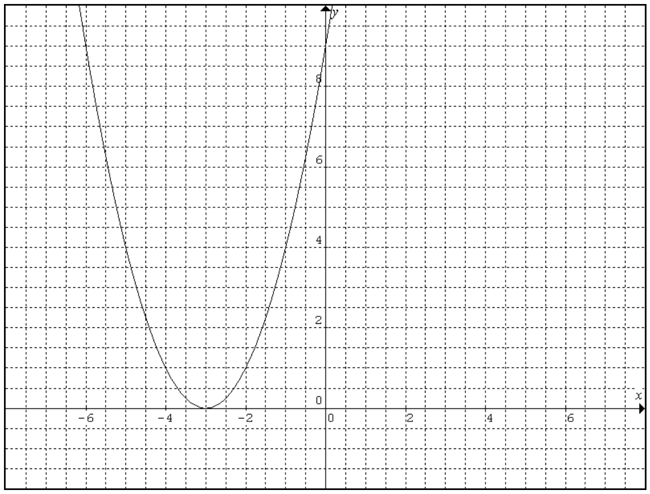
(a) $y = f(x)$



(b) $y = f(x+2)$



(c) $y = f(x+3)$

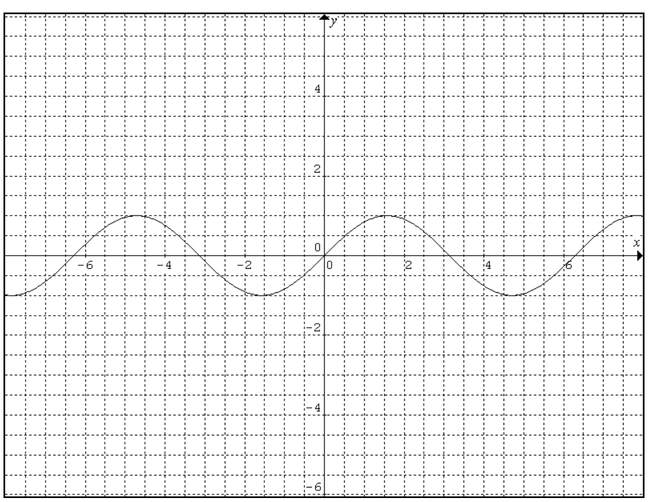


(iii) the transformation of $y = f(x)$ which produces the graphs of $y = f(x+2)$ and $y = f(x+3)$, is **<u>a shift of 2 and 3 units respectively in the negative *x*-direction</u>**. The graphical effect of transforming $y = f(x)$ into $y = f(x+c)$ is **<u>a shift (or translation) of *c* units in the negative *x*-direction</u>**. If *c* is negative, the graph shifts in the positive *x*- direction by *c* units.

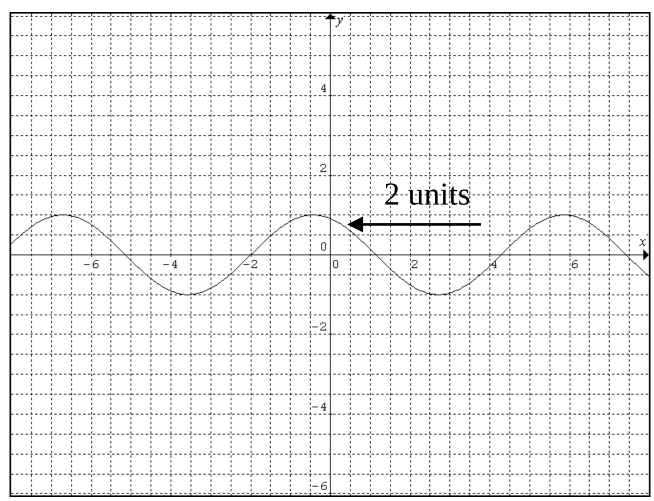
(iv) $g(x) + c = \underline{\sin(x+c)}$.

