*Use a graphing program such as Geogebra with this assignment*

# Transformation of Functions

The function $f(x)$ is defined as $f(x) = x^2$.
The function $g(x)$ is defined as $g(x) = \sin x$.
The function $h(x)$ is defined as $h(x) = x^2 - 1$.
The function $p(x)$ is defined as $p(x) = x^2 - 2x$.

**1.** (i) Deduce an expression in *x* for:

(a) $f(x) + 2$

(b) $f(x) + 3$

(c) $f(x) + c$

(ii) Use your graphing program to draw graphs of:

(a) $y = f(x)$

(b) $y = f(x) + 2$

(c) $y = f(x) + 3$

(iii) By considering the graphs in part (ii), state the transformation of $y = f(x)$ which produces the graphs of $y = f(x) + 2$ and $y = f(x) + 3$ , and deduce the graphical effect of transforming $y = f(x)$ into $y = f(x) + c$. Express your findings as clearly as possible. What happens if *c* is negative?

(iv) Deduce an expression in *x* for $g(x) + c$.

(v) By choosing a suitable value for *c*, check if your answer to (iii) still applies. Sketch the graphs of $y = g(x)$ and $y = g(x) + c$.

**2.** (i) Deduce an expression in *x* for:

(a) $f(x + 2)$

(b) $f(x + 3)$

(c) $f(x + c)$

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(ii) Use your graphing program to draw graphs of:

(a) $y = f(x)$

(b) $y = f(x+2)$

(c) $y = f(x+3)$

(iii) By considering the graphs in part (ii), state the transformation of $y = f(x)$ which produces the graphs of $y = f(x+2)$ and $y = f(x+3)$ , and deduce the graphical effect of transforming $y = f(x)$ into $y = f(x+c)$. Express your findings as clearly as possible. What happens if *c* is negative?

(iv) Deduce an expression in *x* for $g(x+c)$. By choosing a suitable value for *c*, check if your answer to (iii) still applies. Sketch the graphs of $y = g(x)$ and $y = g(x+c)$.

**3.** (i) Deduce an expression in *x* for:

(a) $2h(x)$

(b) $3h(x)$

(c) $kh(x)$

(ii) Use your graphing program to draw graphs of:

(a) $y = h(x)$

(b) $y = 2h(x)$

(c) $y = 3h(x)$

(iii) By considering the graphs in part (ii), state the transformation of $y = h(x)$ which produces the graphs of $y = 2h(x)$ and $y = 3h(x)$ , and deduce the graphical effect of transforming $y = h(x)$ into $y = kh(x)$. Express your findings as clearly as possible.

(iv) Deduce an expression in *x* for $kg(x)$. By choosing a suitable value for *k*, check if your answer to (iii) still applies. Sketch the graphs of $y = g(x)$ and $y = kg(x)$.

**4.** (i) Deduce an expression in $x$ for:

(a) $h(2x)$

(b) $h(3x)$

(c) $h\left(\frac{x}{2}\right)$

(d) $h\left(\frac{x}{3}\right)$

(e) $h(kx)$

(f) $h\left(\frac{x}{k}\right)$

(ii) Use your graphing program to draw graphs of:

(a) $y = h(x)$

(b) $h(2x)$

(c) $h(3x)$

(d) $h\left(\frac{x}{2}\right)$

(e) $h\left(\frac{x}{3}\right)$

(iii) By considering the graphs in part (ii), state the transformation of $y = h(x)$ which produces the graphs of $y = h(2x)$, $y = h(3x)$, $y = h\left(\frac{x}{2}\right)$ and $y = h\left(\frac{x}{3}\right)$, and deduce the graphical effect of transforming $y = h(x)$ into $y = h(kx)$ and $y = h\left(\frac{x}{k}\right)$. Express your findings as clearly as possible.

(iv) Deduce an expression in $x$ for $g(kx)$ and $g\left(\frac{x}{k}\right)$. By choosing a suitable value for $k$, check if your answer to (iii) still applies. Sketch the graphs of $y = g(x)$, $y = g(kx)$ and $y = g\left(\frac{x}{k}\right)$.

**5.** (i) Deduce an expression in $x$ for:

(a) $-h(x)$

(b) $p(-x)$

(ii) Use your graphing program to draw graphs of:

(a) $y = h(x)$

(b) $y = p(x)$

(c) $y = -h(x)$

(d) $y = p(-x)$

(iii) By considering the graphs in part (ii), state the graphical transformation of $y = h(x)$ which produces the graph of $y = -h(x)$ and the graphical transformation of $y = p(x)$ which produces the graph of $y = p(-x)$. Express your findings as clearly as possible.

**6.** Summarise the graphical transformations of $y = f(x)$ given by:

(a) $f(x) + c$
(b) $f(x + c)$
(c) $kf(x)$
(d) $f(kx)$
(e) $f\left(\frac{x}{k}\right)$
(f) $-f(x)$
(g) $f(-x)$

Express your findings as clearly as possible.