



# SECOND PARTIAL





# QUIZZES



# QUIZ 1

82

Prepa Tec  
Calculus I 2nd partial  
Quiz # 1B

Name BARBARA ANCAL Mat. A01570127

I. Circle the right answer. (5 point each)

1) The following functions is not differentiable at  $x = 2$

a)  $f(x) = \frac{x+1}{x-2}$     b)  $f(x) = (x-2)^2$     c)  $f(x) = \frac{x^2}{x+2}$     d)  $f(x) = \sqrt{x+2}$

2) The following function is not differentiable at  $x = 1$

a)  $f(x) = |x+1|$     b)  $f(x) = x^3 - 1$     c)  $f(x) = \frac{1}{x+1}$     d)  $f(x) = \sqrt{x-1}$

3) Find the slope for  $f(x) = -4x^3$  at  $x = 2$

A) 32    B) -32    C) -96     D) -48

4) What is the equation of the tangent line for the curve  $y = x^3 + 2$  at the point  $(-2, 6)$

A)  $y = -12x - 30$     B)  $y = 12x - 30$      C)  $y = 12x + 18$     D)  $y = -12x - 18$

II. Answer the following questions.

1. The concentration of a drug in the bloodstream is given by: (15 points)  
 $C = 870 - 2t^2$ . Where  $t$  is measured in minutes. Find the rate of change in the concentration at 30 minutes.

$C = -4t$      $C = -4(30)$      $C = -120$

2. The following graph shows the function  $y = f(x)$  (20 points)

a) Find the values of "x" where the function is not continuous 1, -2

b) Find the values of "x" where the function is not differentiable 1, 2

III. Find the derivative by definition of the following function: (15 points)

$f(x) = 4x^2 + 7$

$f'(x) = \lim_{h \rightarrow 0} \frac{(4(x+h)^2 + 7) - (4x^2 + 7)}{h}$

$f'(x) = \lim_{h \rightarrow 0} \frac{(4x^2 + 8xh + 4h^2 + 7) - (4x^2 + 7)}{h}$

$f'(x) = \lim_{h \rightarrow 0} \frac{8xh + 4h^2}{h}$

$f'(x) = \lim_{h \rightarrow 0} (8x + 4h)$

$f'(x) = 8x + 4(0)$      $f'(x) = 8x$

IV. Find the derivative of the following:

a)  $f(x) = \frac{7}{2x^2} - 6x^8 + 3\sqrt[3]{x^4}$  (10 points)

$f'(x) = \frac{7x^{-2}}{2} - 48x^7 + 3x^{4/3}$

$f'(x) = \frac{-14x^{-3}}{2} - 48x^7 + \frac{12}{5}x^{-1/3}$

$f'(x) = \frac{-7}{x^3} - 48x^7 + \frac{12}{5\sqrt[3]{x}}$  (simplify!)

b)  $f(x) = \sqrt[3]{7x+2} - 3(1-5x^2)^5$  (10 points)

$f'(x) = (7x+2)^{-2/3} - 18(1-5x^2)^4 \cdot (-10x)$

$f'(x) = \frac{1}{3}(7x+2)^{-2/3} \cdot 18(-10x)(1-5x^2)^4$

$f'(x) = \frac{1}{3} \cdot \frac{18}{\sqrt[3]{(7x+2)^2}} \cdot (-18)(-10x)(1-5x^2)^4$

Simplify!

c)  $f(x) = 5(6x-9x^3)^7$  (10 points)

$f'(x) = 35(6x-9x^3)^6 \cdot (6-27x^2)$

$f'(x) = 35(6-27x^2)(6x-9x^3)^6$

# QUIZ 2

Prepa Tec  
Calculus I 2nd partial  
Quiz # 2B

81

Name Barbara Aneav Mat. A01590137

I. Determine if true or false for each of the following statements (5 points each)

- T The derivative of  $y = \ln(x-5)^{1/2}$  is  $y' = \frac{3}{2(x-5)}$   $\frac{1/2(x-5)^{-1/2}(1)}{2(x-5)}$   $\frac{1}{2\sqrt{x-5}}$   $4 - 2e^{-2x} = 4 + 2e^{-2x}$
- F The derivative of  $y = 4x - e^{-2x}$  is  $y' = 4 - 2e^{-2x}$   $4 - (-2)e^{-2x}$
- F The derivative of  $y = \frac{7x^2 - 3x}{5x^3}$  is  $y' = \frac{14x - 3}{15x^2}$   $\frac{14x - 3}{15x^2}$
- T If the velocity of the car is a function of time, then the derivative of this function with respect to time, describes the acceleration of the car.

II. Circle the right answer. (10 points each)

- The derivative for  $y = 3e^{2/x}$  is:  $3(\frac{-2x^{-2}}{x^2})e^{2/x} = \frac{-6e^{2/x}}{x^2}$ 

A)  $y' = 3e^{2/x}$  B)  $y' = -\frac{6e^{2/x}}{x^2}$  C)  $y' = 3e^2$  D)  $y' = 6x^2e^{2/x}$
- The derivative for  $y = \ln\sqrt{3x-6}$  is:  $y' = \frac{1}{2\sqrt{3x-6}} \cdot \frac{3}{2\sqrt{3x-6}}$   $y' = \frac{3}{2(3x-6)}$ 

A)  $y' = \frac{1}{2}\ln(3x-6)^{1/2}$  B)  $y' = \frac{1}{3x-6}$  C)  $y' = \frac{1}{2}\ln\frac{3}{\sqrt{3x-6}}$  D)  $y' = \frac{1}{2(3x-6)}$
- If the equation that gives the velocity of an object is  $v(t) = 2t^3e^{4t}$ , then the equation that gives the acceleration is:  $a = 2t^3(4e^{4t}) + (e^{4t})(6t^2)$   $a = 12t^3e^{4t} + 6t^2e^{4t} + 2$   $a = 6t^2e^{4t} + 2(2t+1)$ 