The graph of *g*(*x*) is also shifted by 2 units in the negative *x*-direction by the transformation *g*(*x*+2), so the result in (iii) still applies:

$y = g(x)$



$y = g(x+c)$ where $c = 2$

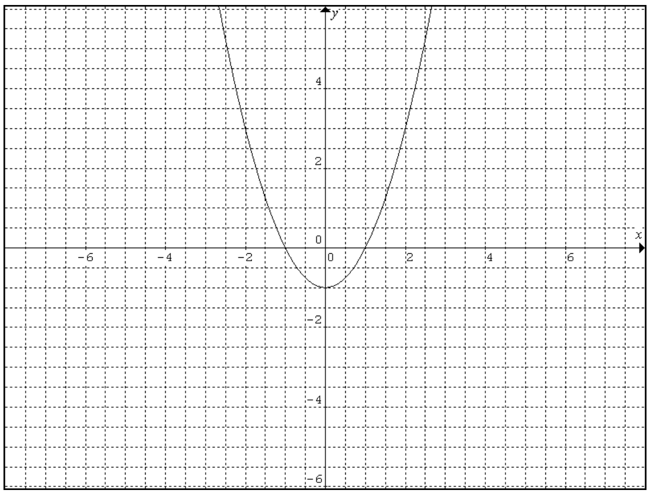


3. (i)(a) $2h(x) = 2(x^2 - 1)$

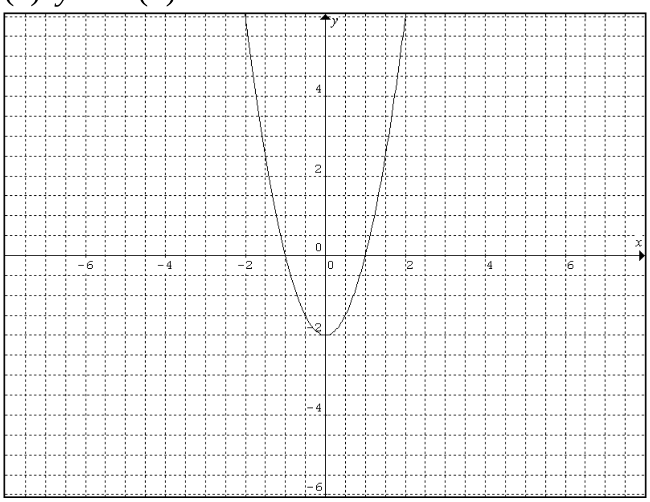
(b) $3h(x) = 3(x^2 - 1)$

(c) $kh(x) = k(x^2 - 1)$

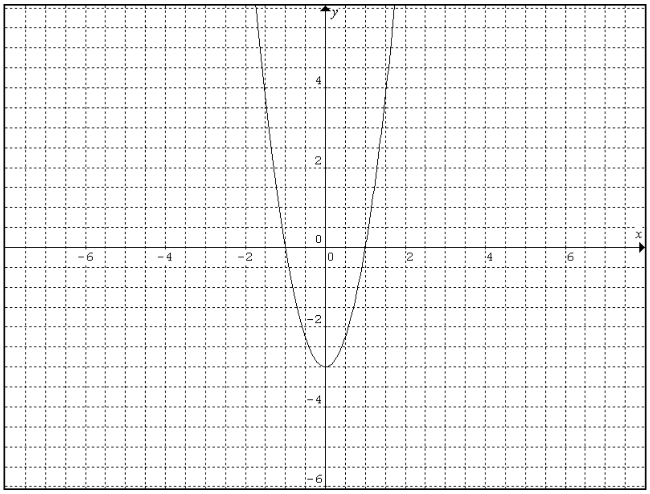
(ii) (a) $y = h(x)$



(b) $y = 2h(x)$



(c) $y = 3h(x)$

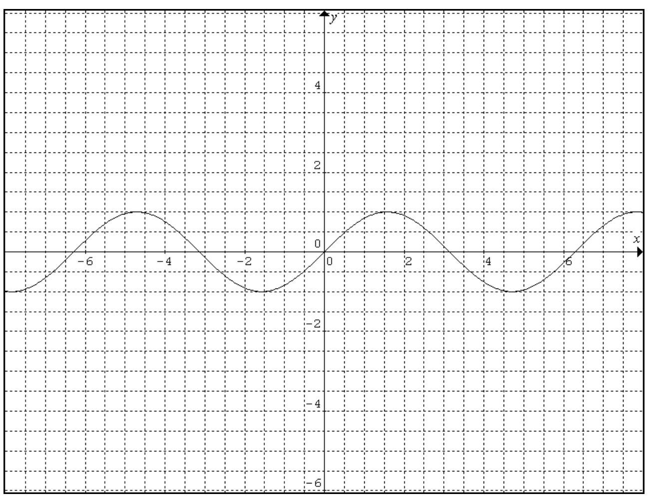


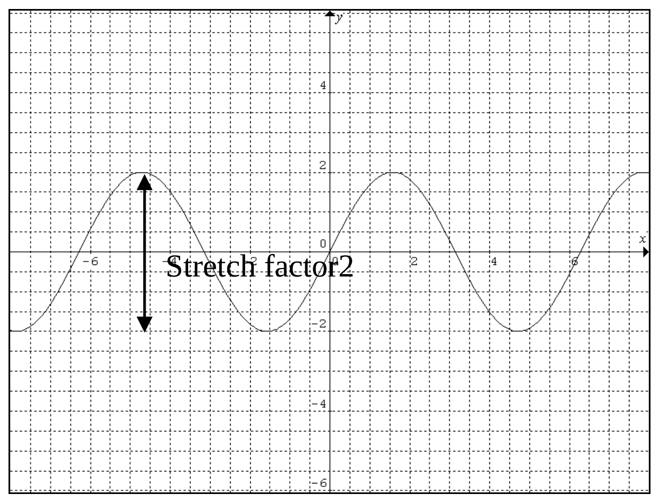
(iii) the transformation of $y = h(x)$ which produces the graphs of $y = 2h(x)$ and $y = 3h(x)$, is **<u>a stretch of magnitude 2 and 3 respectively in the *y*-direction with the *x*-axis invariant</u>**. The graphical effect of transforming $y = h(x)$ into $y = kh(x)$ is **<u>a stretch of magnitude *k* in the *y*-direction with the *x*-axis invariant</u>**.

(iv) $kg(x) = \underline{k \sin x}$.

The graph of $g(x)$ is also stretched by a factor of 2 in the *y*-direction by the transformation $2\sin x$, with the *x*-axis invariant, so the result in (iii) still applies:

$y = g(x)$ $\qquad$ $y = kg(x)$ where $c = 2$





4. (i)

(a) $h(2x) = (2x)^2 - 1$

(b) $h(3x) = (3x)^2 - 1$

(c) $h\left(\frac{x}{2}\right) = \left(\frac{x}{2}\right)^2 - 1$

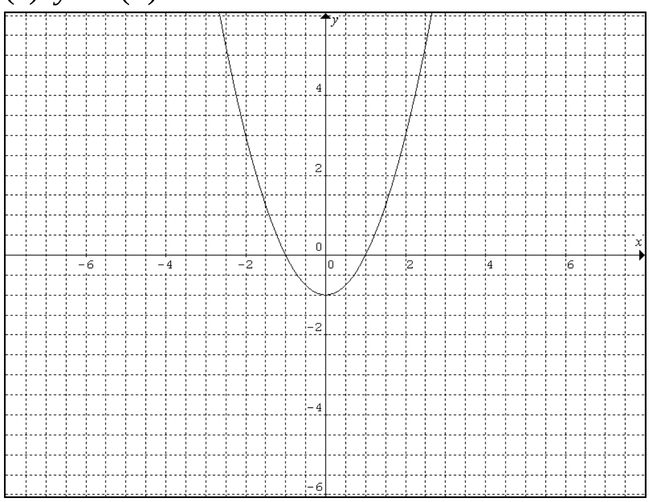
(d) $h\left(\frac{x}{3}\right) = \left(\frac{x}{3}\right)^2 - 1$