A)  $a(t) = 6t^2e^{4t}$  B)  $a(t) = 36t^2e^{4t}$  C)  $a(t) = 6t^2e^{4t}(2t+1)$  D)  $a(t) = 12t^3e^{4t}$

III. Answer the following questions.

- Find the SLOPE of the line tangent to  $y = \frac{e^{-2x}}{8}$  at  $x = 3/2$  (20 points)
 

$y = 8(-2e^{-2x}) - (e^{-3-2x})(0)$   $y = -2e^{-3-2x}$

$y = \frac{8(-2e^{-3-2x})}{64}$   $m = -\frac{1}{4}$

2) Find the derivative of  $f(x) = \frac{(3x-1)^4}{x}$  (15 points)

$f'(x) = \frac{x(4(3x-1)^3(3)) - (3x-1)^4(1)}{x^2}$

$f'(x) = \frac{(3x-1)^3 [12x - (3x-1)]}{x^2}$

$f'(x) = \frac{(3x-1)^3 (12x - 3x + 1)}{x^2}$  *simplify*

$f'(x) = \frac{(3x-1)^3 (9x+1)}{x^2}$

3) Find the derivative  $g(x) = 6x^2 + \frac{1}{e^{4x}} + \ln(7x^2+9) + e^2$  (15 points)

$g'(x) = 12x + e^{-4x} - \frac{14x}{7x^2+9} + 0e^2$

$g'(x) = 12x + 4e^{-4x} - \frac{14x}{7x^2+9}$

$g'(x) = 12x - \frac{4}{e^{4x}} - \frac{14x}{7x^2+9}$  *wrong simplification*



# CLASSWORKS

# More rules on derivatives (with out functions)

More on rules of derivatives  
By: Designing team

91  
21  
23

Name: Barbara Alvear Id: AD1570137 Date: 25/09/17

1. If  $f(5)=1$ ,  $f'(5)=6$ ,  $g(5)=-3$ ,  $g'(5)=2$ . Find the values of

a)  $(f \cdot g)'(5) = f'g + fg' = (1)(2) + (-3)(6) = 2 - 18 = -16$   
 $(f \cdot g)'(5) = \frac{fg' - gf'}{g^2} = \frac{(-3)(6) - (1)(2)}{(-3)^2} = \frac{-18 - 2}{9} = \frac{-20}{9} = (f/g)'(5) = -20/9$   
 $(g/f)'(5) = \frac{fg' - gf'}{f^2} = \frac{(1)(2) - (-3)(6)}{1^2} = \frac{20}{1} = 20$

2. If  $f(3)=4$ ,  $g(3)=2$ ,  $f'(3)=-6$  and  $g'(3)=5$ , find the following values

a)  $(f+g)'(3) = f'(3) + g'(3) = -6 + 5 = -1$   
 $(f \cdot g)'(3) = f'g + fg' = (-6)(2) + (4)(5) = -12 + 20 = 8$   
 $(f/g)'(3) = \frac{fg' - gf'}{g^2} = \frac{(4)(5) - (-6)(2)}{2^2} = \frac{20 + 12}{4} = \frac{32}{4} = 8$

3. If  $h(x) = f(x)g(x)$ , use the table to find  $h'(-1)$ ,  $h'(0)$  and  $h'(1)$

x	f(x)	f'(x)	g(x)	g'(x)
-1	2	1	1	2
0	-1	0	-1	3
1	2	-1	0	5

$h'(x) = fg' + gf'$   
 $h'(-1) = (2)(2) + (1)(1) = 5$   
 $h'(0) = (-1)(3) + (0)(-1) = -3$   
 $h'(1) = (2)(5) + (-1)(0) = 10$

4. If  $h(x) = f(x)/g(x)$ , use the table to find  $h'(-1)$ ,  $h'(0)$  and  $h'(1)$

x	f(x)	f'(x)	g(x)	g'(x)
-1	2	1	1	2
0	-1	0	-1	3
1	2	-1	2	5

$h'(x) = \frac{fg' - gf'}{g^2}$   
 $h'(-1) = \frac{(2)(2) - (1)(1)}{1^2} = 1$   
 $h'(0) = \frac{(-1)(3) - (0)(-1)}{(-1)^2} = -3$   
 $h'(1) = \frac{(2)(5) - (-1)(2)}{2^2} = \frac{12}{4} = 3$

5. Considering that  $P(x) = F(x)G(x)$  and  $Q(x) = F(x)/G(x)$ , where  $F$  and  $G$  are functions whose graphs are shown below.

a) Find  $P'(2)$

$P'(2) = F'G + FG'$   
 $P'(2) = (3)(1/2) + (2)(2) = 1.5 + 4 = 5.5$

b) Find  $Q'(7)$

$Q'(7) = \frac{FG' - F'G}{G^2}$   
 $Q'(7) = \frac{(1)(1/4) - (5)(2/3)}{(2/3)^2} = \frac{1/4 - 10/3}{4/9} = \frac{3/12 - 40/12}{4/9} = \frac{-37/12}{4/9} = -\frac{37}{12} \cdot \frac{9}{4} = -\frac{333}{48} = -\frac{111}{16}$

6. Consider that  $h(x) = f(g(x))$ , find  $h'(-1)$ ,  $h'(0)$  and  $h'(1)$

$h'(x) = f'(g(x))g'(x)$   
 $h'(-1) = f'(g(-1))g'(-1)$   
 $h'(0) = f'(g(0))g'(0)$   
 $h'(1) = f'(g(1))g'(1)$

x	f(x)	f'(x)	g(x)	g'(x)
-1	2	1	1	2
0	-1	0	-1	3
1	2	-1	0	5

$h'(-1) = f'(2)g'(1) = (1)(2) = 2$   
 $h'(0) = f'(-1)g'(-1) = (0)(2) = 0$   
 $h'(1) = f'(2)g'(0) = (1)(3) = 3$

7. Consider that  $h(x) = f(g(x))$ , where  $f$  and  $g$  are functions whose graphs are shown below.

$h(-2) = f(g(-2))$   
 $h(-2) = f(3) = 0.5$   
 $h'(3) = f'(g(3))g'(3)$   
 $h'(3) = f'(1)g'(3) = (1)(2) = 2$

a) Evaluate  $h(-2)$  and  $h(3)$   
 b) Is  $h'(-3)$  positive, negative or zero? Explain your answer. **positive, because it is 2**  
 c) Is  $h'(-1)$  positive, negative or zero? Explain your answer. **positive, because it is 0.3**

8. If  $f(x)$  and  $g(x)$  are the functions whose graphs are shown, let  $u(x) = f(x) \cdot g(x)$  and  $v(x) = f(x)/g(x)$



a) Find  $u'(1)$

$u'(x) = fg' + gf'$   
 $u'(1) = (2)(1) + (1)(2) = 4$

b) Find  $v'(5)$

$v'(x) = \frac{fg' - f'g}{g^2}$   
 $v'(5) = \frac{(2)(3) - (3)(2/3)}{2^2} = \frac{6 - 2}{4} = 1$

# HW: Rules of Differentiation- Exponential Functions

**HW: Rules of Differentiation- Exponential Functions**  
By: Ing Ziad Najjar

Name: Barbara Alwadi ID: 20191019 Date: \_\_\_\_\_

**Find the derivative of the following functions: BOX YOUR ANSWER**

<p>If <math>f(x) = e^u</math> then <math>f'(x) = U' \cdot e^u</math></p>	<p>If <math>f(x) = a^u</math> then <math>f'(x) = U' (\ln a) a^u</math></p>
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<p>1) <math>f(x) = \sqrt{2x + e^{10x}}</math>  <math>f'(x) = (2x + e^{10x})^{1/2}</math>  <math>f'(x) = 1/2 (2x + e^{10x})^{-1/2} (2 + 10e^{10x})</math>  <math>f'(x) = 1/2 (2 + 10e^{10x}) \cdot \frac{1}{2\sqrt{2x + e^{10x}}}</math>  <math>f'(x) = \frac{2 + 10e^{10x}}{2\sqrt{2x + e^{10x}}}</math></p>	<p>2) <math>f(x) = 4e^{x/2} + \frac{5}{2x^2}</math>  <math>f'(x) = \frac{1}{2} \cdot 4 \cdot e^{x/2} + \frac{5}{2} \cdot x^{-3}</math>  <math>f'(x) = 2e^{x/2} + (-5)x^{-3}</math>  <math>f'(x) = 2e^{x/2} - \frac{5}{x^3}</math></p>
<p>3) <math>f(x) = \frac{2x^3}{e^{2x}}</math>  <math>f'(x) = \frac{2x^3 \cdot (-2e^{2x}) - e^{2x} (6x^2)}{(e^{2x})^2}</math>  <math>f'(x) = \frac{2x^2(-7x+3)}{e^{2x}}</math></p>	<p>4) <math>y = x^4(e^{1-2x})</math>  <math>y' = 4x^3(-2e^{-2x})</math>  <math>y' = -8x^3(e^{-2x})</math></p>
<p>5) <math>y = \frac{3}{e^{2x}}</math>  <math>y' = 3(e^{-2x})</math>  <math>y' = 3(-2e^{-4x})</math>  <math>y' = \frac{-12}{e^{4x}}</math></p>	<p>6) <math>y = \frac{e^{2x}}{2x} = 2x \left( \frac{2x}{2x^2} \right) - (e^{2x})(2)</math>  <math>y' = e^{2x} \cdot 2x^{-1} - 2e^{2x}</math>  <math>y' = 2e^{2x} \left( \frac{1}{x} - 2 \right)</math>  <math>y' = \frac{2e^{2x}}{x} - 4e^{2x}</math></p>
<p>7) <math>y = e^{3x}(2x-1)^4</math>  <math>y' = e^{3x} \cdot 4(2x-1)^3 \cdot (2)</math>  <math>y' = e^{3x} \cdot 8(2x-1)^3</math>  <math>y' = 8e^{3x} \cdot (2x-1)^3</math></p>	<p>8) <math>f(x) = \frac{e^{2x}}{6} + 2x^5</math>  <math>f'(x) = \frac{e^{2x}}{6} \cdot (2) + 10x^4</math>  <math>f'(x) = \frac{2e^{2x}}{6} + 10x^4</math>  <math>f'(x) = \frac{e^{2x}}{3} + 10x^4</math></p>



# Partial Project

<https://www.geogebra.org/m/ru7QjnYN>





# THIRD PARTIAL





# QUIZZES



# QUIZ 1

Prepa Tec  
Calculus I 3rd partial  
Quiz # 1B

70  
No Procedures  
-10

Name Barbara Alvarez Mat. A01570137

I. Determine if true or false for each of the following statements (10 points each)

- The third derivative of  $y = 4e^{3x}$  is  $\frac{d^3y}{dx^3} = 36e^{3x}$ .  $y' = 12e^{3x}$   $y'' = 36e^{3x}$   $y''' = 108e^{3x}$
- The derivative of  $4x^2y - 8x = 6y^2 + 5$  is  $\frac{dy}{dx} = \frac{2}{2x-3y} = \frac{(4x^2y' + 8xy) - 12yy' = 8}{4x^2 - 12y}$   $y' = \frac{8-8xy}{4(x^2-3y)}$
- The derivative of  $y = x^{3 \ln x}$  is  $y' = x^{3 \ln x} (3 \ln x + 3x + 1)$
- The area of a circle is decreasing at a rate of  $40 \text{ cm}^2$  per hour. Then the rate at which its radius is changing when its radius measures  $50 \text{ cm}$  is  $\frac{dr}{dt} = \frac{2}{3\pi} \left[ \frac{\text{cm}}{\text{hour}} \right]$ .  $A = \pi r^2$   $A = \pi (2r)$   $40 = \pi (2) \frac{dr}{dt} = \frac{2}{3\pi}$