(e) $h(kx) = (kx)^2 - 1$

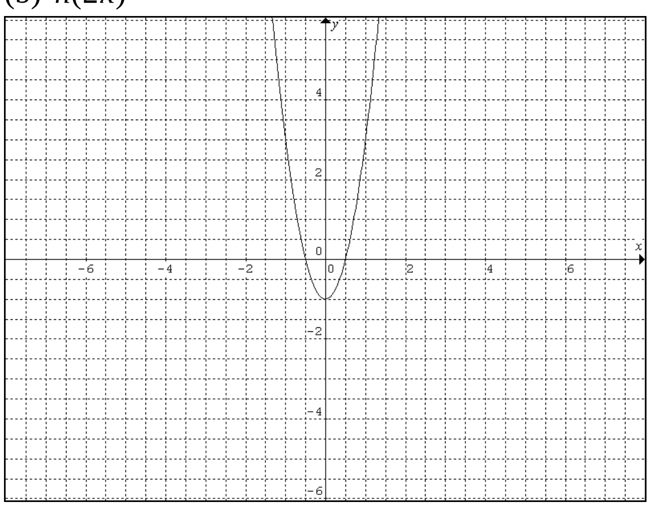
(f) $h\left(\frac{x}{k}\right) = \left(\frac{x}{k}\right)^2 - 1$

(ii)