II. Answer the following problem. (10 points each letter)

A baseball is thrown upward while being in the moon (hypothetically), with an initial velocity of  $80 \text{ meters/second}$ . The height of the ball is given by  $s = 80t - 8t^2$

- The equation that gives the velocity of the ball at any time.  
 $v = 80 - 16t$
- The time when velocity is zero (that is the time to reach the maximum height)  
 $t = 5s$   $0 = 80 - 16(5)$
- The maximum height of the ball (that is when velocity is zero)  
 $h = 200m$   $80(5) - 8(5)^2$
- The times (on the way up and on the way down) when the height is at  $128 \text{ feet}$ .  
 $t = 25s$   $t = 85s$   $128 = 80t - 8t^2$   $v = 0$
- The velocities of the ball when the height is  $128 \text{ feet}$ .  
 $v = 48 \text{ m/s}$   $v = -48 \text{ m/s}$
- The equation that gives the acceleration of the ball at any time.  
 $a = -16$

# QUIZ 2

Prepa Tec Calculus I 3rd partial Quiz # 2B

Name BARBARA ANEAV Mat. A01570137 50

**Determine if true or false for each of the following statements. (5 points each)**

- ~~X~~ The function  $f(x) = -3x^3 - 3x^2 + 14$  has only one critical point.  $-9x^2 - 6x - 3x(2x-2)$
- ~~X~~ Let  $f$  be a function whose second derivative exists on an open interval, if  $f''(x) > 0$  for all  $x$  in that interval, then the graph of  $f$  is concave downward on that interval.
- ~~X~~ If " $a$ " is a critical point of the function  $f(x)$  that is continuous, and if  $f'(x) < 0$  at  $(-\infty, a)$  and  $f'(x) > 0$  at  $(a, \infty)$ , then,  $f(x)$  has a relative maximum at  $(a, f(a))$ .
- ~~X~~ The function  $f(x) = x^4 - 4$  has a relative maximum at  $(0, -4)$   
 $0 = 4x^3$   $0 = 4(x^3)$   $(-\infty, 0), (0, 1), (1, \infty)$

**Choose the right answer (10 points each)**

- ~~X~~ If  $(c, f(c))$  is a critical point, then  
 A)  $f'(c) = 0$  B)  $f'(c) < 0$  C)  $f'(c) = 0$  D)  $f'(c) > 0$
- ~~X~~ According to the second derivative test if  $f''(c) < 0$ , then  
 A)  $f(c)$  is concave upward.  
 B)  $f(c)$  is relative maximum.  
 C)  $f(c)$  is a critical point.  
 D)  $f(c)$  is a relative minimum.
- ~~X~~ The function  $f(x) = 20x - x^3$  has a critical point:  
 $f'(x) = 20 - 2x^2$   $x = 10$   
 $2(10 - x)$   
 A)  $x = -10$  B)  $x = 10$  C)  $x = 1$  D)  $x = 0$
- ~~X~~ If  $f''(c) > 0$  then  $f(x)$  is:  
 A) concave downward B) concave upward C) decreasing D) increasing
- ~~X~~ It can be determined if the curve of  $y=f(x)$  has a change of concavity:  
 A) inflection point B) Critical point C) x-intercept D) y-intercept
- ~~X~~ The function  $y = -x^3 + 6x^2$  has a relative maximum at:  
 $-3x^2 + 12x$   $(-\infty, 0), (0, 4), (4, \infty)$   
 $-3x(x-4)$   $-1$   $3$   $S$   
 A)  $(4, 32)$  B)  $(0, 0)$  C)  $(4, 32)$  D)  $(6, 0)$   
 $+ \quad + \quad -$   
 $(4, 32)$
- ~~X~~ The function  $y = x^3 - 3x^2$  has a relative minimum at:  
 $3x^2 - 6x$   $(-\infty, 0), (0, 2), (2, \infty)$   
 $3x(x-2)$   $-1$   $1$   $3$   
 A)  $(1, -2)$  B)  $(3, 0)$  C)  $(0, 0)$  D)  $(2, -4)$   
 $+ \quad - \quad +$   
 $2(2, -4)$

**Answer the following showing your entire procedure.**

1. The following graph represents  $f(x)$  use it to sketch the graphs of  $f'(x)$ . (10 points)



# CLASSWORKS



# Applications of derivatives:

## Problems involving position, velocity and acceleration

Applications of derivatives  
Problems involving position, velocity and acceleration

Name: Barbara Aweav ID: A015107 Date: 20/10/17

Consider each of the following situations and answer clearly. Remember to use the appropriate mathematical notation and to frame your final answer.

76  $\frac{33}{43}$

- An object is moving along a straight line, and its position (in meters) is given by the function  $s(t) = 80t - t^2$ . Determine
  - The velocity of the object when  $t = 2$  sec.  $v'(t) = 80 - 2t$   $v'(2) = 80 - 2(2) = 76 \text{ m/s}$
  - The acceleration when  $t = 3$  sec.  $a'(t) = -2 \text{ m/s}^2$
  - The time when the velocity is zero and the position of the object at that time.  $t = 40 \text{ s}$   $s(t) = 1600 \text{ m}$
- An object is moving along a straight line, and its position (in meters) is given by the function  $s(t) = 3t + \frac{48}{t+1}$ . Determine
  - The velocity of the object when  $t = 2$  sec.  $v'(t) = 3 - 48/(t+1)^2$   $v'(2) = 3 - 48/9 = -2.33 \text{ m/s}$
  - The acceleration when  $t = 2$  sec.  $a'(t) = 96/(t+1)^3$   $a'(2) = 96/27 = 3.5 \text{ m/s}^2$
  - The time when the velocity is zero and the position of the object at that time.  $t = 3 \text{ s}$   $s(t) = 21 \text{ m}$
- A dynamite charge blows a rock up with a velocity of 160 feet/sec. The height of the rock is given by  $h(t) = 160t - 16t^2$  where "h" is measured in feet and "t" in seconds. Find
  - The equation that gives the velocity of the rock at any time.  $v(t) = 160 - 32t$
  - The time when the velocity is zero.  $t = 5 \text{ s}$
  - The height of the rock when the velocity is zero (maximum height).  $h = 400 \text{ ft}$
  - The times (on the way up and on the way down) when the height is 256 feet.  $t = 2 \text{ s}$  and  $t = 10 \text{ s}$
  - The velocities of the rock when the height is 256 feet.  $v = 96 \text{ ft/s}$  and  $v = -24 \text{ ft/s}$
  - The equation that gives the acceleration of the rock at any time.  $a(t) = -32$
  - How long does it take the rock to fall back down?  $t = 10 \text{ s}$
- A baseball is thrown upward while being in the moon (hypothetically), with an initial velocity of 24 meters/second. The height of the ball is given by  $s = 24t - 0.8t^2$ .
  - Find the equations of velocity and acceleration at any time.  $v = 24 - 1.6t$   $a = -1.6$
  - How long does it take the ball to reach its maximum height?  $t = 15 \text{ s}$
  - Find the maximum height of the ball.  $h = 180 \text{ m}$
  - How long was the ball in the air?  $t = 30 \text{ s}$
- The position of an object is given by  $S(t) = t^3 - 6t^2 + 9t$  where "t" is measured in seconds and "s" in meters.
  - Find the equations of velocity and acceleration as a function of time.  $v(t) = 3t^2 - 12t + 9$   $a(t) = 6t - 12$
  - Find the time when the velocity is zero.  $t = 2 \text{ s}$  and  $t = 3 \text{ s}$
  - Find the acceleration when the velocity is zero.  $a(t) = 6t - 12$   $a(2) = 0$   $a(3) = 6$
  - Find the time when the acceleration is zero and then give the velocity at that time.  $t = 2 \text{ s}$   $v = -3 \text{ m/s}$
- The height of a certain tree (starting from being 1 year old) is modeled by  $H(t) = 5\sqrt{t} + 2t^2 + 10$ , where height is measured in cm and time in years. Find:
  - The height of the tree in its 5th year (hint  $t=4$ ).  $82 \text{ cm}$
  - The function that models the rate of change of its height.  $H'(t) = \frac{5\sqrt{t}}{2} + 4t$
  - The rate of change when  $t=4$ .  $5$
  - The rate of change when  $t=9$ .  $8.5$
  - When is the tree growing faster? at  $t=4$  or  $t=9$  years? Why?   
At 4 years, because it grows more per year.

**CHALLENGE.** The following graph shows the position of a particle that moves along a straight line (author: Lic. Norma Patricia Salinas Martinez).

- In which interval or intervals is the velocity of the particle positive?
- In which interval or intervals is the velocity of the particle negative?
- In which interval or intervals of time is the position increasing faster?
- In which interval or intervals of time is the position increasing slower?
- In which interval or intervals of time is the position decreasing faster?
- In which interval or intervals of time is the position decreasing slower?
- In which interval or intervals of time is the velocity increasing?
- In which interval or intervals of time is the velocity decreasing?

# Related rates

**Related Rates**  
 By: Arq. Monica M. Paniagua, Ing. Ziad Najjar, Lic. Lucy Solis, Lic. Carmela de la fuente  
 Name Barbara Aweav ID # A01570137 Date 27/10/17

**Related rates**

- A spherical balloon is being filled at a rate of  $50 \text{ in}^3/\text{sec}$ , at what rate does the radius increase when the radius is  $5 \text{ in}$ ?  

$$\frac{dv}{dt} = 50 \frac{\text{in}^3}{\text{s}} \quad \frac{dr}{dt} = ? \quad V = \frac{4}{3} \pi r^3 \quad \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$50 = 4\pi (5)^2 \frac{dr}{dt} \quad \frac{dr}{dt} = \frac{50}{40\pi (5)^2} = \frac{dr}{dt} = 0.159 \text{ in/s}$$
- The area of a circle is increasing at a rate of  $20 \text{ in}^2/\text{min}$ . Find the rate at which the radius is increasing when the radius is  $4 \text{ in}$ .  

$$\frac{dA}{dt} = 20 \frac{\text{in}^2}{\text{min}} \quad A = \pi r^2 \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$20 = \pi (4)^2 \frac{dr}{dt} \quad \frac{dr}{dt} = \frac{20}{2\pi (4)^2} = 0.796 \text{ in/min}$$
- A stone is thrown into a lake and a circular ripple moves out at a constant rate of  $0.5 \text{ meters/sec}$ . Find the rate at which the circle's area is increasing at  $r = 0.4 \text{ meters}$ .  