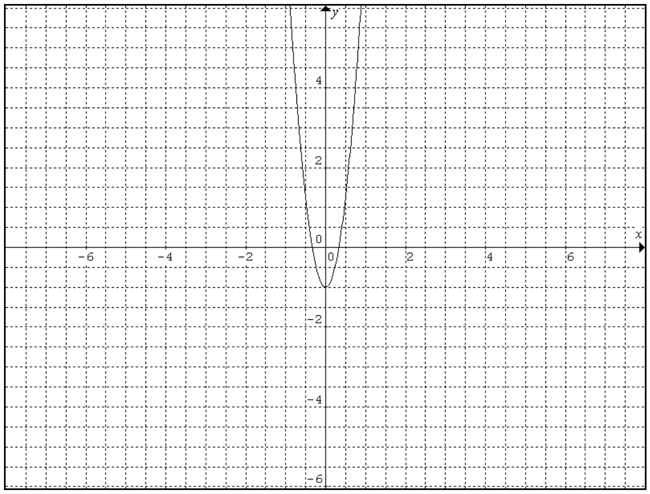
(a) $y = h(x)$



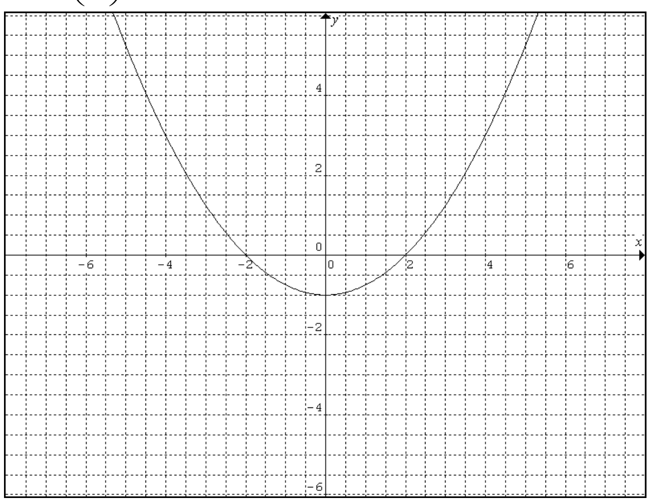
(b) $h(2x)$



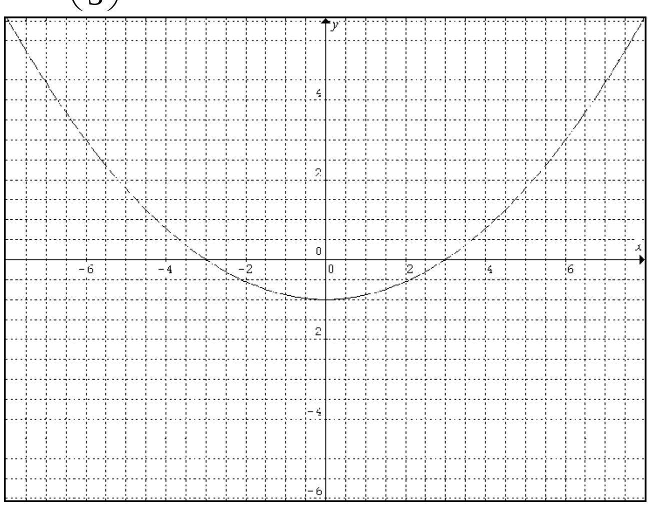
(c) $h(3x)$



(d) $h\left(\dfrac{x}{2}\right)$



(e) $h\left(\dfrac{x}{3}\right)$



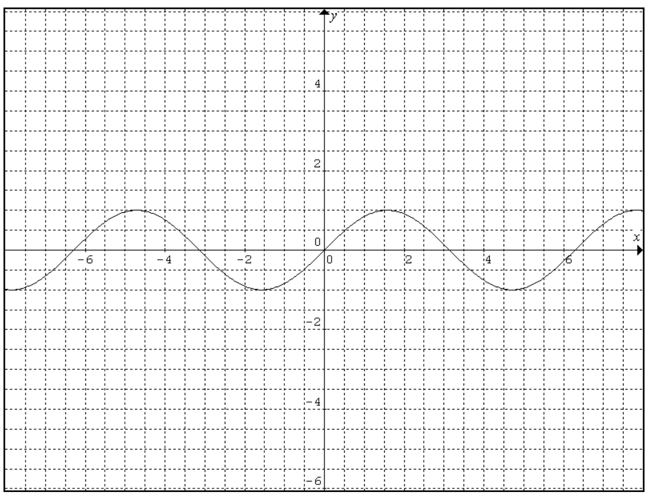
(iii) the transformation of $y = h(x)$ which produces the graphs of $y = h(2x)$, $y = h(3x)$, $y = h\left(\frac{x}{2}\right)$ and $y = h\left(\frac{x}{3}\right)$, is **a stretch of magnitude ½, ⅓, 2, and 3 units respectively in the x-direction with the y-axis invariant**. The graphical effect of transforming $y = h(x)$ into $y = h(kx)$ and $y = h\left(\frac{x}{k}\right)$ is **a stretch of magnitude $1/k$ and k units respectively in the x-direction with the y-axis invariant**.

(iv) $g(kx) = \sin kx$.

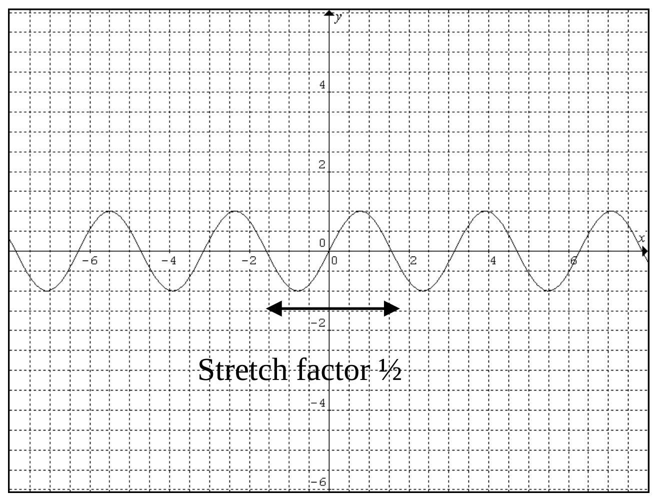
$g\left(\frac{x}{k}\right) = \sin\left(\frac{x}{k}\right)$.

The graph of $g(x)$ is also stretched by a factor of ½ in the x-direction by the transformation $\sin 2x$, with the y-axis invariant, and the graph of $g(x)$ is also stretched by a factor of 2 in the x-direction by the transformation $\sin\left(\frac{x}{2}\right)$, with the y-axis invariant, so the result in (iii) still applies:

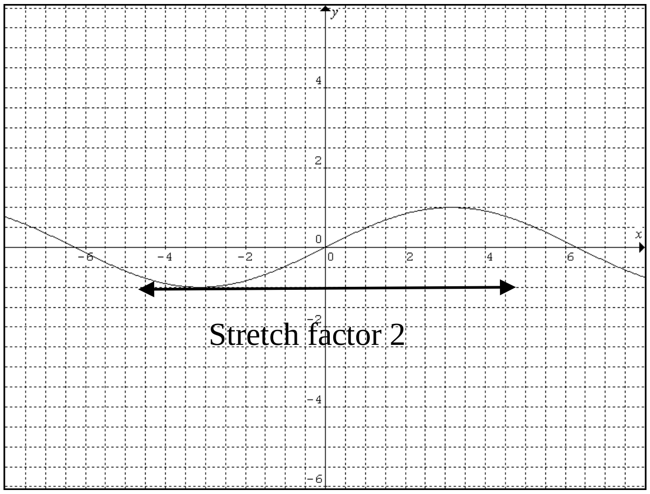
$y = g(x)$



$y = g(kx)$ where $c = 2$



$y = g\left(\frac{x}{k}\right)$ where $c = 2$

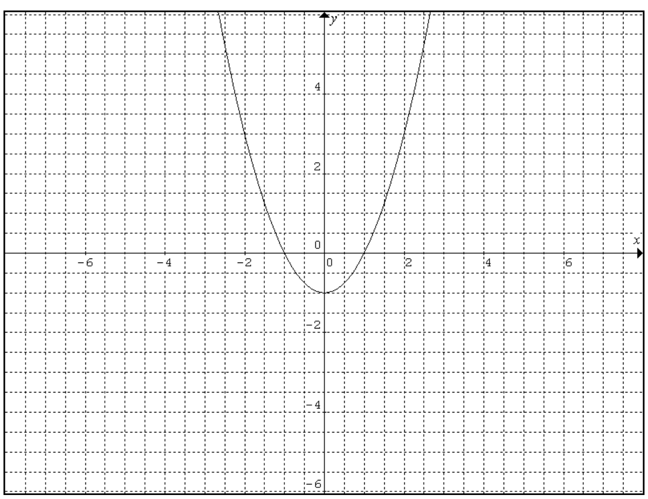


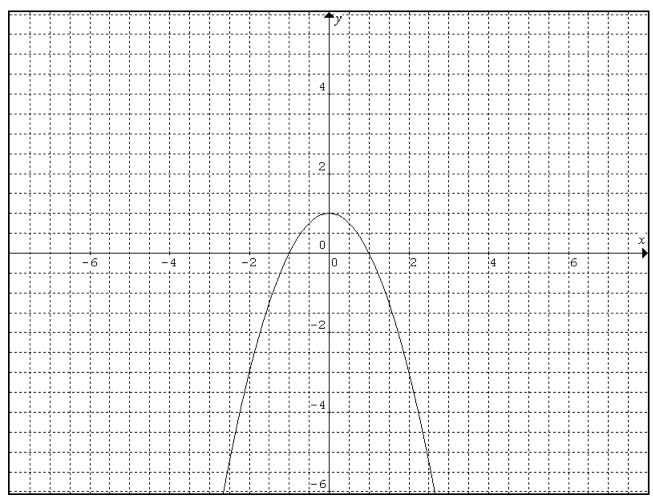
5. (i)