$$v = 0.5 \text{ m/s} \quad A = \pi r^2 \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi (0.4) (0.5) = 1.257 \text{ m}^2/\text{s}$$
- Air is being pumped into a spherical balloon making the radius change at a constant rate of  $0.5 \text{ cm/sec}$ . Find the rate of change of the volume and the rate of change of the surface area when the radius is  $10 \text{ cm}$  ( $V = \frac{4}{3}\pi r^3$ ,  $A = 4\pi r^2$ ).  

$$r = 10 \text{ cm} \quad \frac{dr}{dt} = 0.5 \text{ cm/s} \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi (10)^2 (0.5) = 200\pi = 628.32 \text{ cm}^3/\text{s}$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi (10) (0.5) = 40\pi = 125.66 \text{ cm}^2/\text{s}$$
- A cone is increasing in size as time goes by in such a way that the volume is changing at a constant rate of  $75 \text{ cm}^3/\text{min}$ . The height is twice the radius. Determine the rate of change of the height, when the height is  $5 \text{ cm}$ . ( $V = \frac{1}{3}\pi r^2 h$ )  

$$V = \frac{1}{3}\pi r^2 h \quad \frac{dV}{dt} = \frac{\pi}{12} 2h^2 \frac{dh}{dt}$$

$$75 = \frac{\pi}{12} (5)^2 \frac{dh}{dt} \quad \frac{dh}{dt} = \frac{75 \cdot 12}{\pi (5)^2} = \frac{12}{\pi} = 3.82 \text{ cm/min}$$

- A farmer has a water reservoir that has the shape of a cylinder of  $5 \text{ ft}$  in diameter and  $14 \text{ ft}$  in height. Water is pumped from it at a constant rate of  $0.25 \text{ ft}^3/\text{min}$ . Find the rate at which the water level is lowering, when the water already has a level of  $10 \text{ ft}$ .  

$$\frac{dv}{dt} = -0.25 \text{ ft}^3/\text{min} \quad V = \pi r^2 h \quad \frac{dv}{dt} = \pi (2.5)^2 \frac{dh}{dt}$$

$$-0.25 = \pi (2.5)^2 \frac{dh}{dt} \quad \frac{dh}{dt} = \frac{-0.25}{25\pi} = -0.013 \text{ ft/min}$$
- A cube is increasing its size at a rate of  $3 \text{ cm}^3/\text{min}$ . Determine the rate of change of the volume and the rate of change of the surface area when the edge measures  $6 \text{ cm}$ .  

$$\frac{dv}{dt} = 3 \text{ cm}^3/\text{min} \quad a = 6 \text{ cm} \quad V = a^3 \quad \frac{dV}{dt} = 3a^2 \frac{da}{dt}$$

$$3 = 3(6)^2 \frac{da}{dt} \quad \frac{da}{dt} = \frac{1}{12} = 0.083 \text{ cm/min}$$
- A spherical balloon is being filled with air in such a way that the radius is increasing at a constant rate of change  $0.2 \text{ cm/s}$ . Find the rate of change of the volume when the radius is  $12 \text{ cm}$ .  

$$\frac{dr}{dt} = 0.2 \text{ cm/s} \quad V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi (12)^2 (0.2) = 115.2\pi = 361.9 \text{ cm}^3/\text{s}$$
- A cone is increasing in such a manner that its height is always three times its radius. Find the rate at which the radius is changing, at the moment when radius measures  $6 \text{ cm}$ , knowing that its volume is incrementing at a rate of  $60 \text{ cm}^3$  per minutes (Hint:  $V = \frac{1}{3}\pi r^2 h$ )  

$$h = 3r \quad V = \frac{1}{3}\pi r^2 h \quad \frac{dV}{dt} = \frac{\pi}{18} 2h^2 \frac{dr}{dt}$$

$$60 = \frac{\pi}{18} (18)^2 \frac{dr}{dt} \quad \frac{dr}{dt} = \frac{60}{9\pi} = 0.177 \text{ cm/min}$$
- A triangle is increasing in such a way that its height is always half of its base. Find the rate at which the area is changing, when the base measures  $12 \text{ cm}$ , if the base is incrementing at a rate of  $3 \text{ cm}$  per minute.  

$$h = \frac{b}{2} \quad A = \frac{b \times h}{2} \quad \frac{dA}{dt} = \frac{3b}{2} \frac{db}{dt}$$


$$\frac{dA}{dt} = \frac{3(12)}{2} (3) = 18 \text{ cm}^2/\text{min}$$

**ANSWERS**


- $\frac{1}{2\pi} = 0.159 \text{ in/sec}$
- $\frac{5}{2\pi} = 0.796 \text{ in/min}$
- $0.4\pi = 1.257 \text{ m}^2/\text{s}$
- $\frac{dV}{dt} = 200\pi = 628.32 \text{ cm}^3/\text{s}$ ,  $\frac{dA}{dt} = 40\pi = 125.66 \text{ cm}^2/\text{s}$
- $\frac{12}{\pi} = 3.82 \text{ cm/min}$
- $-\frac{1}{25\pi} = -0.013 \text{ ft/min}$
- $\frac{dV}{dt} = 324 \text{ cm}^3/\text{min}$ ,  $\frac{dA}{dt} = 216 \text{ cm}^2/\text{min}$
- $115.2\pi = 361.9 \text{ cm}^3/\text{sec}$
- $\frac{dr}{dt} = \frac{5}{9\pi} = 0.177 \text{ cm/min}$
- $\frac{dA}{dt} = 18 \text{ cm}^2/\text{min}$