(a) $-h(x) = -(x^2 - 1) = 1 - x^2$

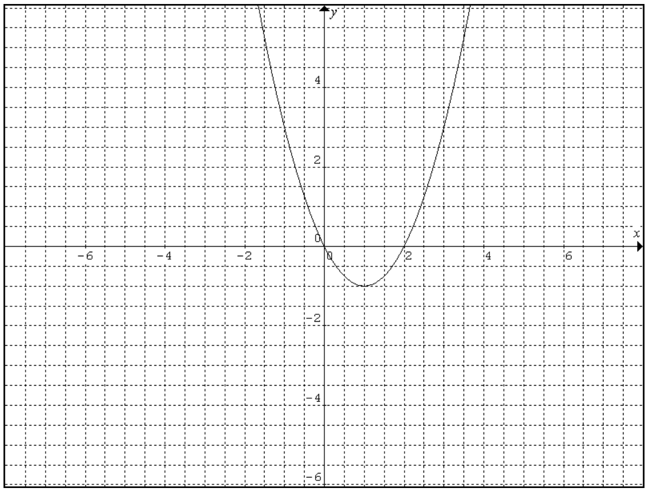
(b) $p(-x) = (-x)^2 - 2(-x) = x^2 + 2x$

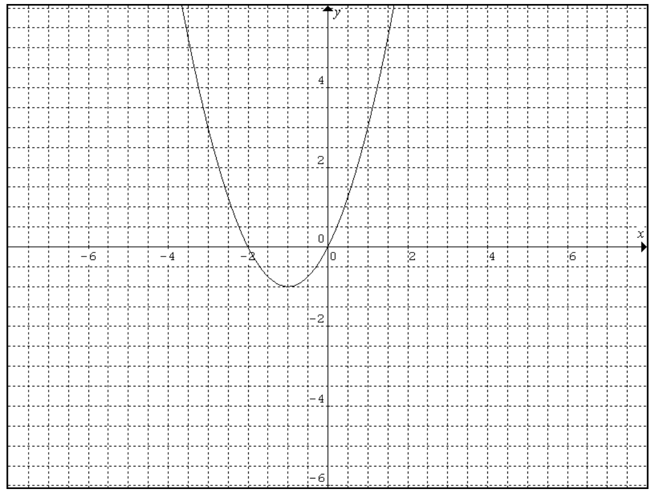
(ii) (a) $y = h(x)$ (c) $y = -h(x)$





(b) $y = p(x)$ (d) $y = p(-x)$





(iii) the transformation of $y = h(x)$ which produces the graph of $y = -h(x)$, is **<u>a reflection in the *x*-axis</u>**. The transformation of $y = p(x)$ which produces the graph of $y = p(-x)$, is **<u>a reflection in the *y*-axis</u>**.

6. In summary:

(a) $f(x) + c$ is a translation (or shift) of c units in the positive *y*-direction.

(b) $f(x + c)$ is a translation (or shift) of c units in the negative *x*-direction.

(c) $kf(x)$ is a stretch of magnitude k in the *y*-direction with the *x*-axis invariant.

(d) $f(kx)$ is a stretch of magnitude $\frac{1}{k}$ in the *x*-direction with the *y*-axis invariant.

(e) $f\left(\frac{x}{k}\right)$ is a stretch of magnitude k in the *x*-direction with the *y*-axis invariant.

(f) $-f(x)$ is a reflection in the *x*-axis.

(g) $f(-x)$ is a reflection in the *y*-axis.