# Applying the First derivative Test

Late

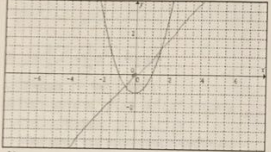


Applying the First Derivative Test  
By: Lucy Solís

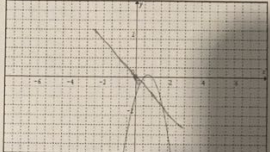


Name Barbara Alvarez Group 402 Date 10/11/17

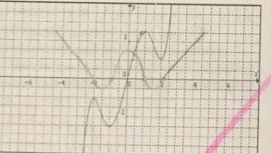
I. Use the graph to find the intervals in which the graph of  $f(x)$  is increasing or decreasing and sketch the derivative

1) 

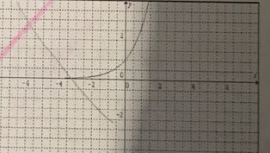
f(x) is increasing:  $(0, \infty)$

2) 

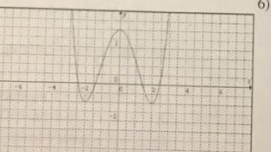
f(x) is increasing:  $(-\infty, -5)$

3) 

f(x) is decreasing:  $(-\infty, 0)$

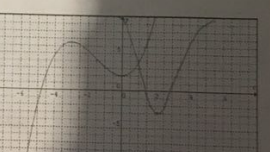
4) 

f(x) is decreasing:  $(2, \infty)$

5) 

f(x) is increasing:  $(-2, 0) \cup (2, \infty)$

f(x) is decreasing:  $(-\infty, -2) \cup (0, 2)$

6) 

f(x) is increasing:  $(-\infty, 1) \cup (3, \infty)$

f(x) is decreasing:  $(1, 3)$

II. For each of the following functions find:

1)  $f(x) = x^3 - 6x^2 + 9x + 1$

a) Domain  $(-\infty, \infty) \subset \mathbb{R}$

b) Derivative of  $f(x)$   $f'(x) = 3x^2 - 12x + 9$

c) Critical Values  $f'(x) = 0$  a decrease + exist

d) Maximum and minimum coordinates  $0 = 3x^2 - 12x + 9$

e) Intervals where the function is increasing  $0 = x^2 - 4x + 3$

f) Intervals where the function is decreasing  $0 = (x-3)(x-1)$

e)  $(-\infty, 1) \cup (3, \infty)$

f)  $(1, 3)$

int	$(-\infty, 1)$	$(1, 3)$	$(3, \infty)$
value	0	2	4
sign	+	-	+
inc	inc	dec	inc

2)  $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x + 8$

a) Domain  $(-\infty, \infty)$

b) Derivative of  $f(x)$   $f'(x) = x^2 + x - 6$

c) Critical Values  $f'(x) = 0$

d) Maximum and minimum coordinates  $0 = x^2 + x - 6$

e) Intervals where the function is increasing  $0 = (x+3)(x-2)$

f) Intervals where the function is decreasing

e)  $(-\infty, -3) \cup (2, \infty)$

f)  $(-3, 2)$

int	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
value	-5	0	4
inc	inc	dec	inc

3)  $f(x) = x^3 + x^2 - 5x - 5$

a) Domain  $(-\infty, \infty)$

b) Derivative of  $f(x)$   $f'(x) = 3x^2 + 2x - 5$

c) Critical Values  $f'(x) = 0$

d) Maximum and minimum coordinates  $0 = 3x^2 + 2x - 5$

e) Intervals where the function is increasing  $0 = x^2 + 2/3x - 5/3$

f) Intervals where the function is decreasing  $0 = (x-1)(x+5/3)$

e)  $(-\infty, -5/3) \cup (1, \infty)$

f)  $(-5/3, 1)$

int	$(-\infty, -5/3)$	$(-5/3, 1)$	$(1, \infty)$
value	-2	0	2
inc	inc	dec	inc

4)  $f(x) = x^4 - 8x^2 + 1$

a) Domain  $(-\infty, \infty)$

b) Derivative of  $f(x)$   $f'(x) = 4x^3 - 16x$

c) Critical Values  $f'(x) = 0$

d) Maximum and minimum coordinates  $0 = 4x^3 - 16x$

e) Intervals where the function is increasing  $0 = x^2 - 4x$

f) Intervals where the function is decreasing  $0 = x(x-4)$

e)  $(-2, 0) \cup (2, \infty)$

f)  $(-\infty, -2) \cup (0, 2)$

x=0, x=2, x=-2

5)  $g(x) = \frac{x^4}{4} - \frac{x^3}{3} - 3x^2 + 1$

a) Domain  $(-\infty, \infty)$

b) Derivative of  $f(x)$   $f'(x) = x^3 - x^2 - 6x$

c) Critical Values  $f'(x) = 0$

d) Maximum and minimum coordinates  $0 = x(x-3)(x+2)$

e) Intervals where the function is increasing  $x=3$

f) Intervals where the function is decreasing  $x=-2$


e)  $(-\infty, -2) \cup (3, \infty)$

f)  $(-2, 3)$

int	$(-\infty, -2)$	$(-2, 3)$	$(3, \infty)$
value	-3	1	4
inc	inc	dec	inc



# Graphing 1st & 2nd derivative

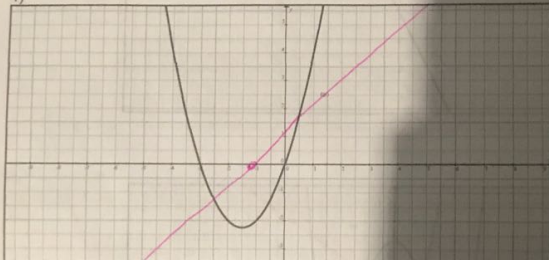
 **Graphing 1<sup>st</sup> & 2<sup>nd</sup> derivative**  
By: Ing. Ziad Najjar

PREPA Tec

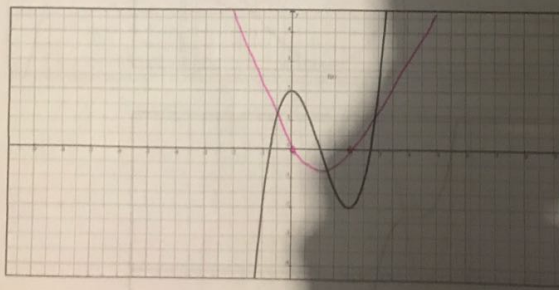
Name: Barbara Al-eav ID Number: A01570137

1. Given the graph of  $f(x)$ , sketch the graph of  $f'(x)$  on the same Cartesian plane (in a distinct color)

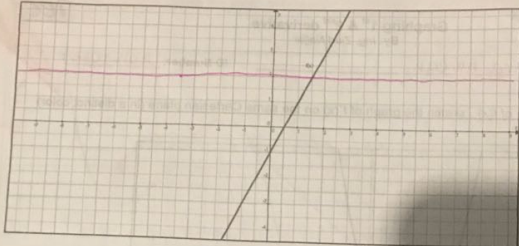
1)



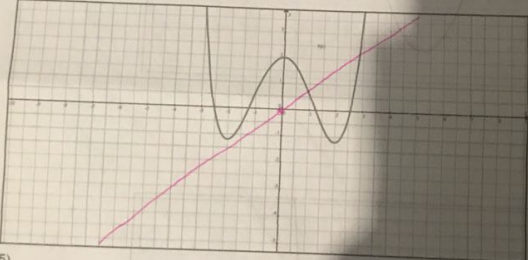
2)



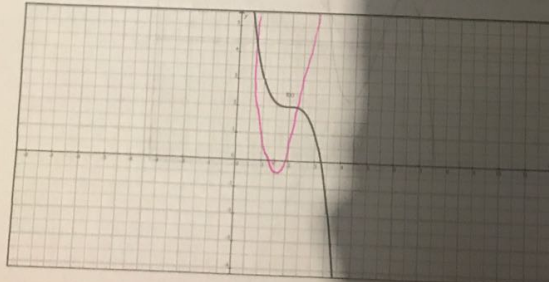
3)



4)

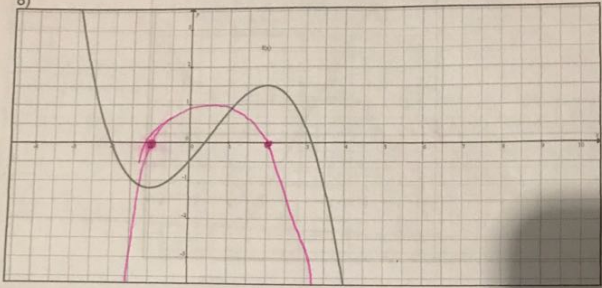


5)

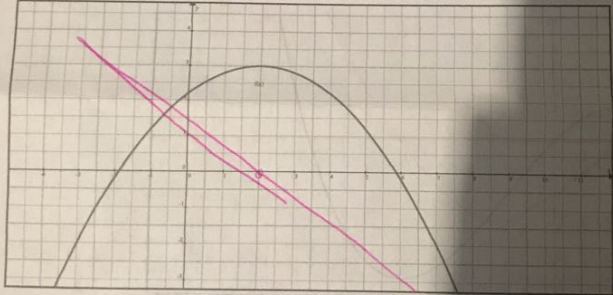


II. Given the graph of  $f(x)$ , sketch the graph of  $f'(x)$  and  $f''(x)$  on the same Cartesian plane (in two distinct colors)

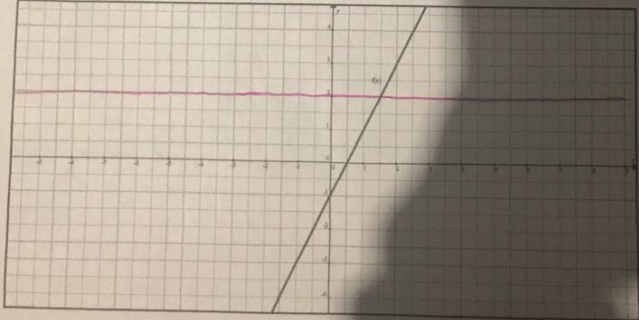
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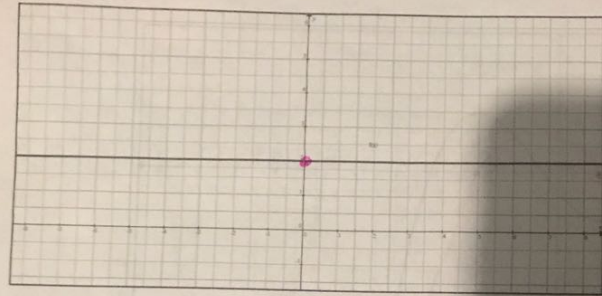
9)



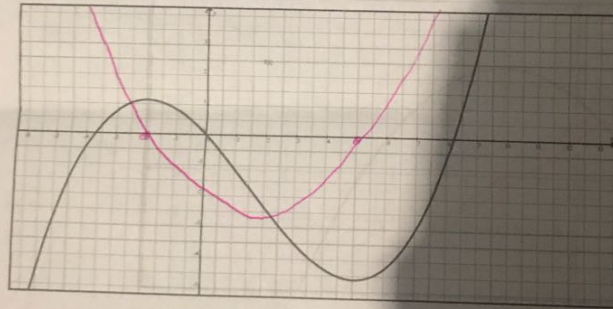
10)



6)



7)



# ACTIVITY: VALOR DE LA DIVERSIDAD

<https://www.geogebra.org/m/eDwdfAfq>